MA4825

Robotics

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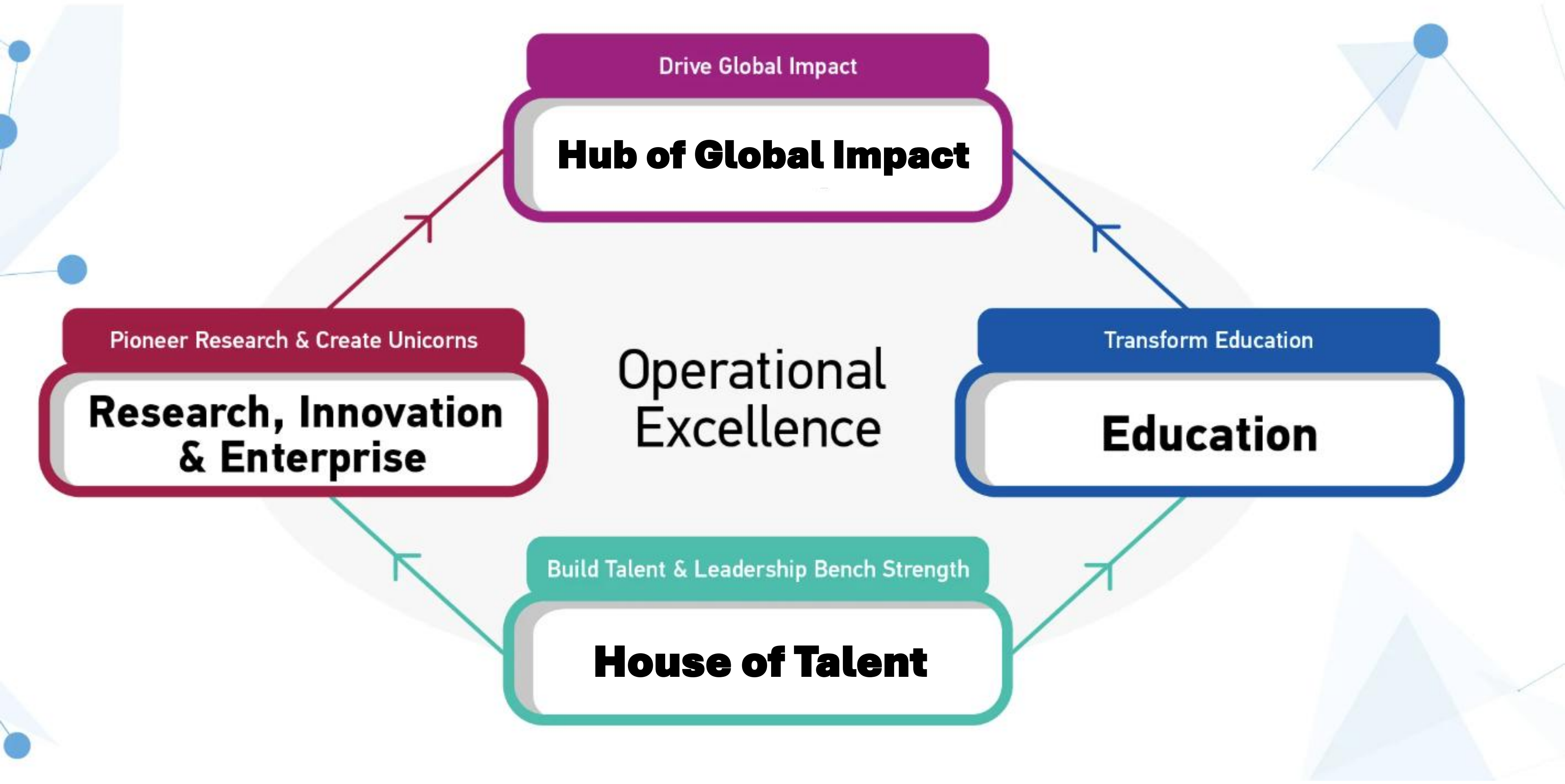
Outline

- Module 1: Robot’s Advanced Body
- Module 2: Robot’s Advanced Perception
- Module 3: Robot’s Advanced Planning
- Module 4: Robot’s Advanced Control

About NTU

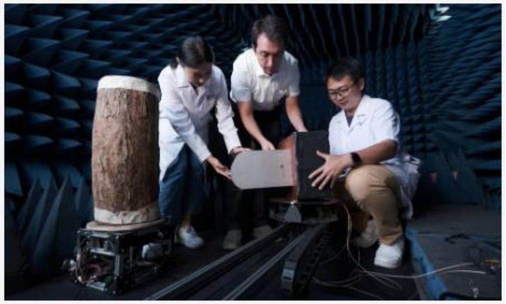
Remember NTU's Vision …



Remember NTU's Mission …

Education

We deliver transformative educational experiences that make our students both future- and AI-ready, so they are sought after by employers.



House of Talent

We attract, develop, and retain the very best people to drive excellence across the University.

Research, Innovation and Enterprise

We pursue breakthrough discoveries. We integrate technology and the humanities to address global challenges. We accelerate cutting edge innovation and create promising new enterprises.

Hub of Global Impact

We drive global impact in all that we do. We pursue long-lasting global partnerships with like-minded institutions across the world.

Education is to help citizens to fulfill their missions on Earth, which include: to understand the world and to improve the world …

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7 新加坡

李总理：排名不应是办大学终极考量

黄伟曼　报道
ngwaimun@sph.com.sg

本地大专学府的主要服务对象是新加坡学生，大学除了追求学术表现之外，也扮演重要的国家与社会角色，李显龙总理提醒本地大学不应为追求名次而忽略所肩负的重任，必须持续改善本地教育，为新加坡人提供精益求精的教育。

李总理昨晚主持新加坡国立大学大学城（University Town，简称UTown）开幕仪式时指出，本地大学的世界排名一直受到关注，但在衡量大学的表现时，不应只看名次先后，排名也不应是办大学的终极考量。

他向现场近1000名国大学生、教师与嘉宾解释说，许多在世界大专教育排行榜上名列前茅的大国顶尖大学收生广泛，例如美国哈佛大学、斯坦福大学、英国牛津大学、剑桥大学、中国北京大学和清华大学等每年录取的学生中，只有一小部分是当地学生，而本地的情况却不同，大专学府主要收本地生，必须履行更广泛的国家任务。

他说："本地大学的责任在于发展学生的品格与社会良知、鼓励他们在求学时期建立长久的友谊与默契。大学要以灌输新加坡价值观和精神特质为出发点，教导学生取之社会用之社会的精神，让他们知道自己有责任推动新加坡前进。"

"要达到这个宗旨，每一所大学都应该建立自己独特的教育模式，不要一味复制其他大学。"

占地19公顷的大学城是国大搬到肯特岗校园现址后，最大型的一次扩展计划。设于大学城的四所寄宿型学院和为研究生所设的大学城宿舍，将为4000多名学生提供住宿，将教学融入住宿环境里，让来自不同背景和科系的学生一起生活、一起学习。

在大学城学院课程中，学生需在所住的学院里修读五个单元，包括两个写作单元和三个研讨课单元。他们也有机会聆听外国学者发表专题演讲，并参加各种不同的课外活动。

李总理昨天肯定国大为提升学生学习体验所做的努力，他认为大学城为学生营造了一个有利于相互学习的空间。他指出，大学城是国大的"新聚集点"，它将国大社群缩小，进一步拉近学生与同学和教授的关系。

他也以陈爱丽丝与彼德学院（College of Alice & Peter Tan)学生之前在"整合式课程"（Capstone Programme）下为乐龄人士制定健康饮食计划为例，表示希望看到大学城继续推行类似社会计划，鼓励学生积极参与这些关怀社会的活动。

国大校长陈祝全教授致辞时指出，除了设施与空间设计吸引人之外，大学城为国大社群以及国大的运作和成长带来了深刻且积极的影响。"大学城可说已成为国大校园的核心，在一些方面补足了之前肯特岗校园所缺少的生气。"

相关新闻刊第6页

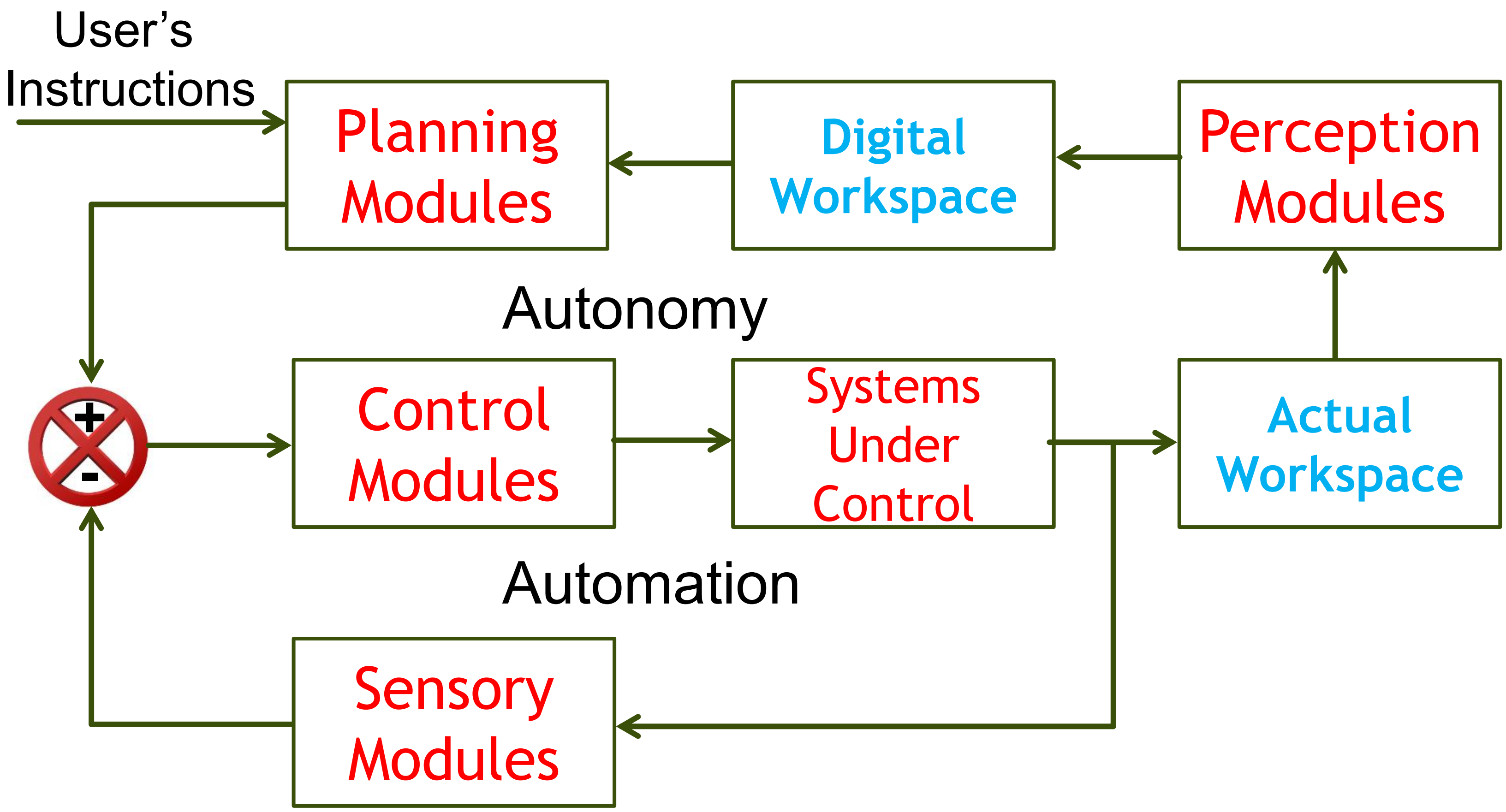
About You

Remember your mission as MAE undergraduates …

- You are here to grow your knowledge and skills so as to be able to design machines with controllable behaviors and hopefully in some intelligent ways.

How to fulfill your mission?

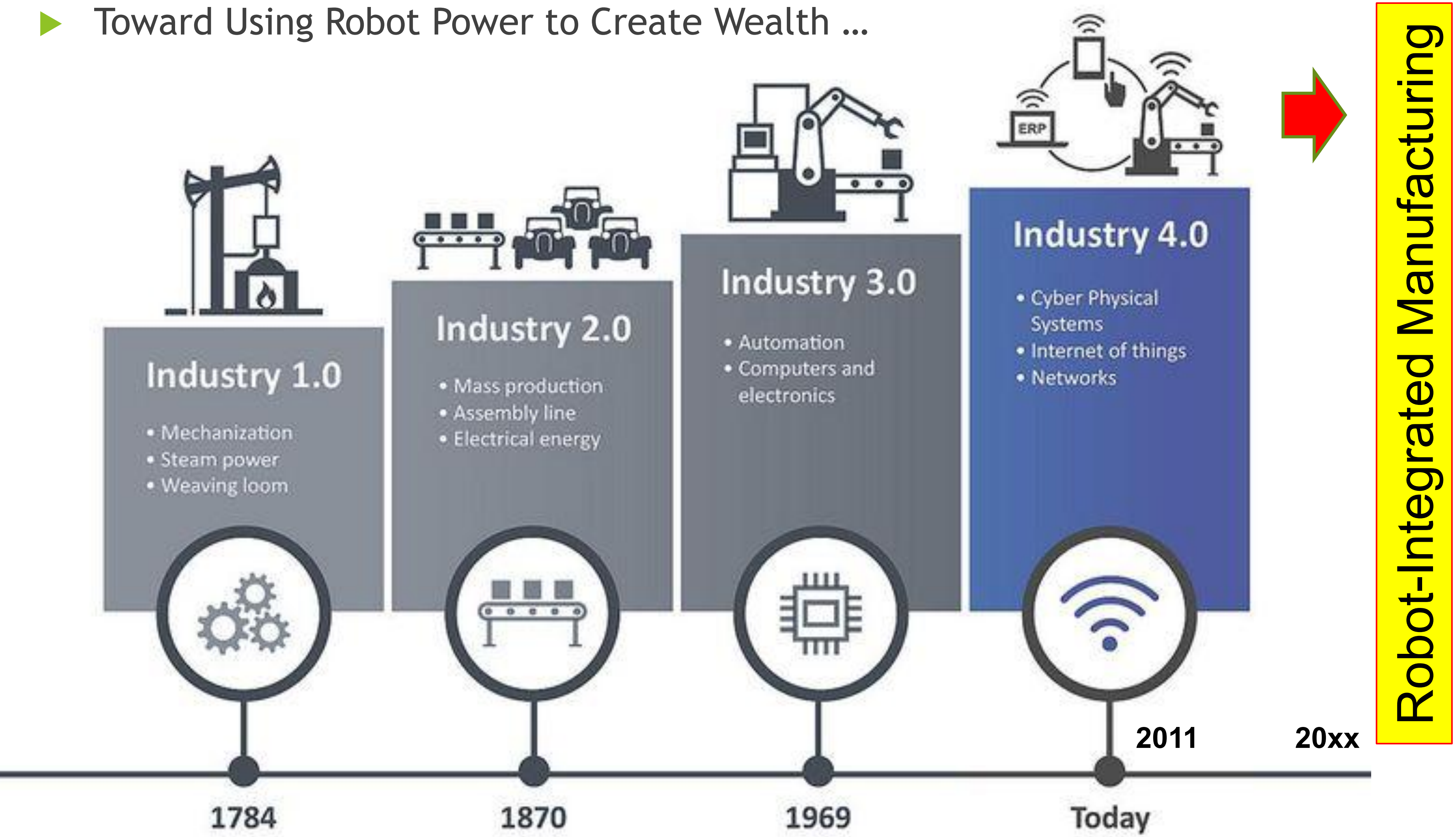
- To apply learnt knowledge and skills into the implementation of the following universal blueprint underlying all the intelligent machines or systems.



About Course

Why to study this course?

- Toward Using Robot Power to Create Wealth …



How to study this course?

- To put yourselves into the mindset of designers of robots as products:
  - Who are the users?
  - What are the needs of users?
  - What are your robots which could meet the needs of your users or buyers?
  - What are the solutions behind the design of your robots?

Market Demands or Needs
↓
Product Specifications
↓
Design Specifications
↓
Conceptual Design
↓
Selection of Materials/Components/Devices
↓
Embodiment Design
↓
Prototyping
↓
Optimizing
↓
Production
↓
Marketing

What to Learn?

Q1: What is the energy flow?

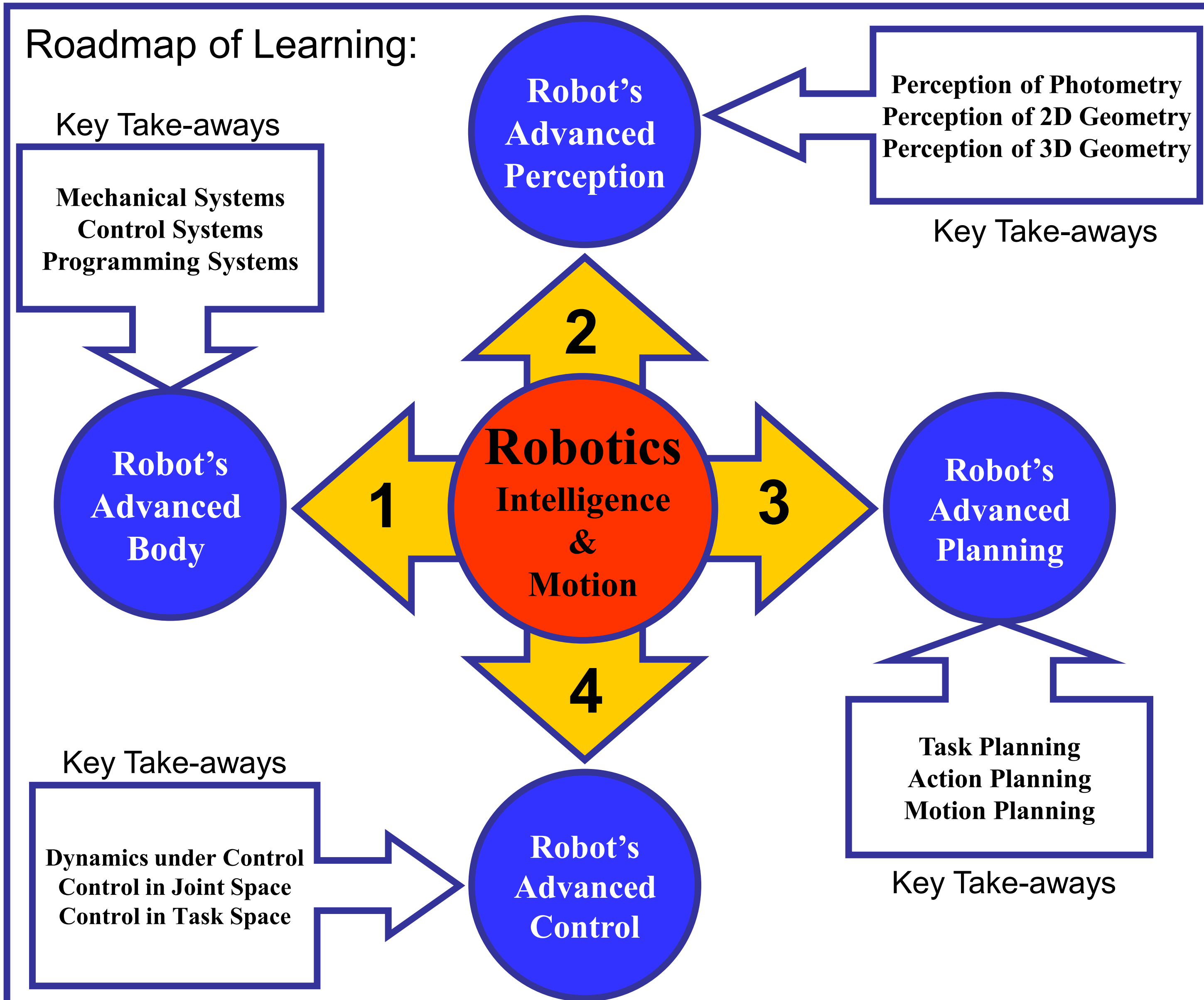
Q2: What is the signal flow?

Q3: What is the knowledge flow?

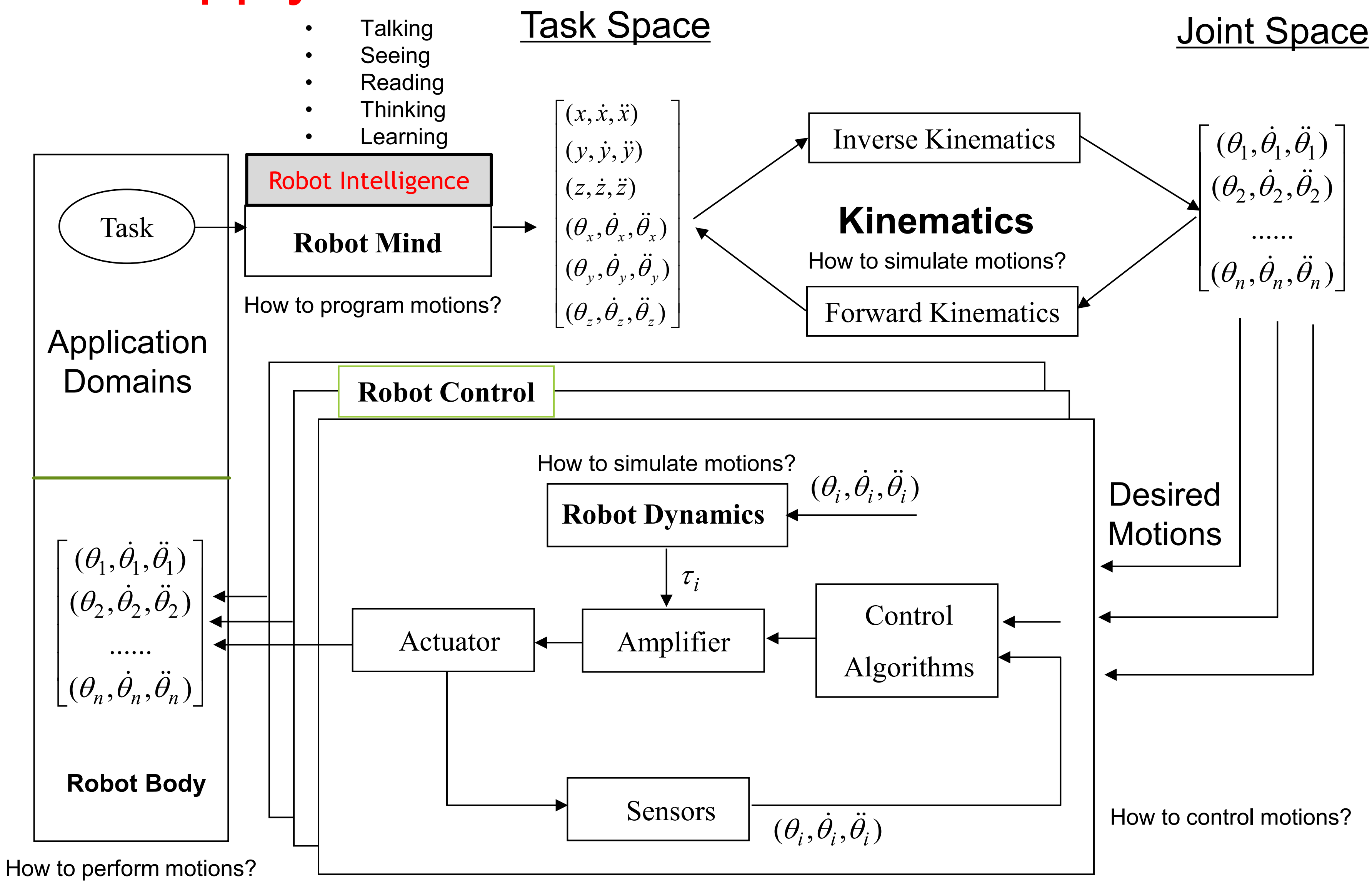
Q4: What is the relationship between energy flow and signal flow?

Q5: What is the relationship between signal flow and knowledge flow?

1. One Machine
2. Two Capabilities
3. Three Benefits
4. Four Pillars



How to Apply?



Talking
Seeing
Reading
Thinking
Learning
Task Space
Joint Space
Robot Intelligence
Task
Robot Mind
How to program motions?
Application Domains
$(x,\dot{x},\ddot{x})$
$(y,\dot{y},\ddot{y})$
$(z,\dot{z},\ddot{z})$
$(\theta_x,\dot{\theta}_x,\ddot{\theta}_x)$
$(\theta_y,\dot{\theta}_y,\ddot{\theta}_y)$
$(\theta_z,\dot{\theta}_z,\ddot{\theta}_z)$
Inverse Kinematics
Kinematics
How to simulate motions?
Forward Kinematics
$(\theta_1,\dot{\theta}_1,\ddot{\theta}_1)$
$(\theta_2,\dot{\theta}_2,\ddot{\theta}_2)$
......
$(\theta_n,\dot{\theta}_n,\ddot{\theta}_n)$
Robot Control
How to simulate motions?
$(\theta_i,\dot{\theta}_i,\ddot{\theta}_i)$
Robot Dynamics
τ_i
Desired Motions
Control Algorithms
Amplifier
Actuator
Sensors
$(\theta_i,\dot{\theta}_i,\ddot{\theta}_i)$
How to control motions?
$(\theta_1,\dot{\theta}_1,\ddot{\theta}_1)$
$(\theta_2,\dot{\theta}_2,\ddot{\theta}_2)$
......
$(\theta_n,\dot{\theta}_n,\ddot{\theta}_n)$
Robot Body
How to perform motions?

Terminology Alert

- Advanced Robotics is about the study of advanced robots which could perform tasks in some intelligent ways.

- Advanced Robot is a machine which has
  - two capabilities (automatic control and autonomous control),
  - three benefits and
  - four pillars.

Today's Lectures …

- Module 1: Robot's Advanced Body
- Module 2: Robot's Advanced Perception
- Module 3: Robot's Advanced Planning
- Module 4: Robot's Advanced Control

Module 2

MA4825 Robotics

Robot's Advanced Perception

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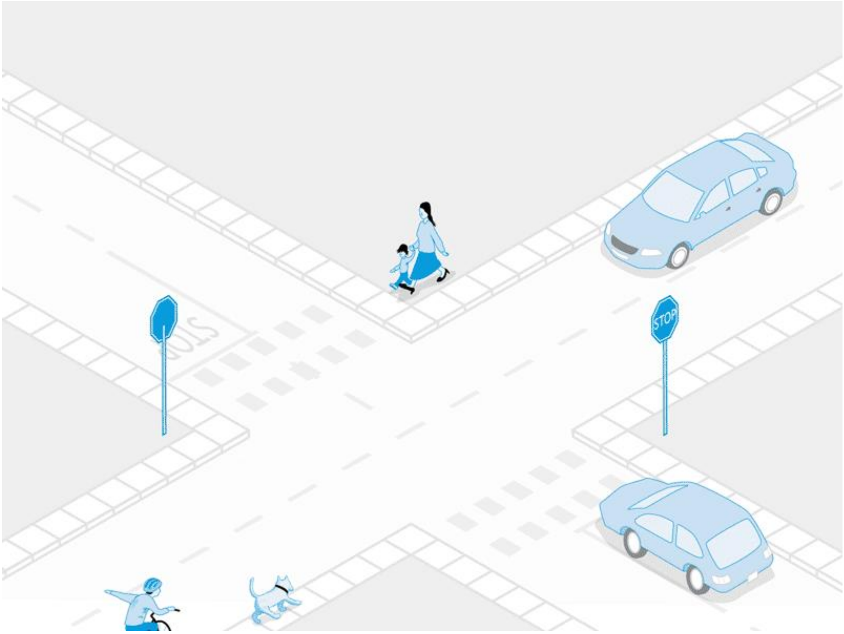
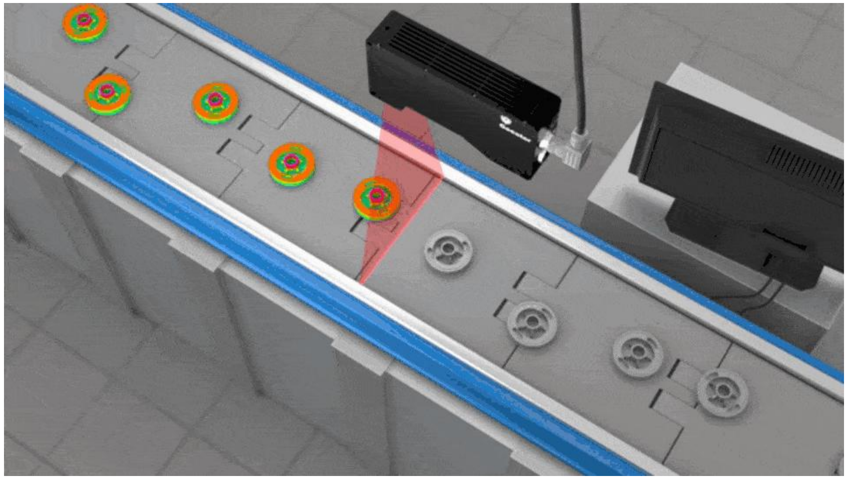
Outline of Module 2

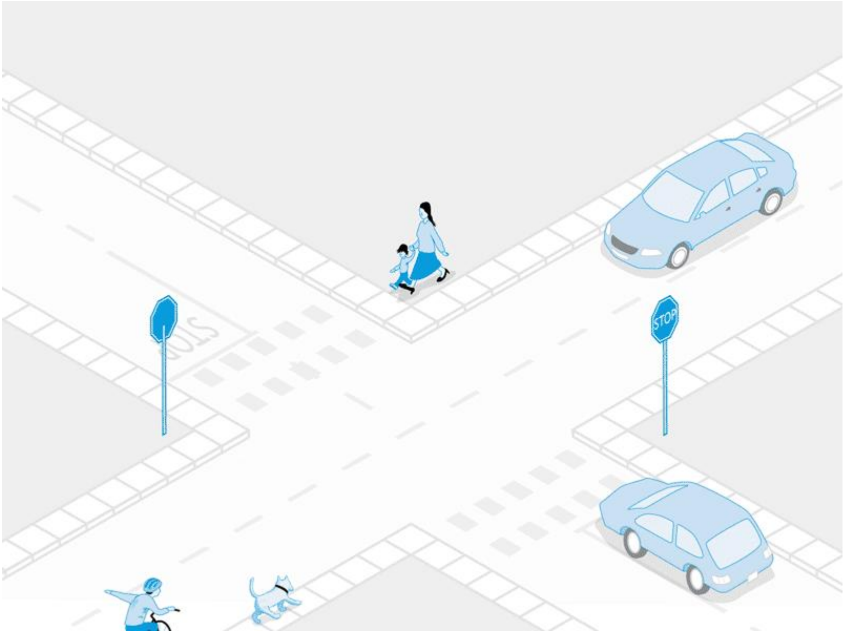
- Perception of Photometry
- Perception of 2D Geometry
- Perception of 3D Geometry

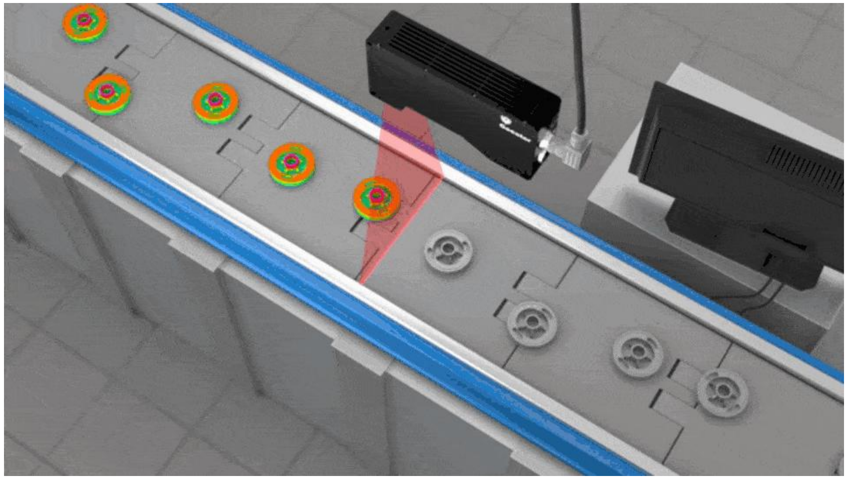




Outline of Module 2

- Perception of Photometry
- Perception of 2D Geometry
- Perception of 3D Geometry





Module 2

MA4825 Robotics

Lecture 1

Perception of Photometry

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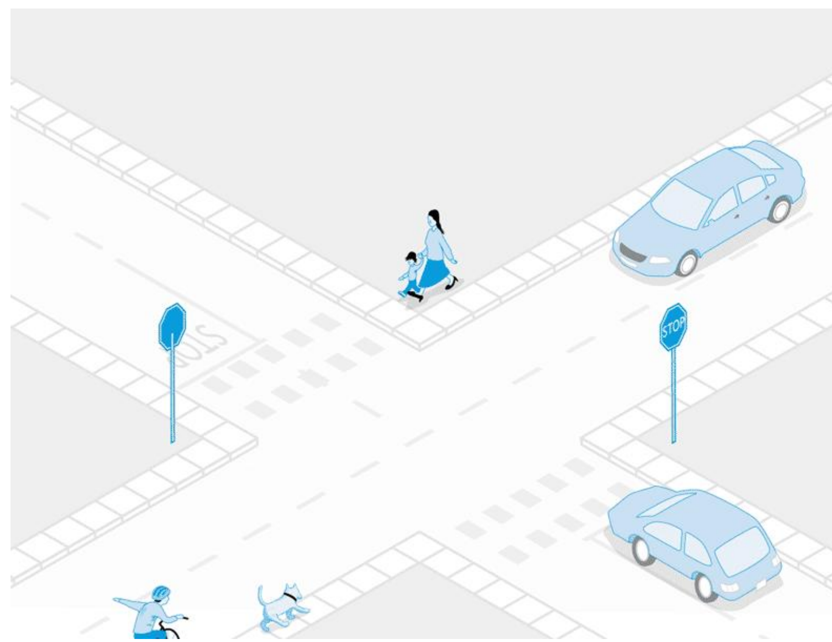
Outline of Lecture 1

- Basics of Sensing and Measurement
- Basics of Visual Signals
- Parameter(s) of Visual Signals
- Measurement of Photometry



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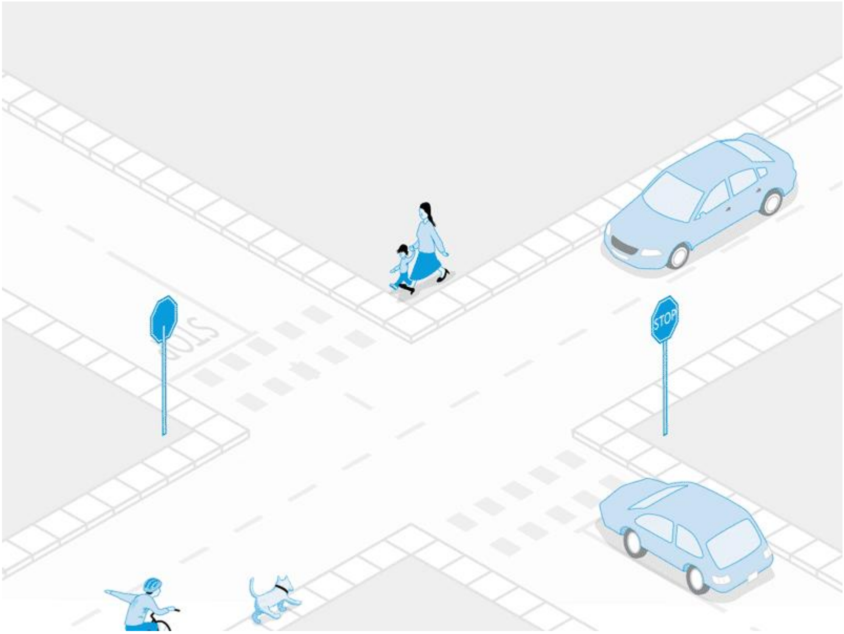
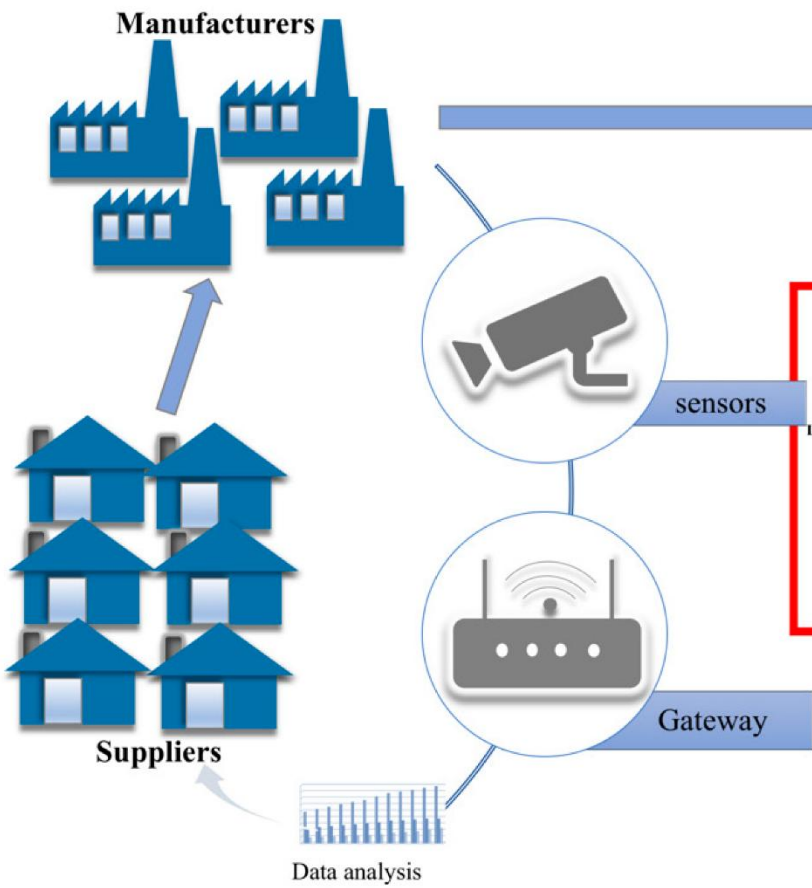
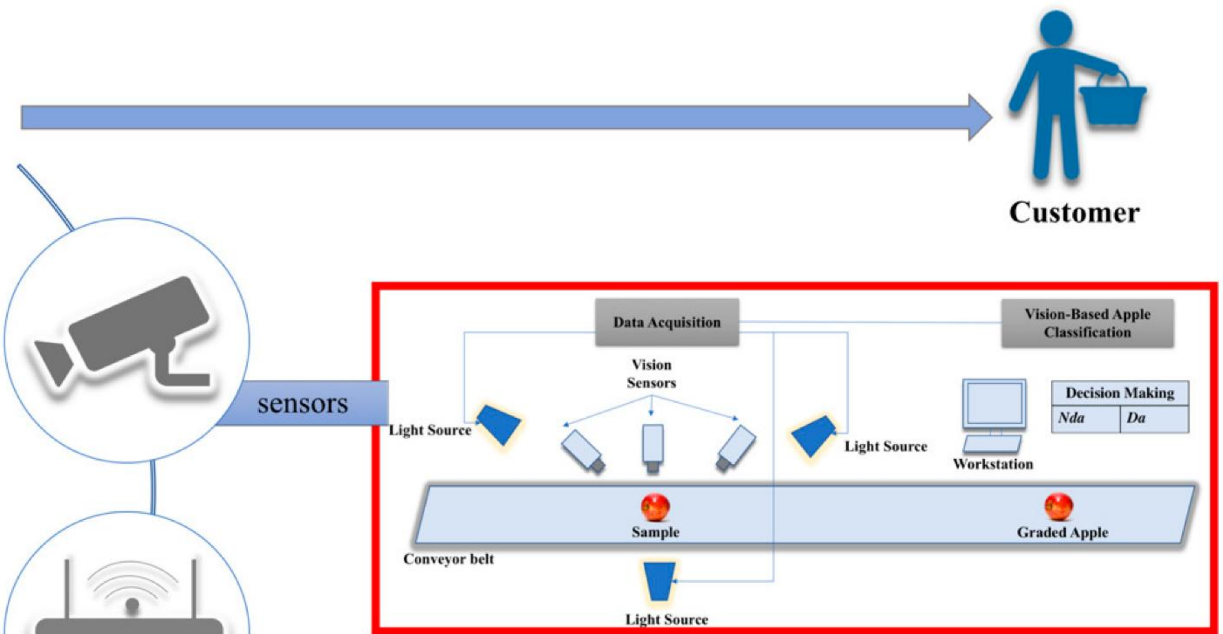


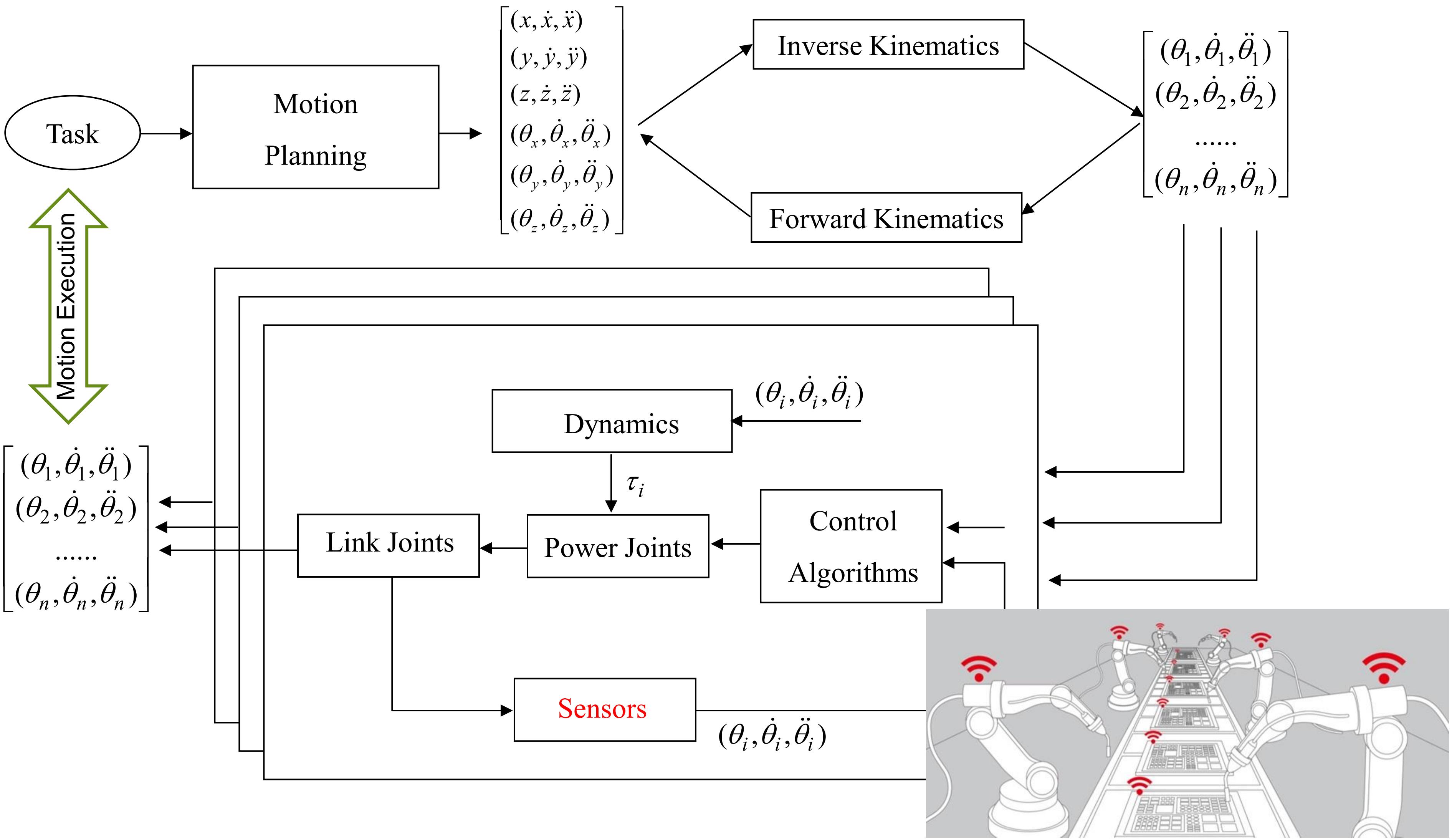
Sensing in Manufacturing Systems

Sensing in Automated Production Lines

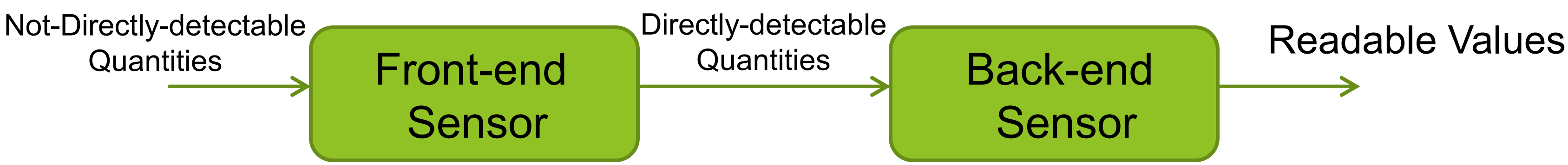


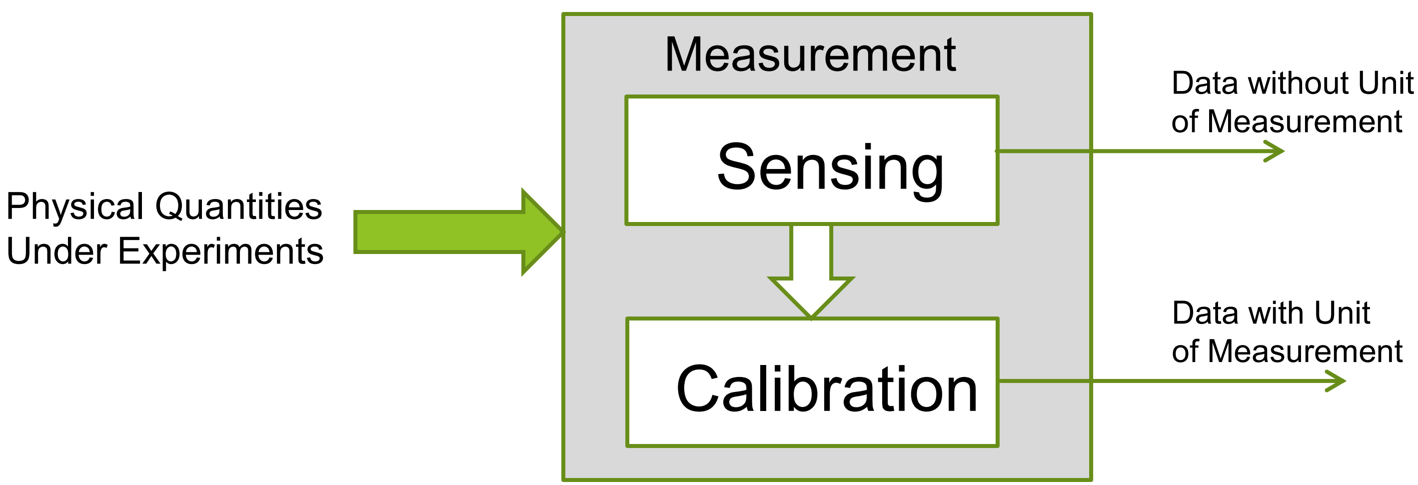


Sensing in Automated Equipment



How to do sensing and measurement?





Outline of Lecture 1

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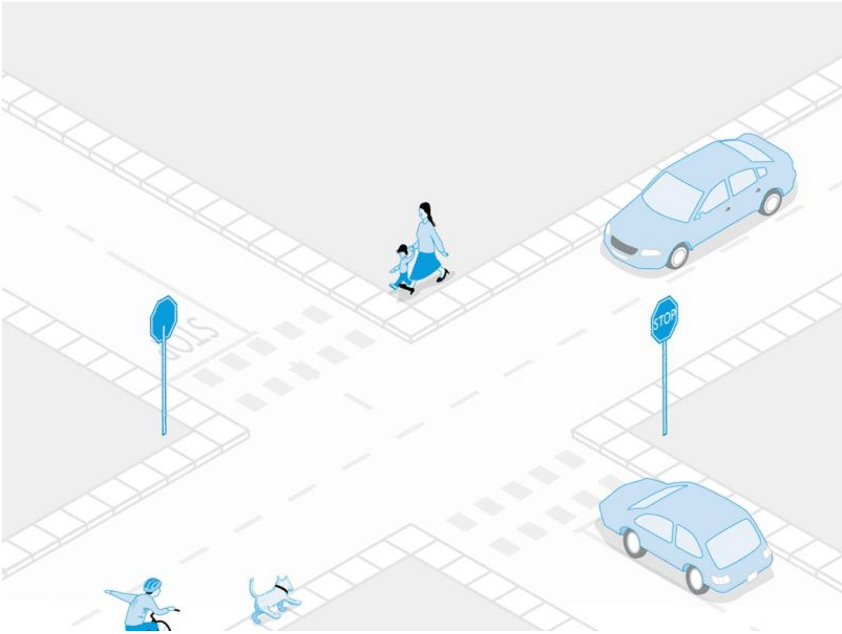
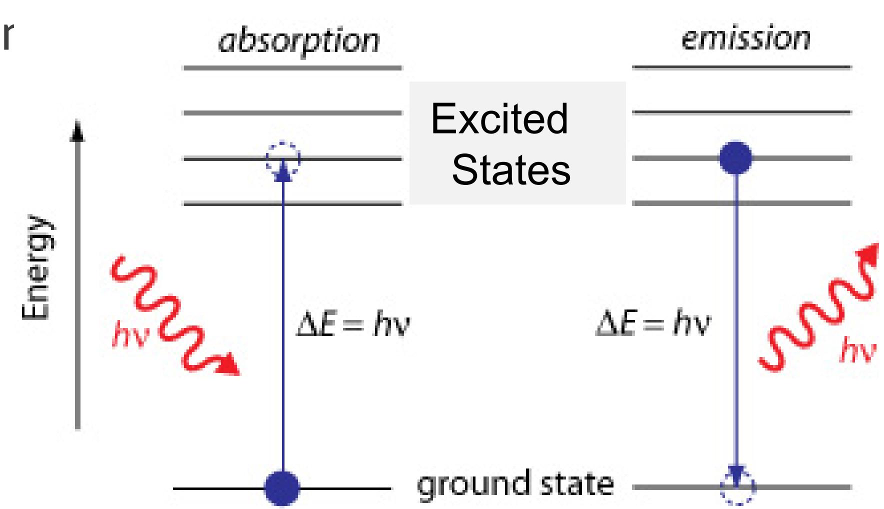
- Measurement of Photometry



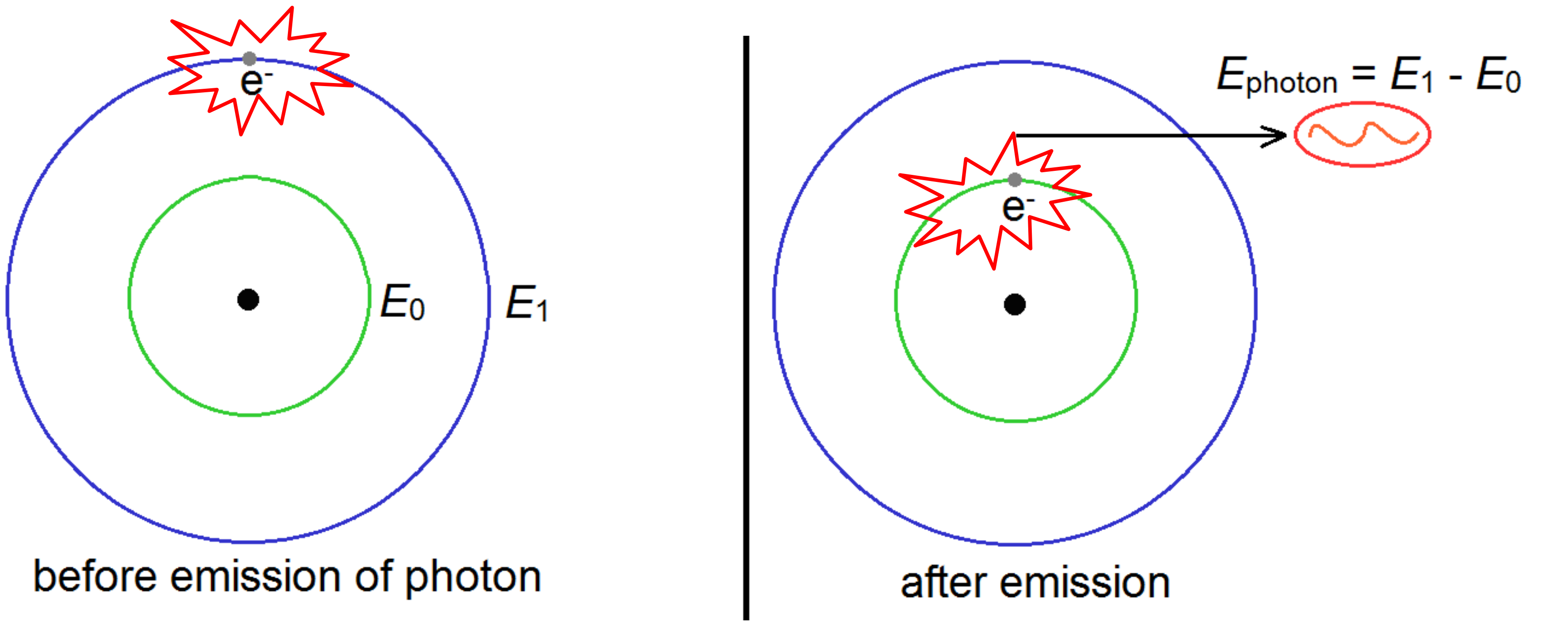
Emission of Photons

- An electron inside atom or molecule can transit between the ground state and excited states.

- When an electron inside atom or molecule transits from an excited state to the ground state, a photon is emitted.

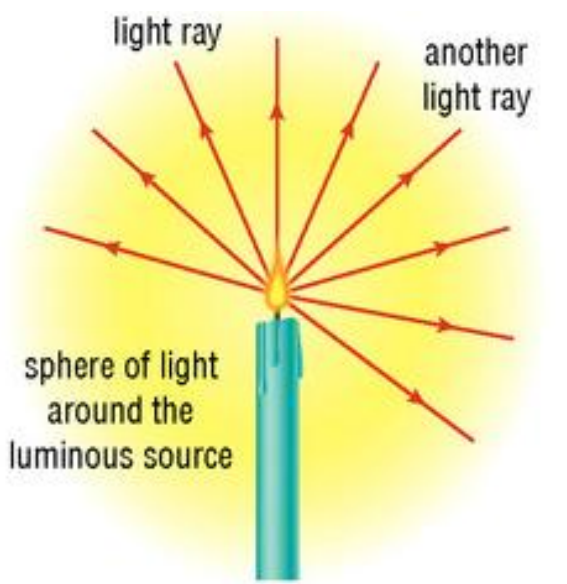


Example


e-
E0
E1
before emission of photon
Ephoton = E1 - E0
e-
after emission

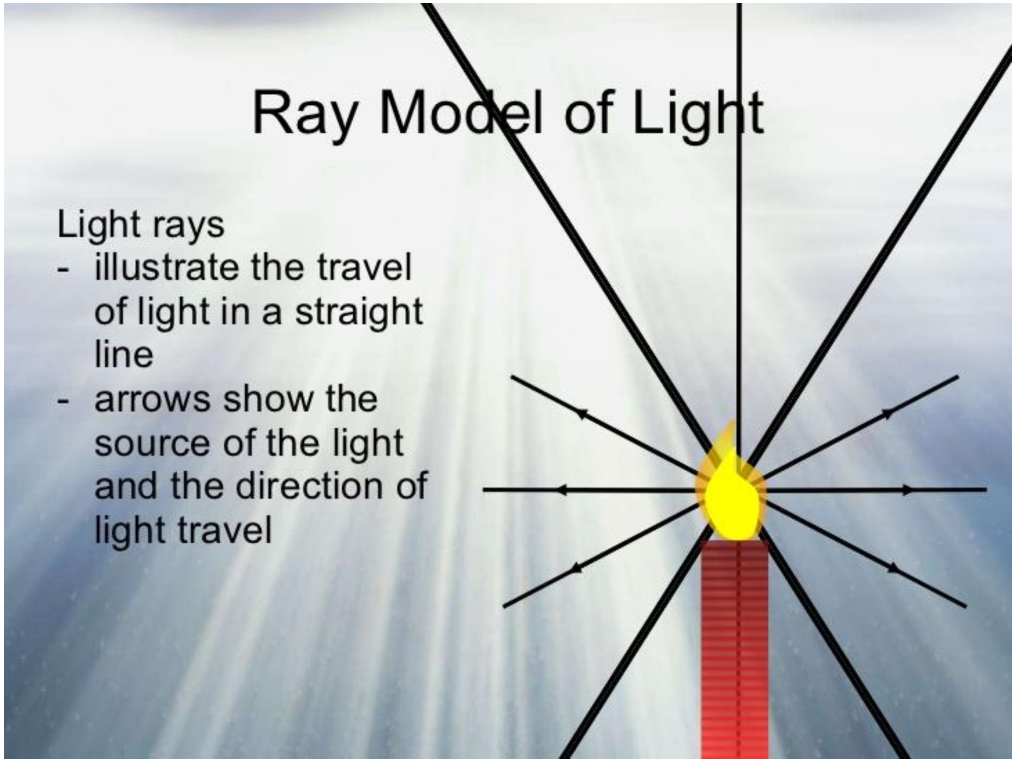
Sources of Light

- Any object, which emits continuously photons, is a light source.



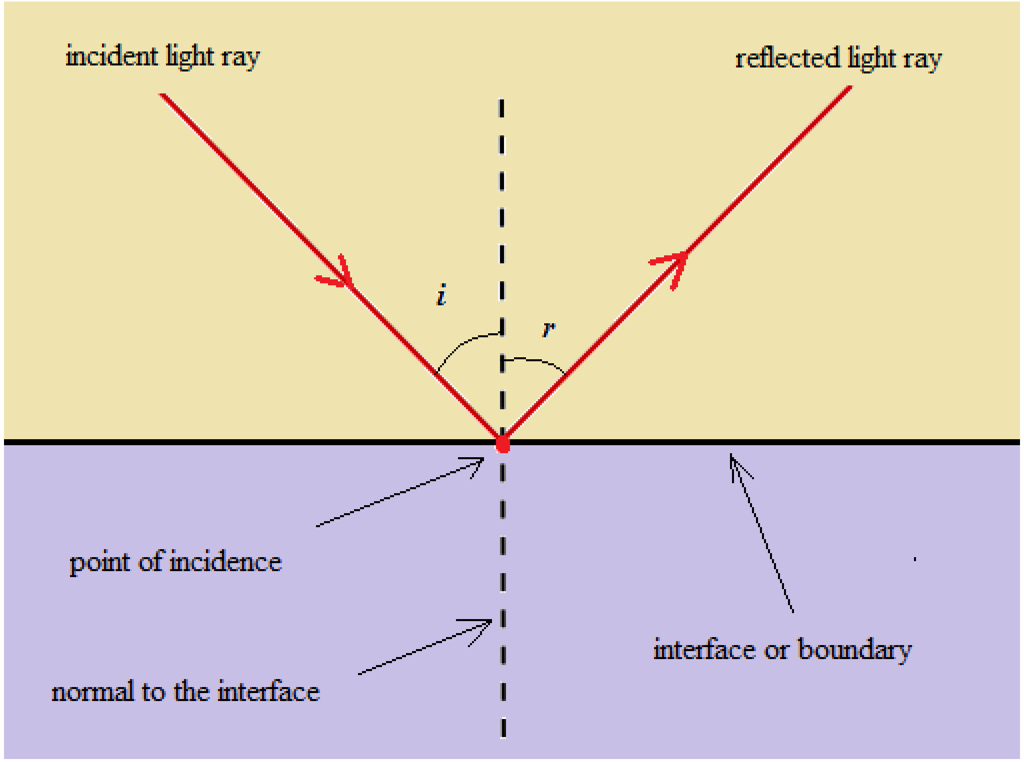
Understanding Lights (1)

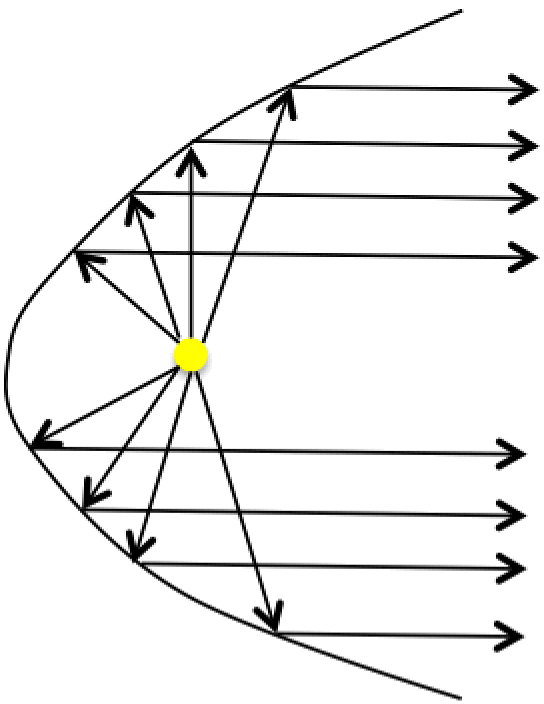
- Lights travel along a straight line from a source. The path being travelled by photons along a line are called a light ray.



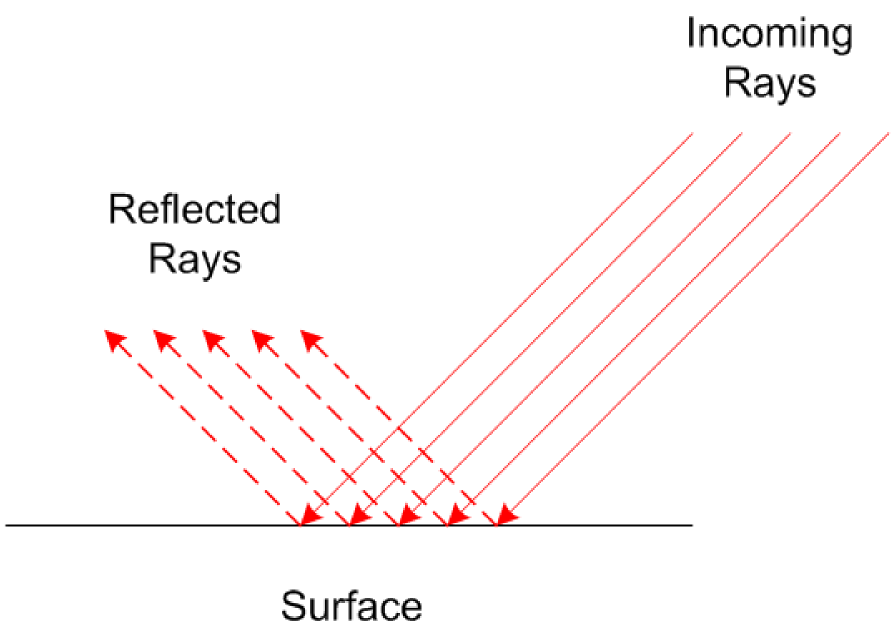
Understanding Lights (2)

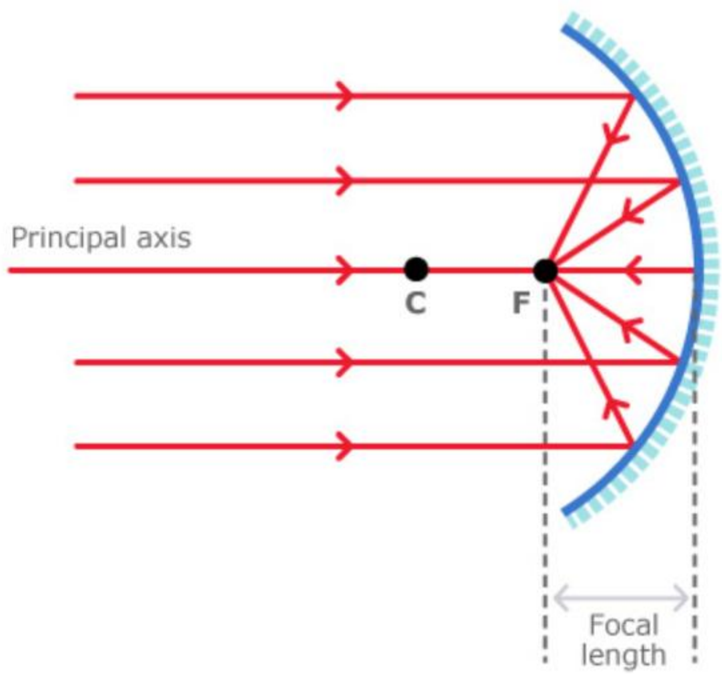
- Light rays can be reflected so as to change the directions of travel.



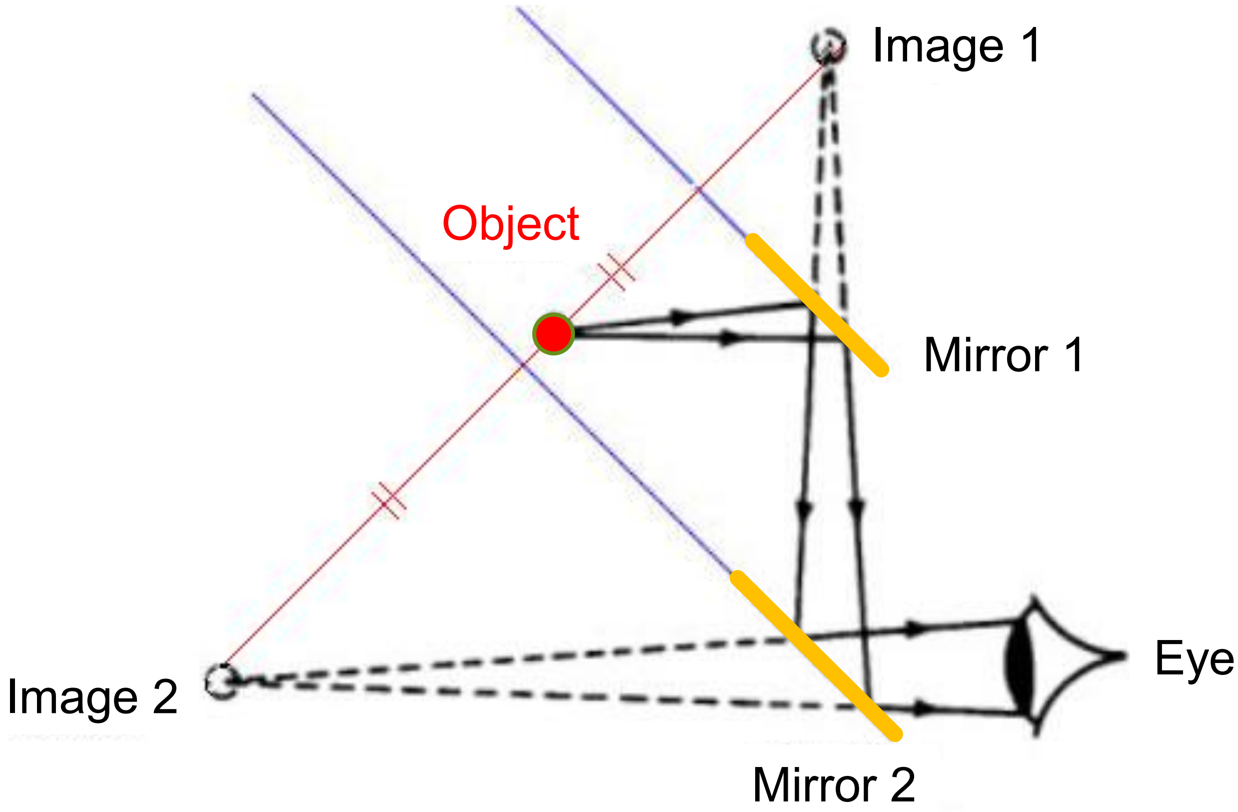


Example


Incoming
Rays
Reflected
Rays
Surface

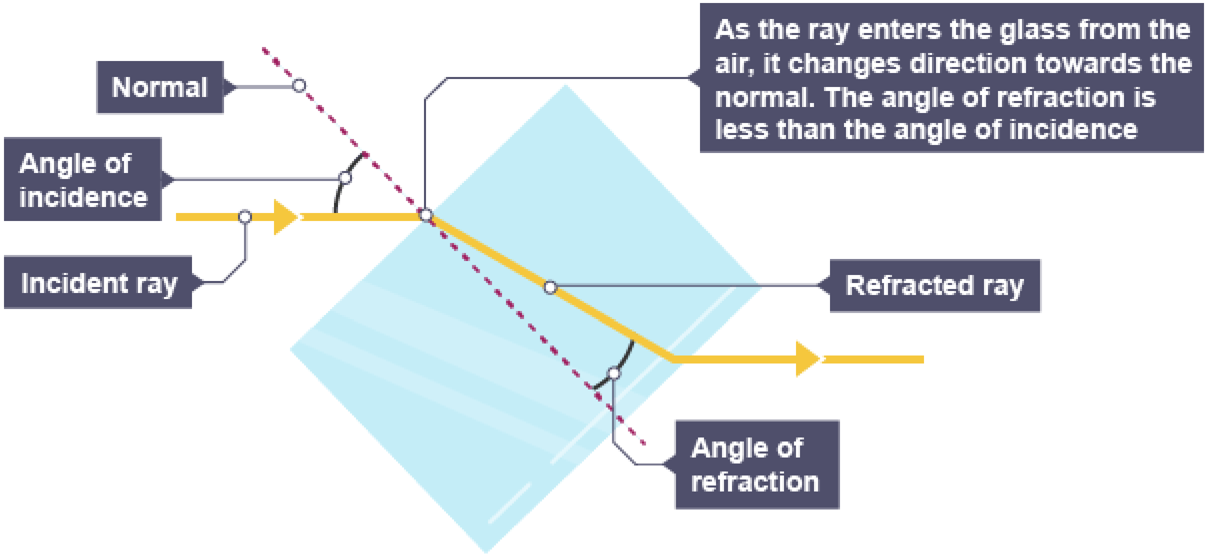

Principal axis
C
F
Focal
length

Use of Mirrors to Create the Effect of: one object, two mirrored images

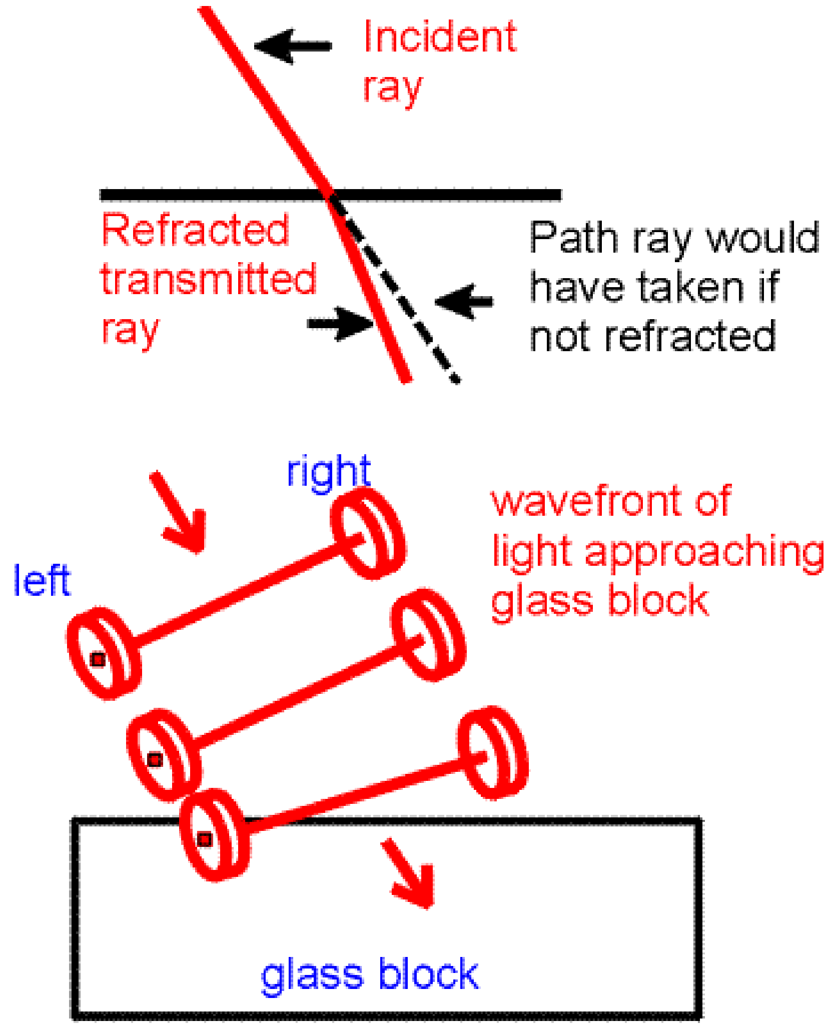


Understanding Lights (3)

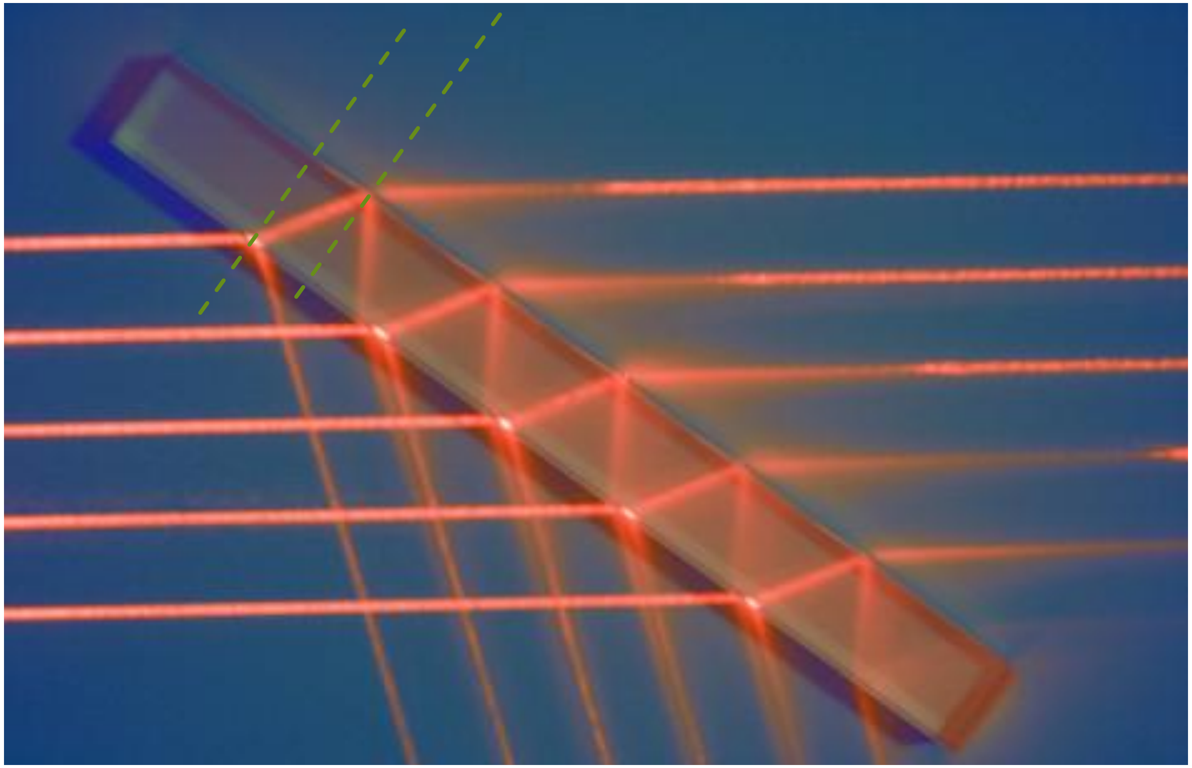
- Light rays can travel from one media into another media. Such change of media will cause light rays to change direction of travel. Such phenomenon is called refraction.



Example

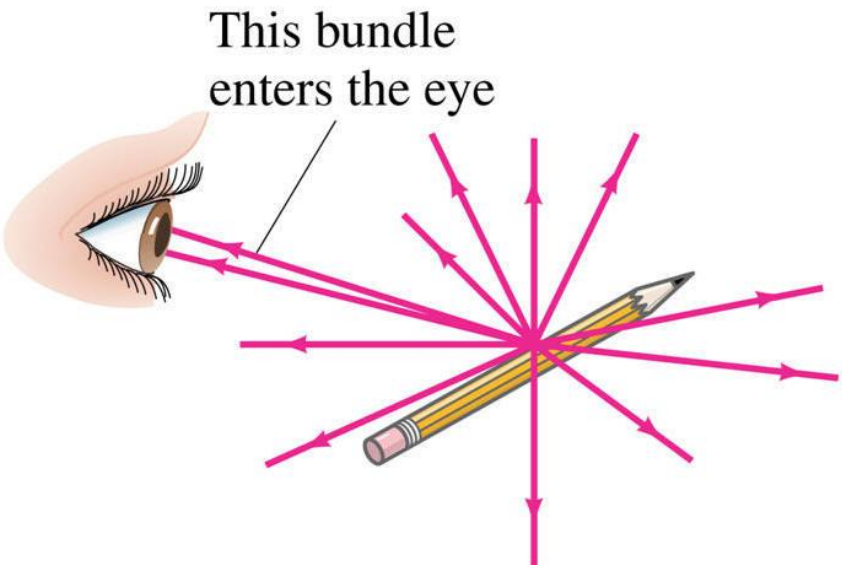

Incident
ray
Refracted
transmitted
ray
Path ray would
have taken if
not refracted
right
left
wavefront of
light approaching
glass block
glass block

Example

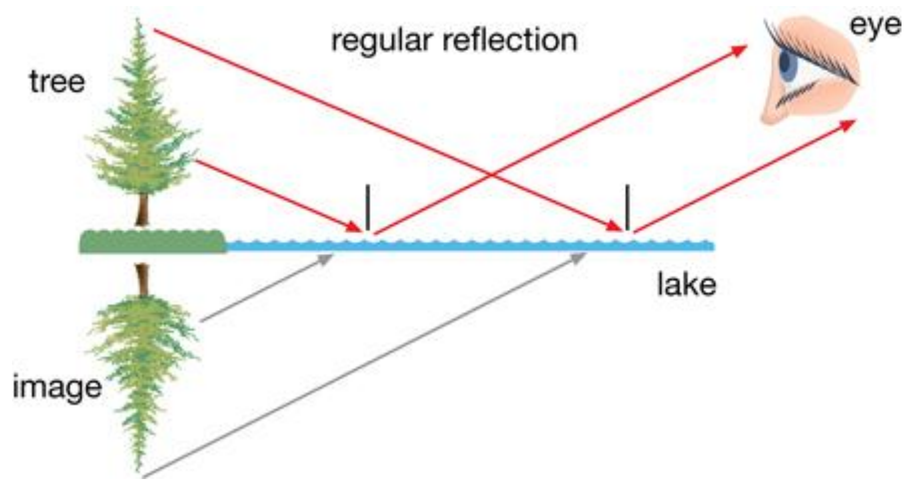


Understanding Lights (4)

- Light rays can end the journey by entering light receivers such as eyes or cameras.

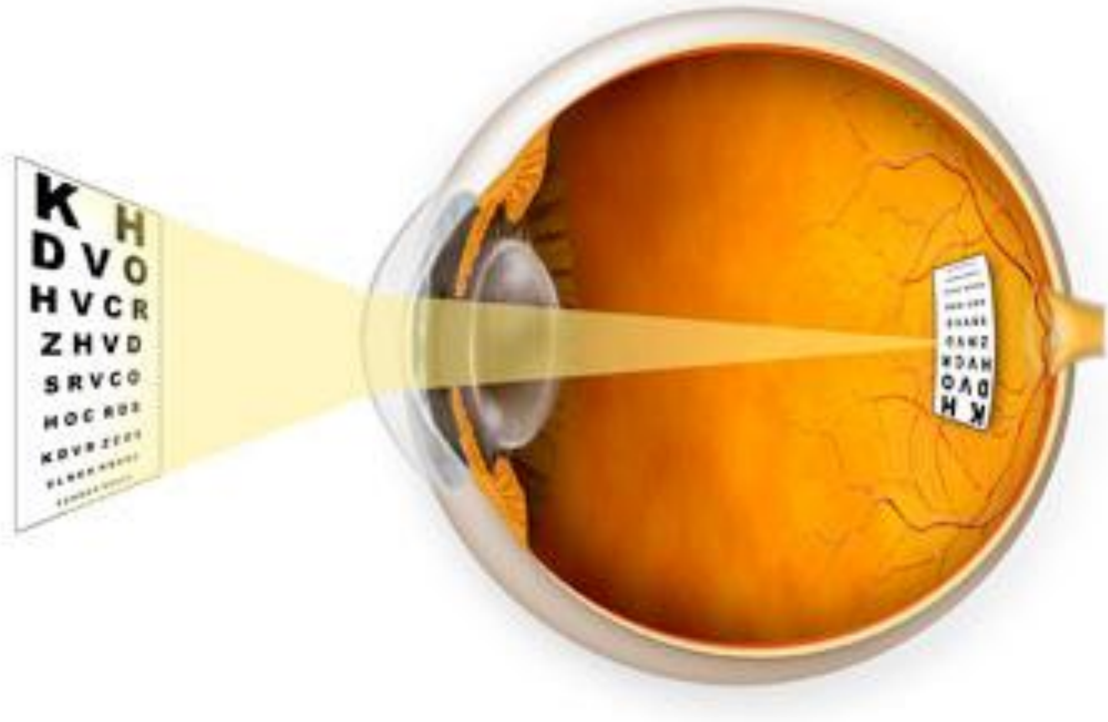


Direct Light Rays

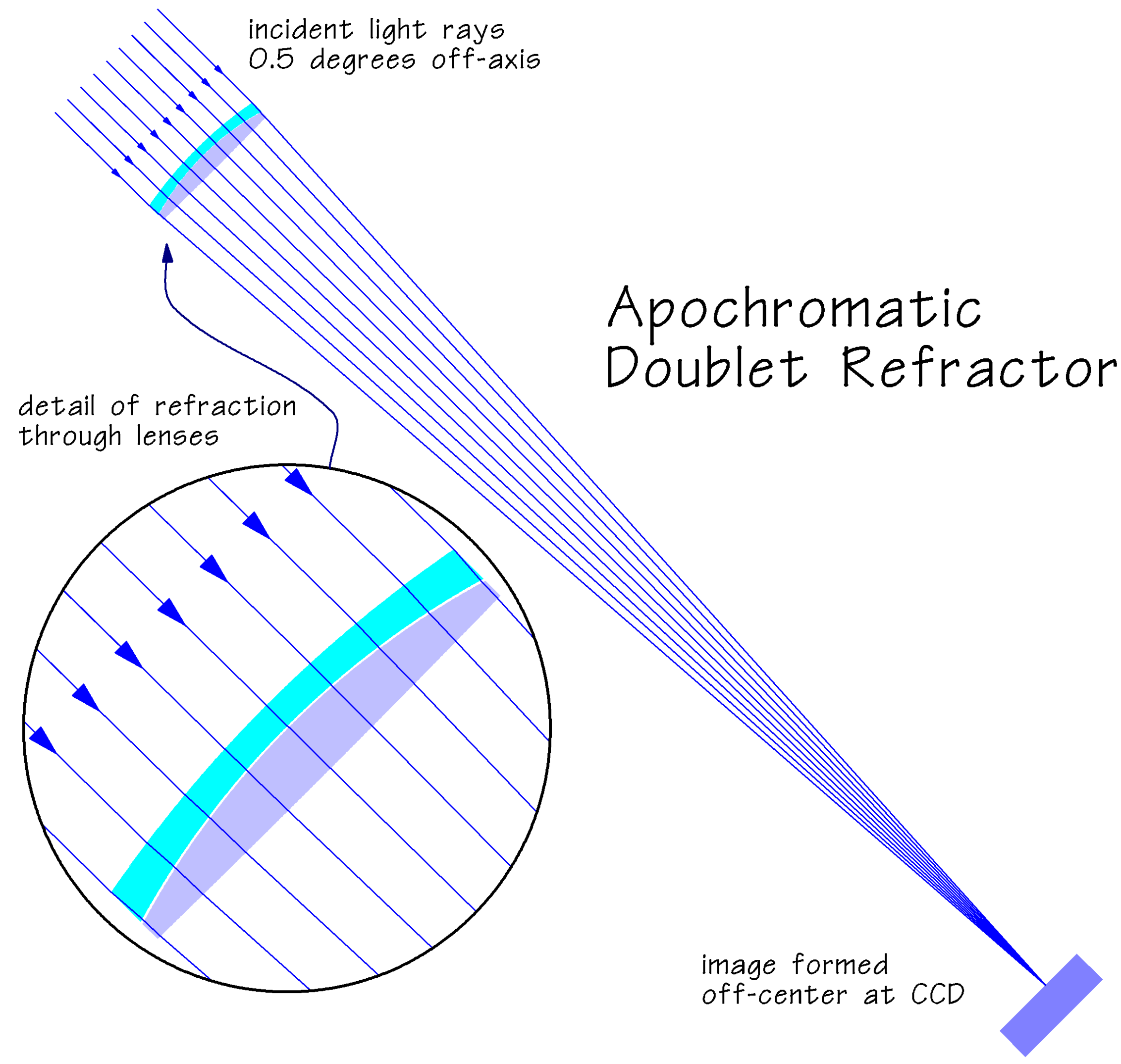


Reflected Light Rays

Example

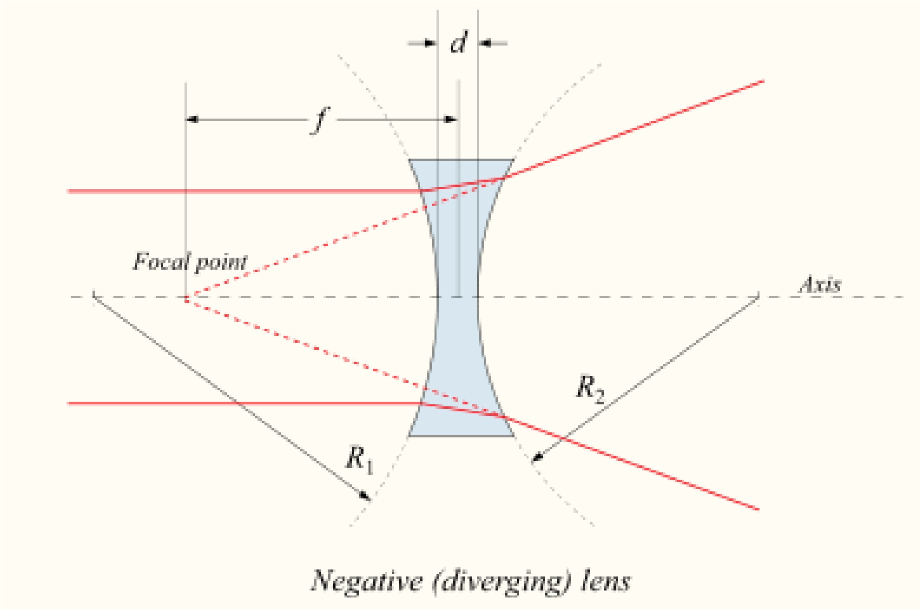


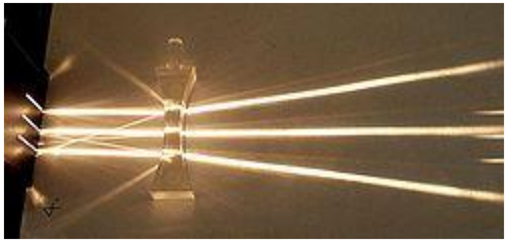
Example


incident light rays
0.5 degrees off-axis
Apochromatic
Doublet Refractor
detail of refraction
through lenses
image formed
off-center at CCD

Understanding Lights (5)

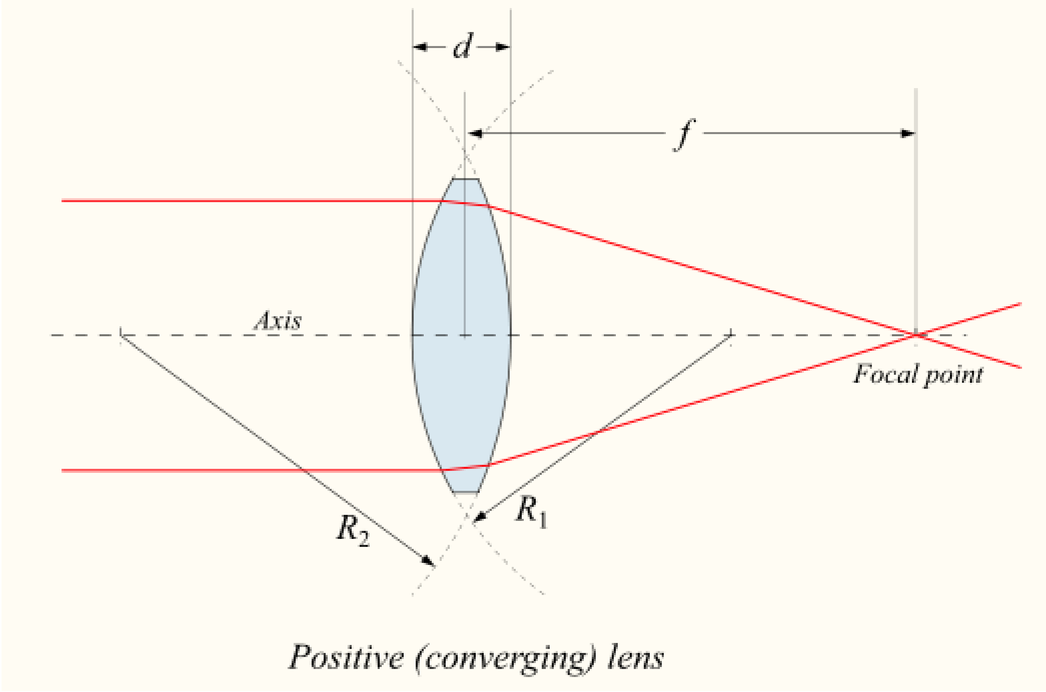
- A transparent device, with two spherical surfaces sharing a common axis, is called a lens. If the spherical surfaces are concave, the incoming light rays to such lens will be diverged.

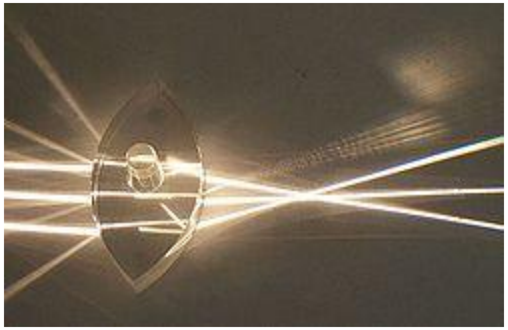




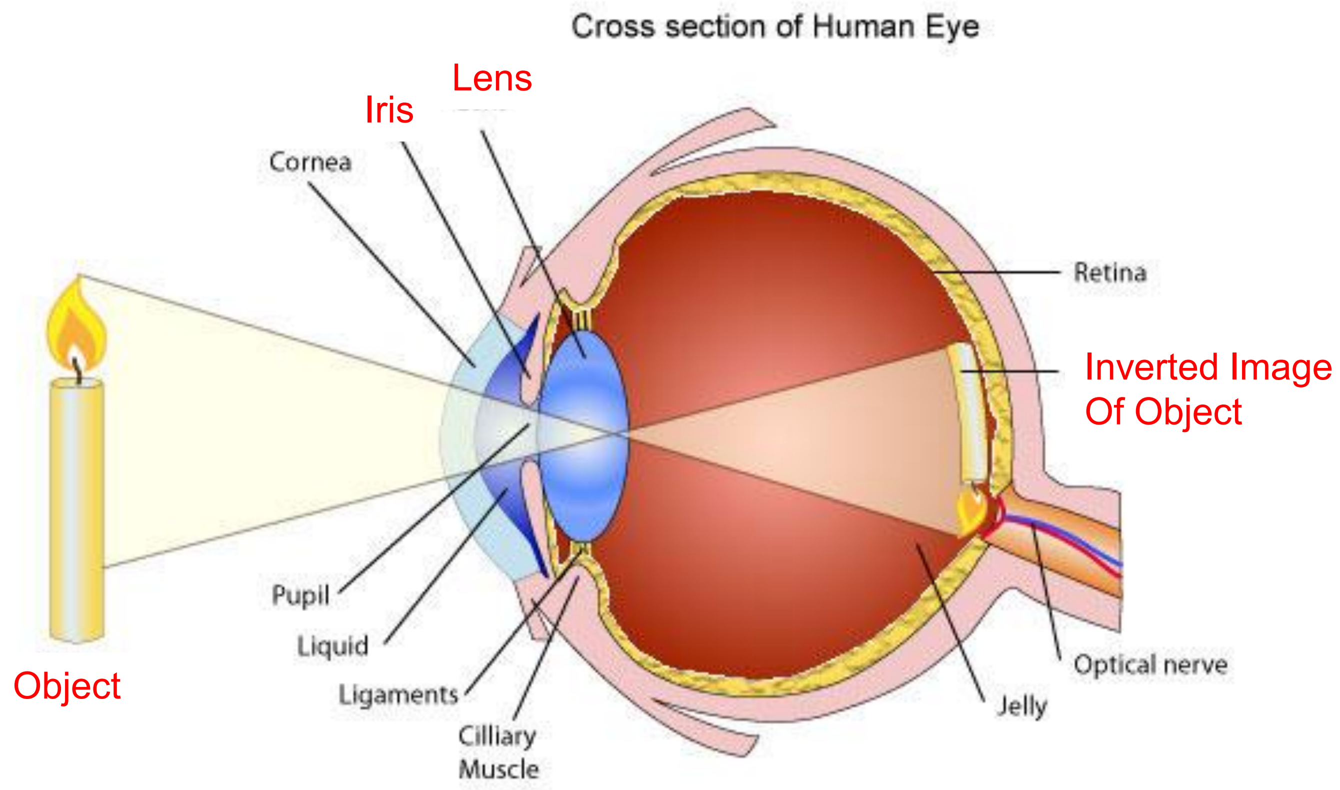
Understanding Lights (6)

- A transparent device, with two spherical surfaces sharing a common axis, is called a lens. If the spherical surfaces are convex, the incoming light rays to such lens will be focused.

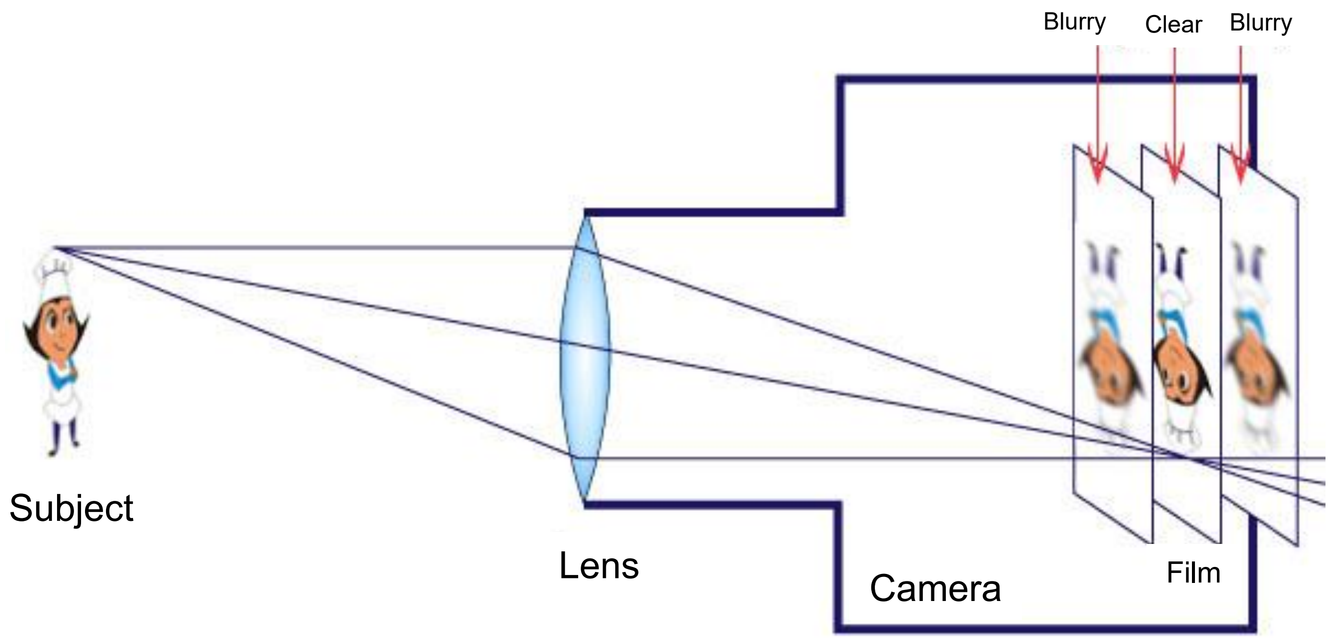




Example

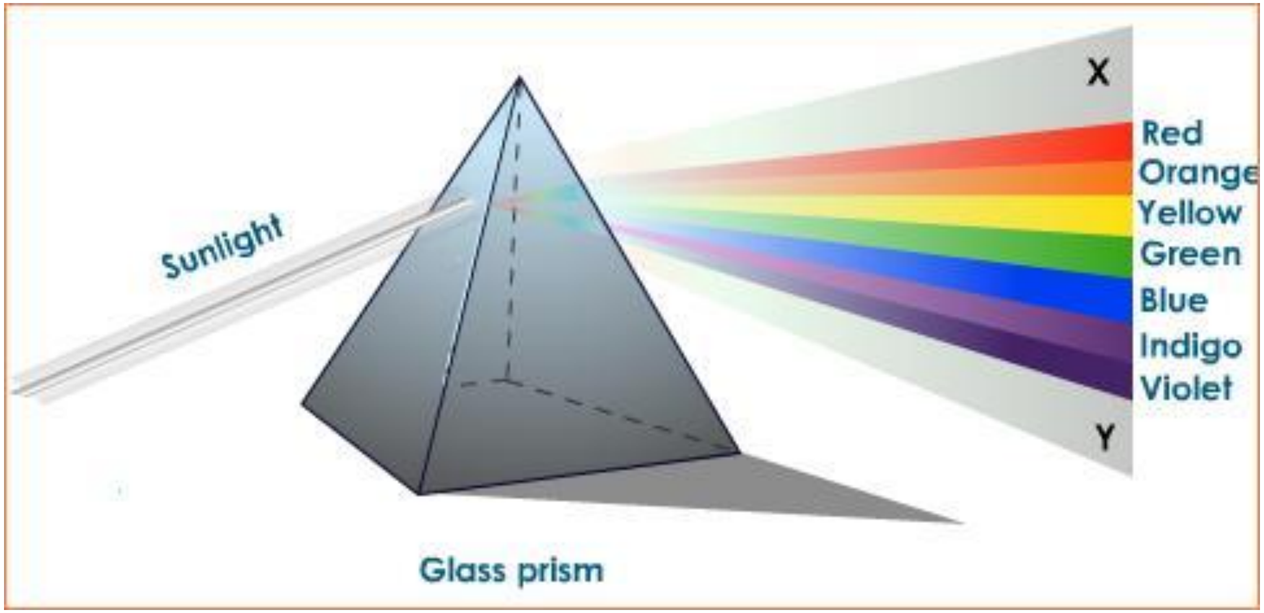

Cross section of Human Eye
Lens
Iris
Cornea
Retina
Inverted Image
Of Object
Pupil
Liquid
Ligaments
Cilliary
Muscle
Optical nerve
Jelly
Object

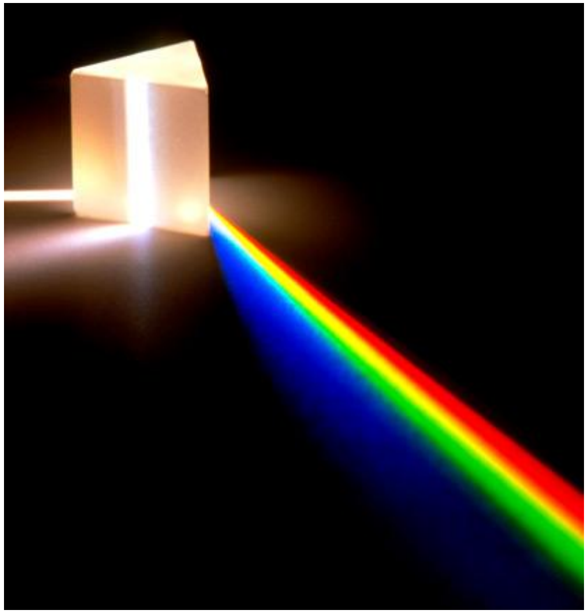
Example



Understanding Lights (7)

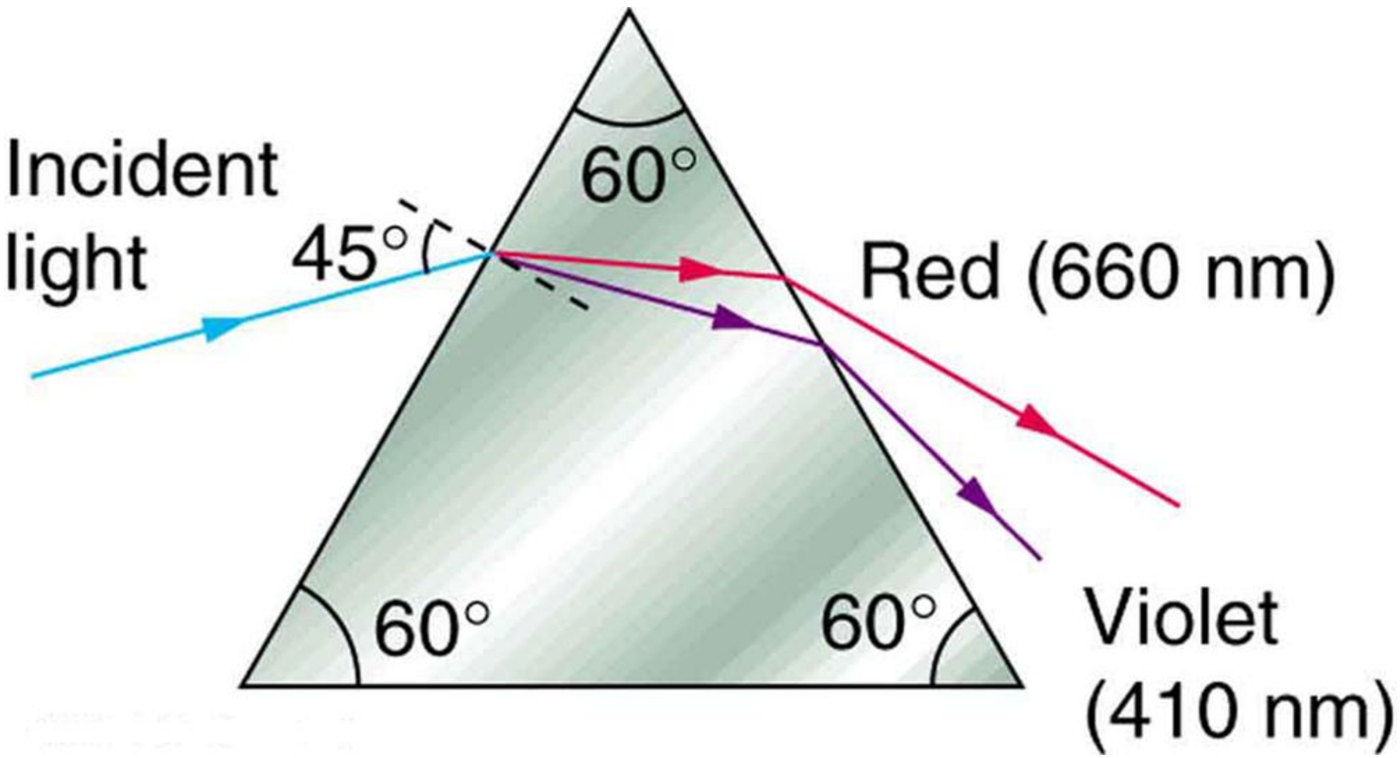
- A light ray is composed of photon waves of different wave lengths or frequencies, which create the sensation of colourful light rays.





Colorful is colorless while colorless is colorful

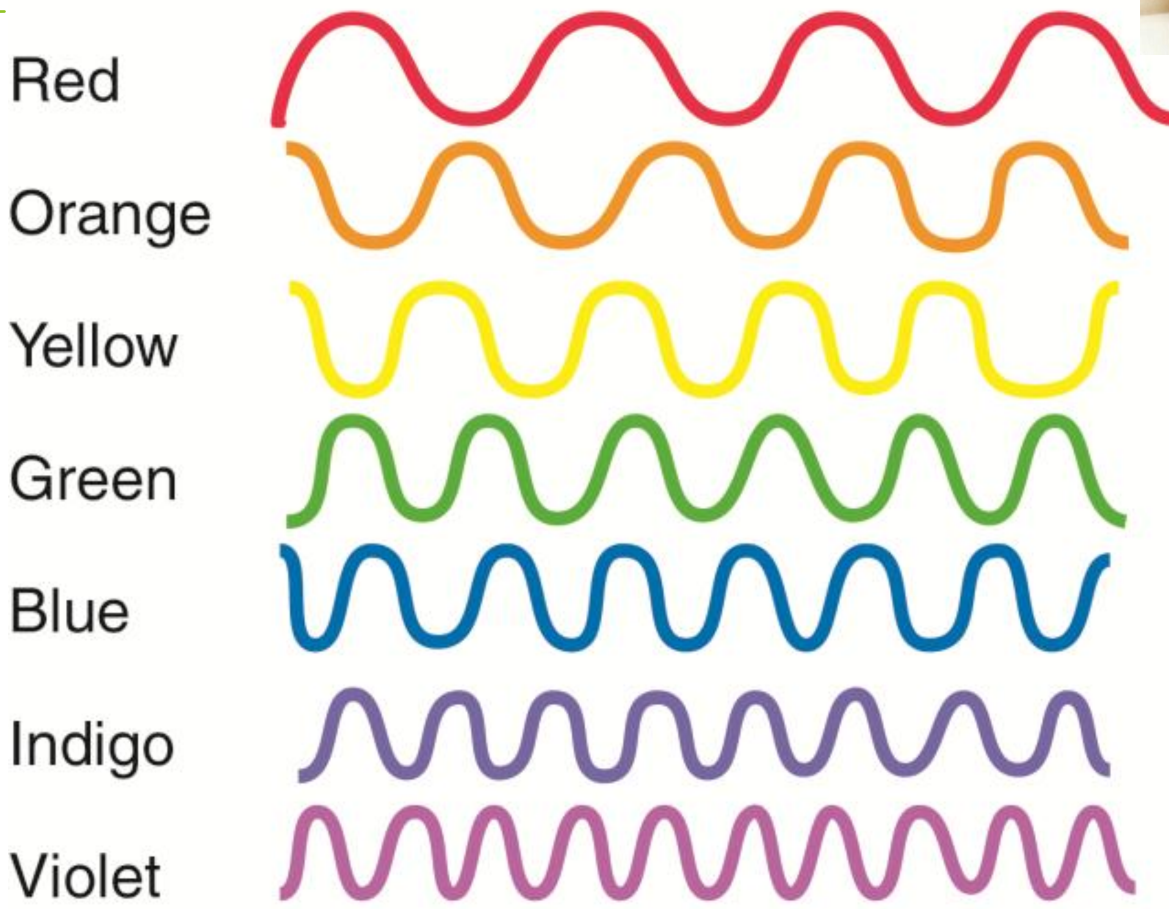
Example



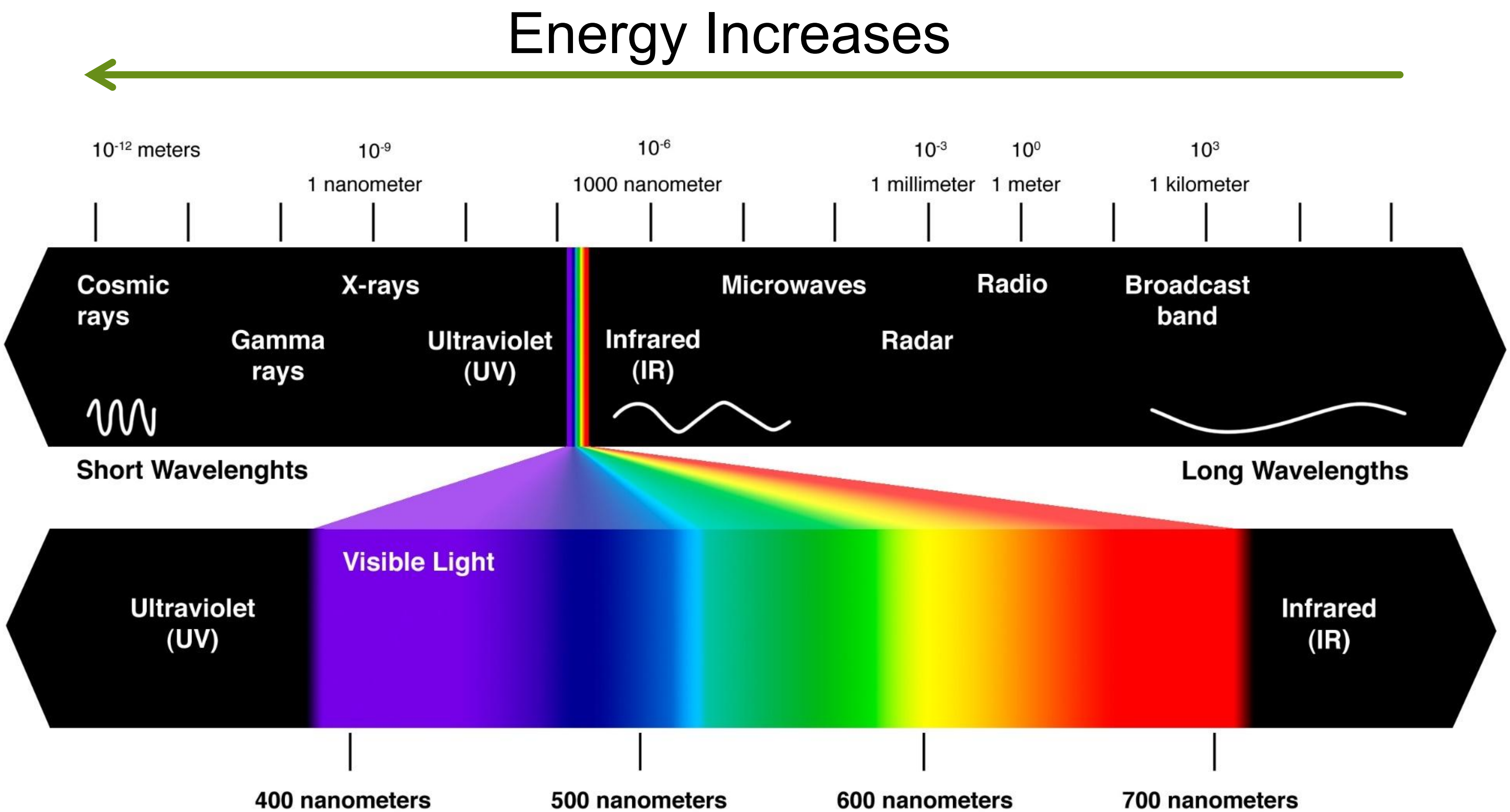
Example

Colorful is colorless while colorless is colorful

Human Being's Sensation



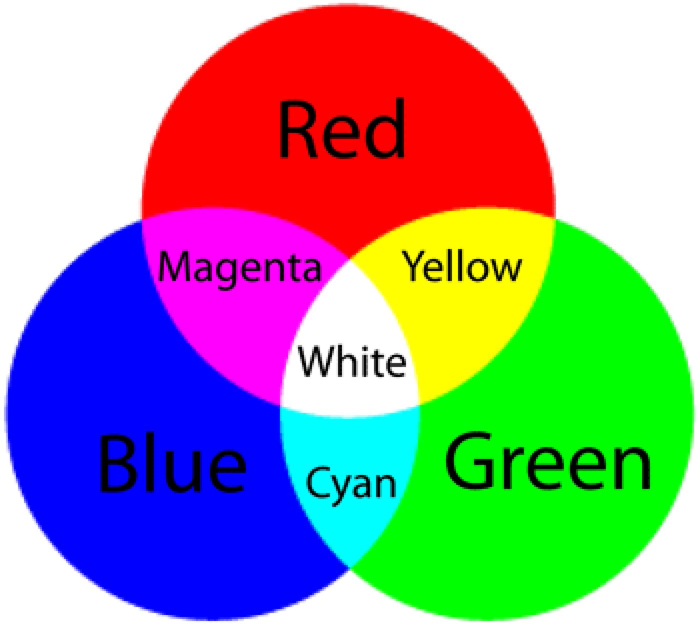
Example



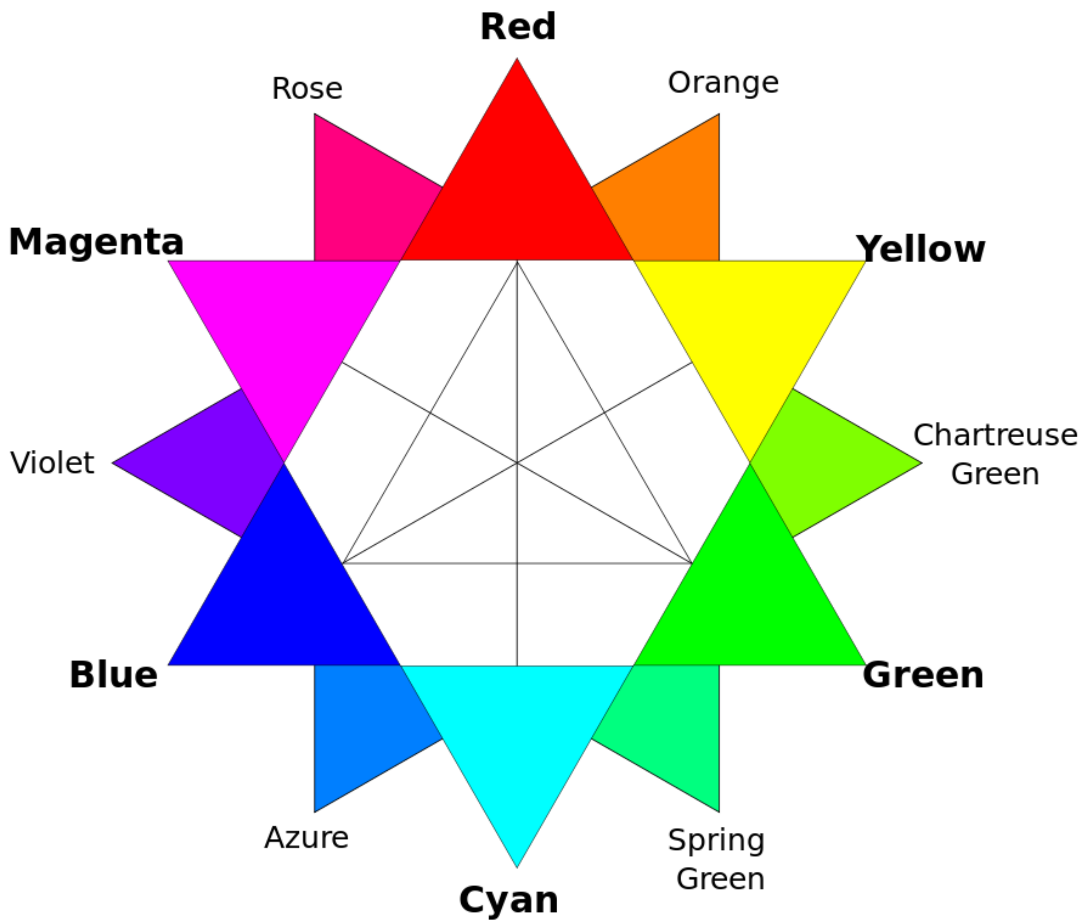
Colorful is colorless while colorless is colorful

Understanding Lights (8)

- Red, Green and Blue are three primary additive colours. The mixture of the primary additive colours can produce other colours.

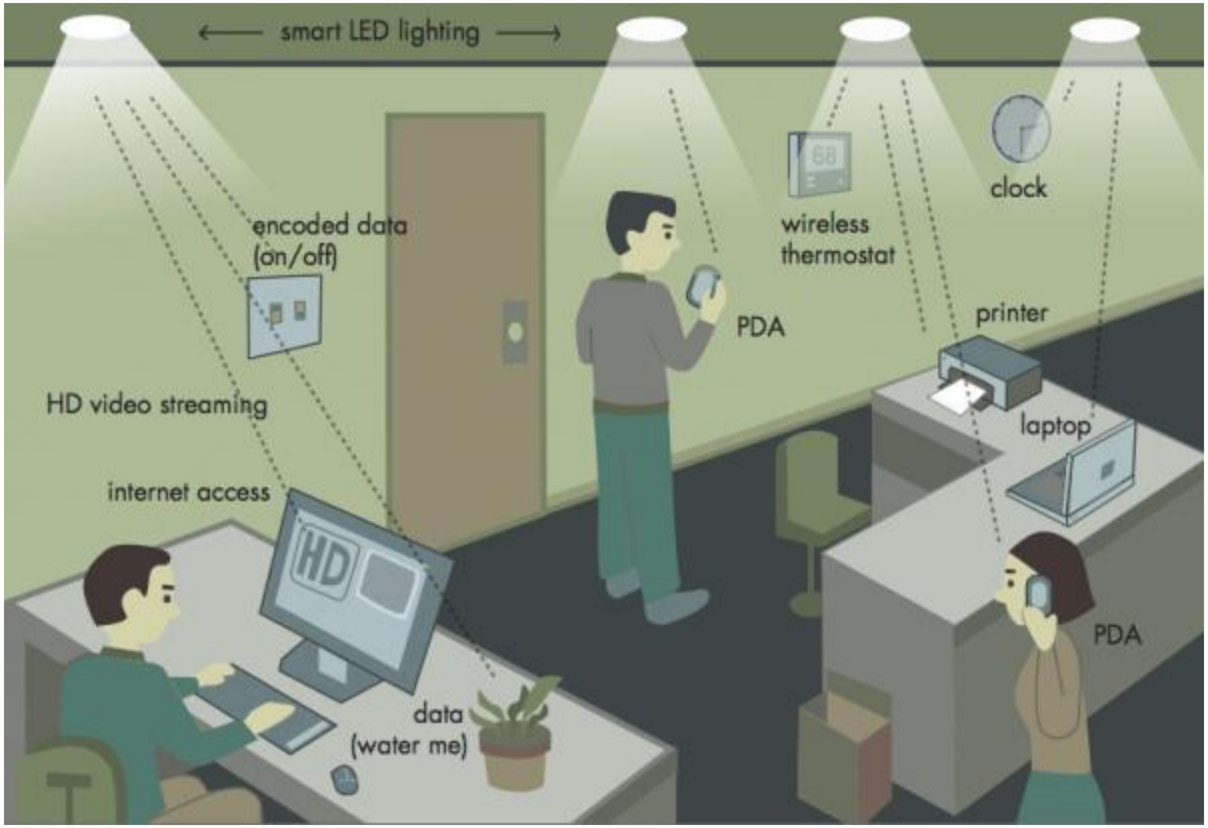


Example

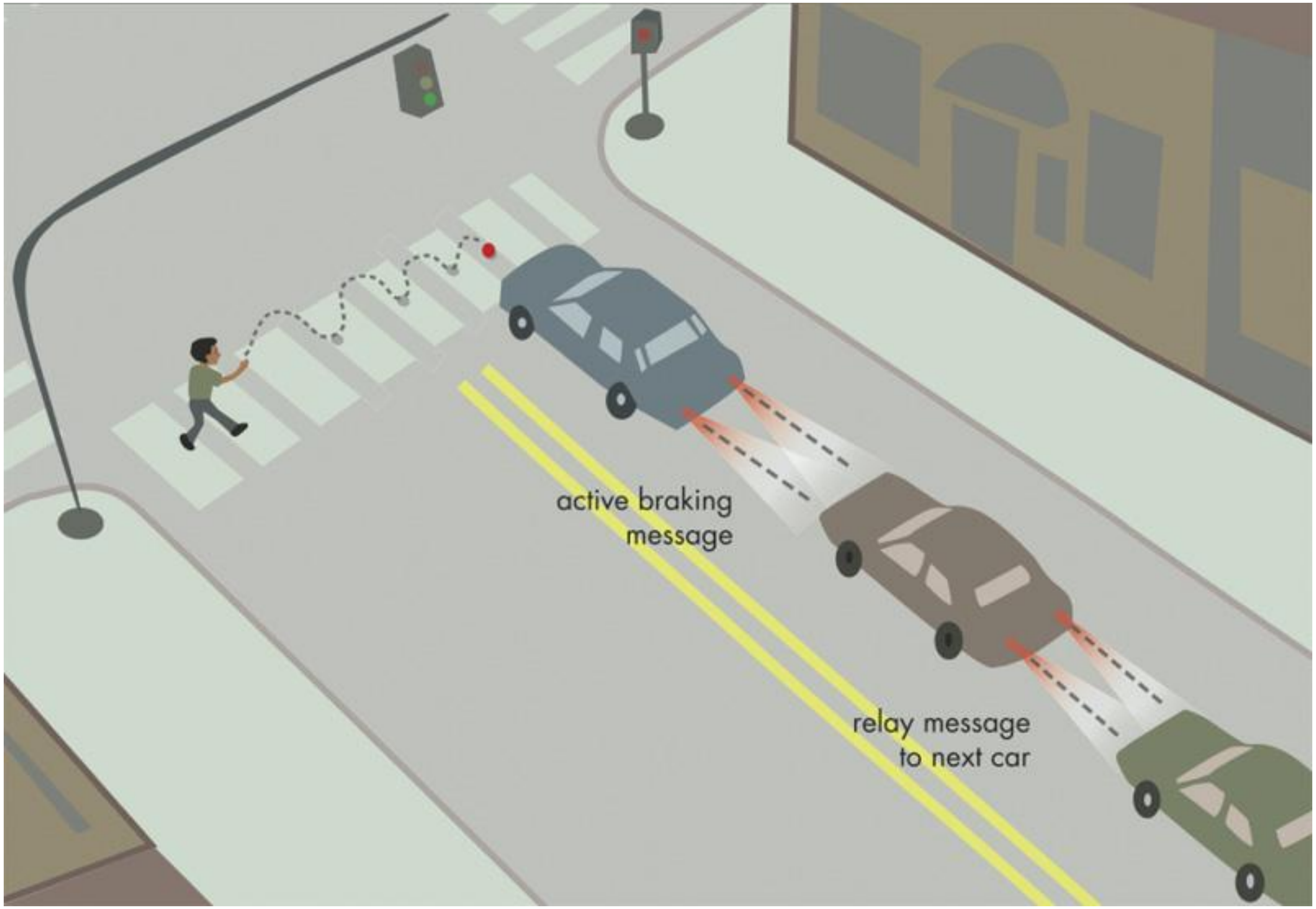

Red
Rose
Orange
Magenta
Yellow
Violet
Chartreuse Green
Blue
Green
Azure
Spring Green
Cyan

Understanding Lights (9)

- Light rays are electro-magnetic waves which can serve as signal carriers for data communications.

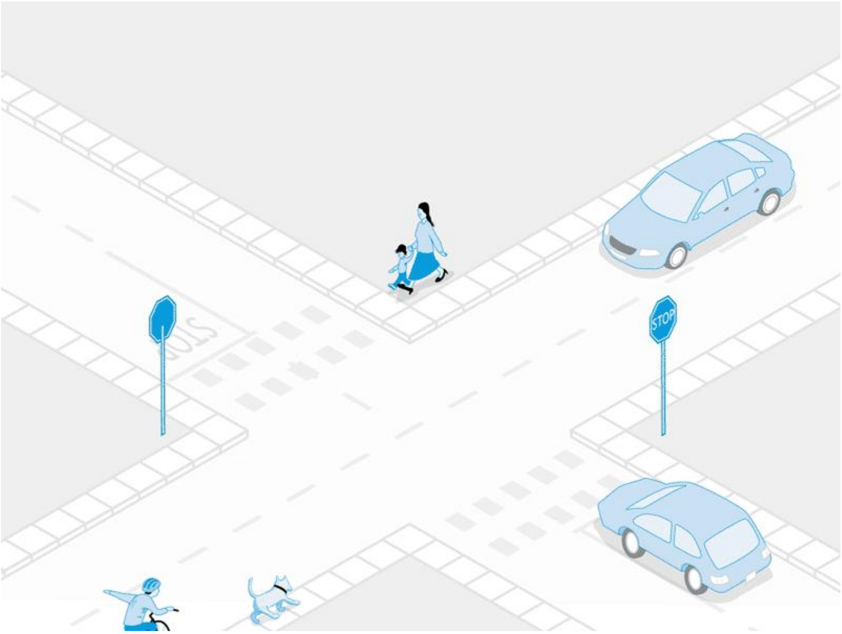


Example



Outline of Lecture 1

- Basics of Sensing and Measurement
- Basics of Visual Signals
- Parameter(s) of Visual Signals
- Measurement of Photometry



What could be computed from visual signals?

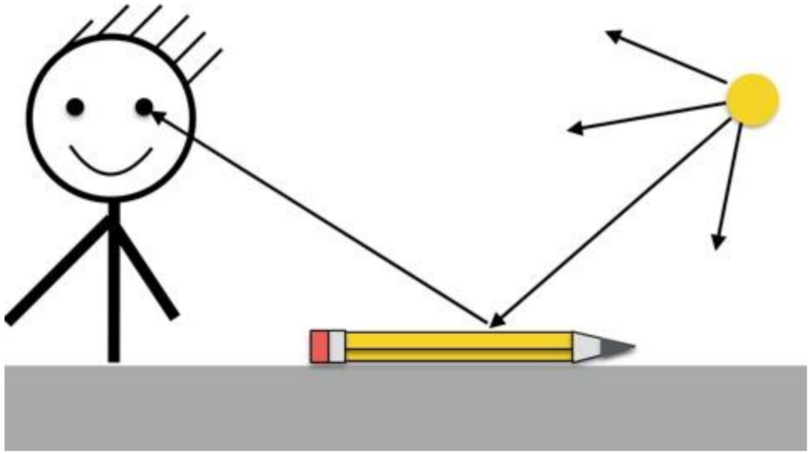
Geometry

- Path (Motion)
- Direction (Appearance)

Photometry

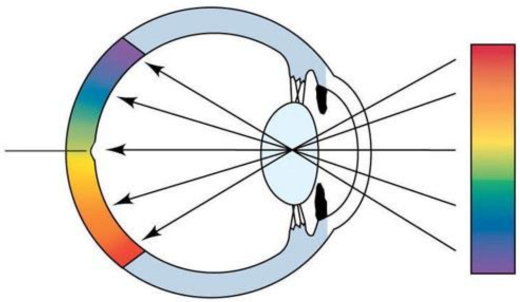
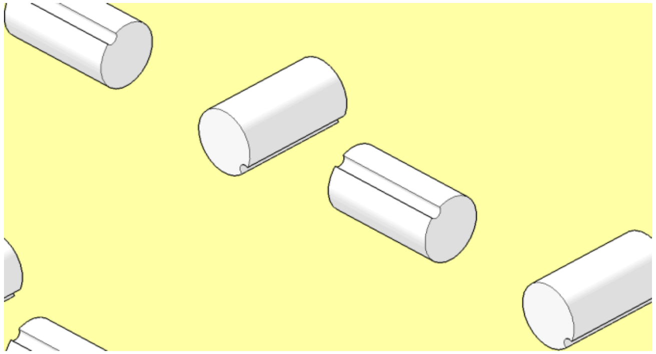
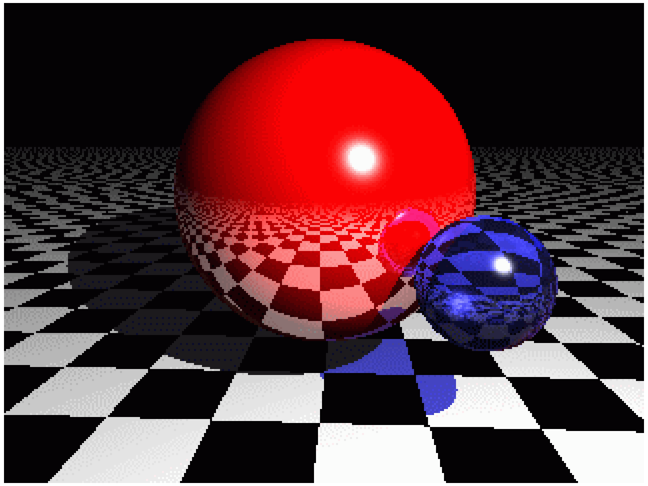
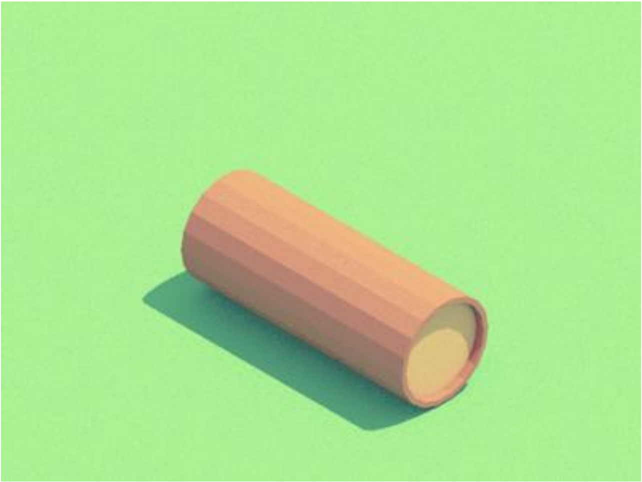
- Luminance (or Intensity)
- Chrominance (or Colour)

Colour Space



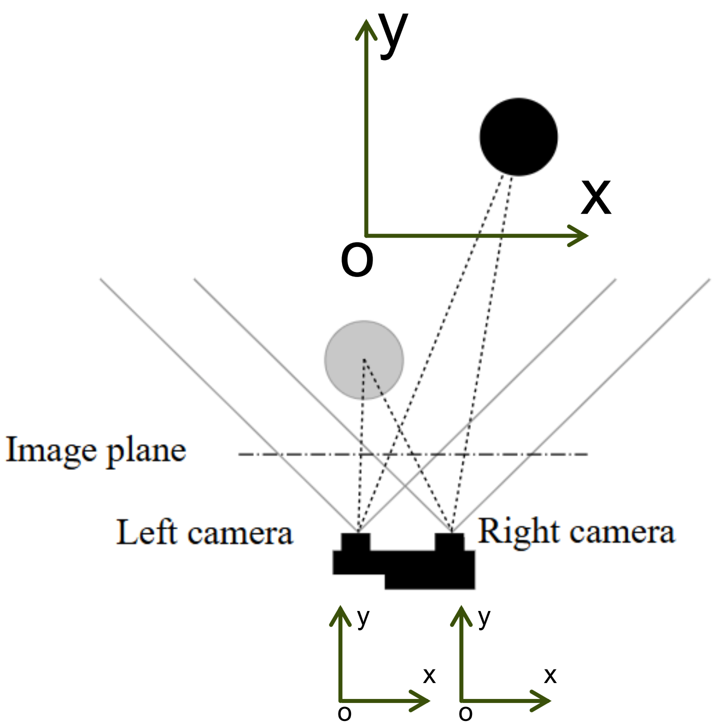
What is Path of Visual Signals?

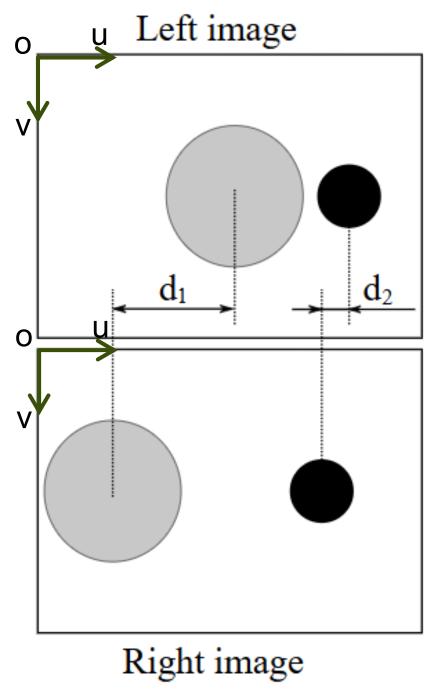
- Paths refer to the spatial locations that light sources pass through.

What is Direction of Visual Signals?

- Directions refer to the angles of light rays with respect to reference coordinate systems.

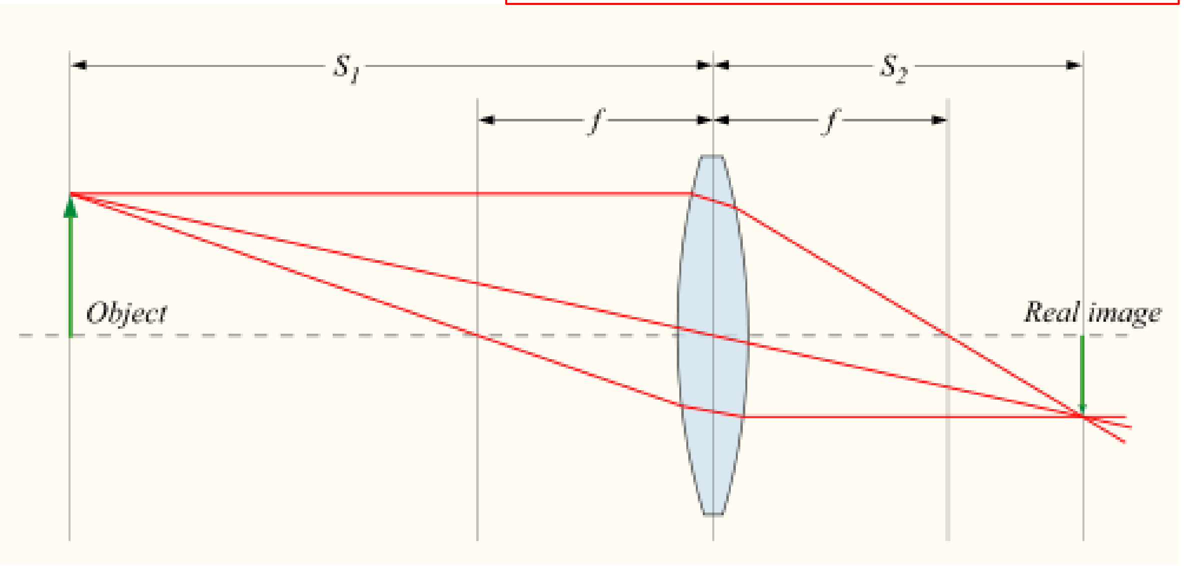




Geometric Equation of Thin Lens (1)

$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$$

f:	Focal Length
S1:	Distance to Object
S2:	Distance to Image

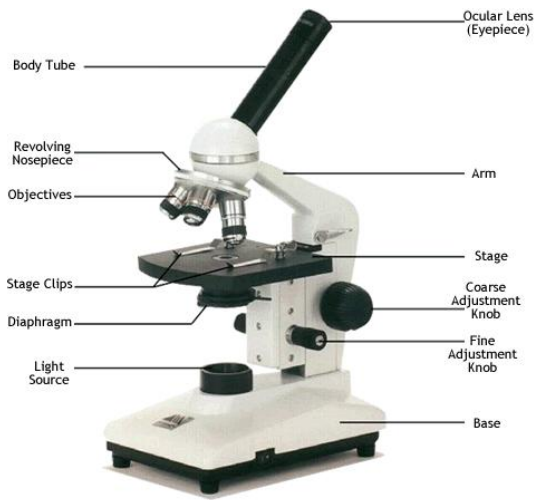


Normal Scope

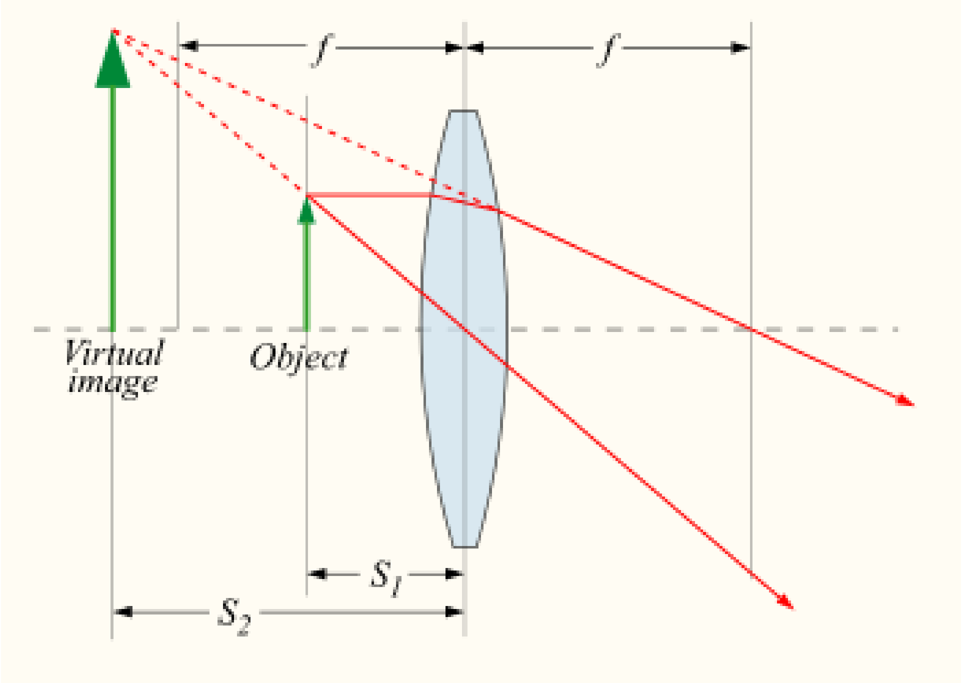
Geometric Equation of Thin Lens (2)

$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$$

f:	Focal Length
S1:	Distance to Object
S2:	Distance to Image

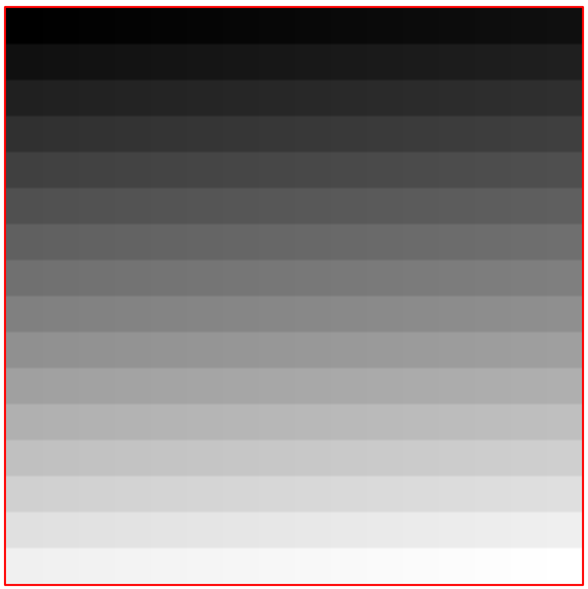


Micro Scope



What is Luminance of Visual Signals?

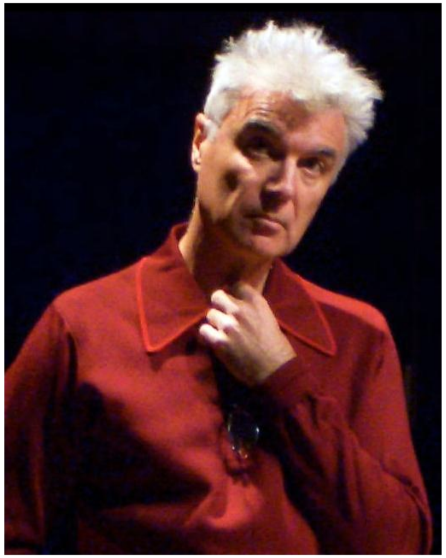
- Luminance refers to a single value which represents the energy levels of light rays.



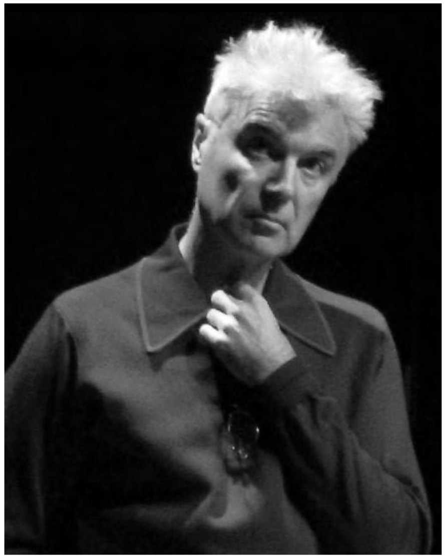
Grey Scale Levels

What is Chrominance of Visual Signals?

- Chrominance refers to a set of two values which represent the colour information of light rays.



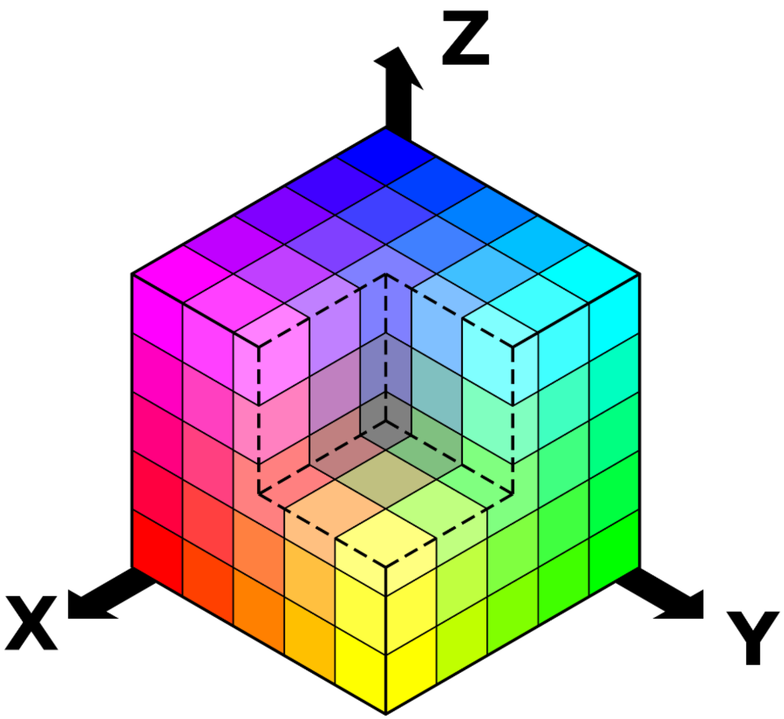
With Colours



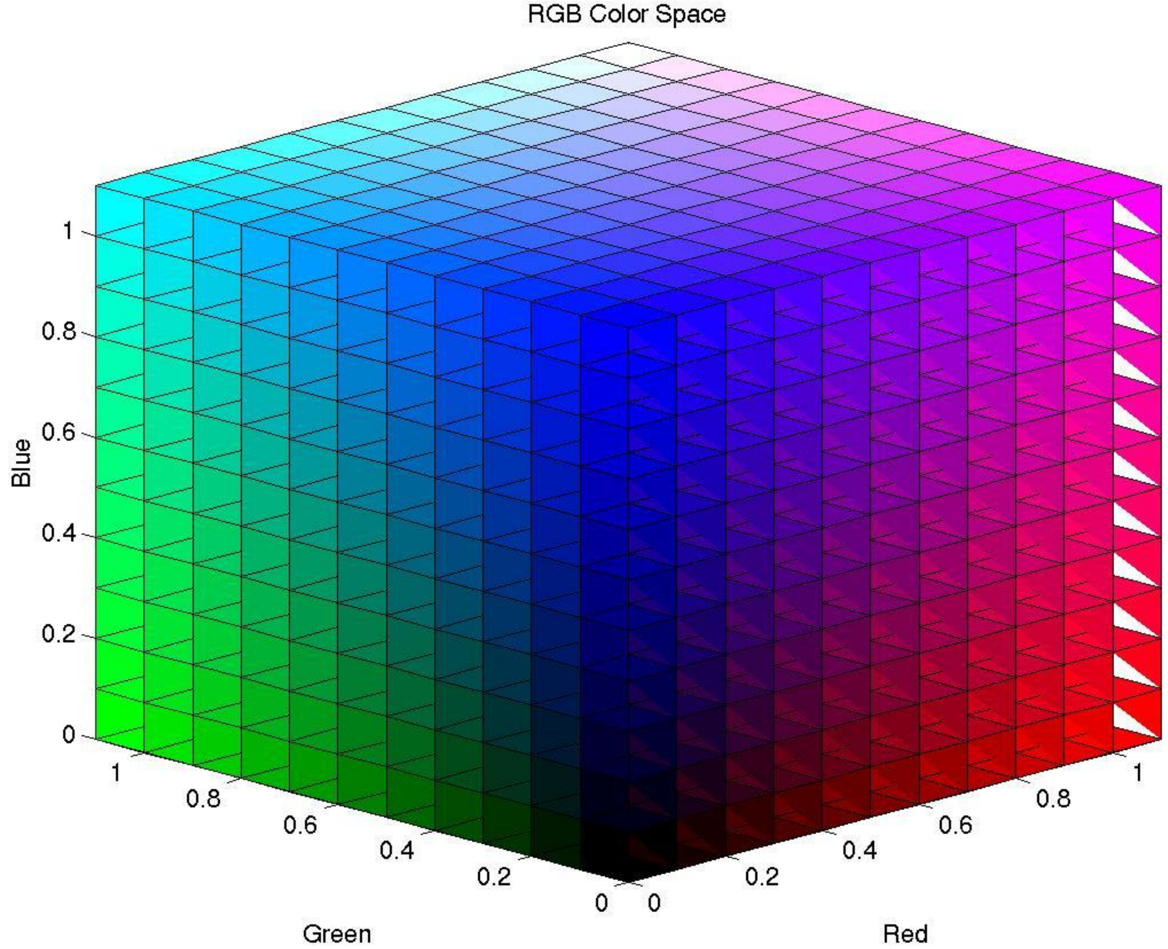
Without Colours

Colour Spaces of Visual Signals

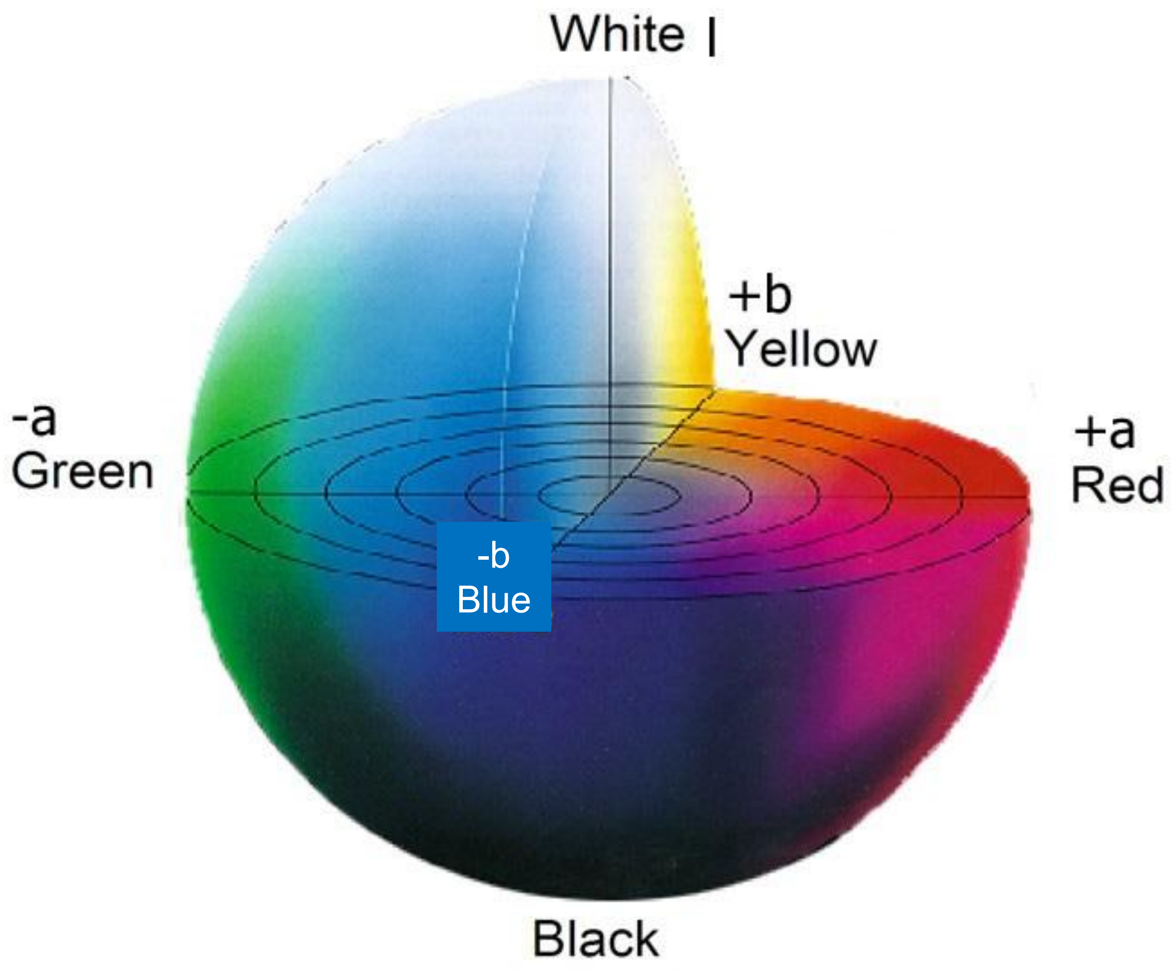
- The values of luminance and chrominance form three-dimensional colour spaces.



Example of RGB Colour Space

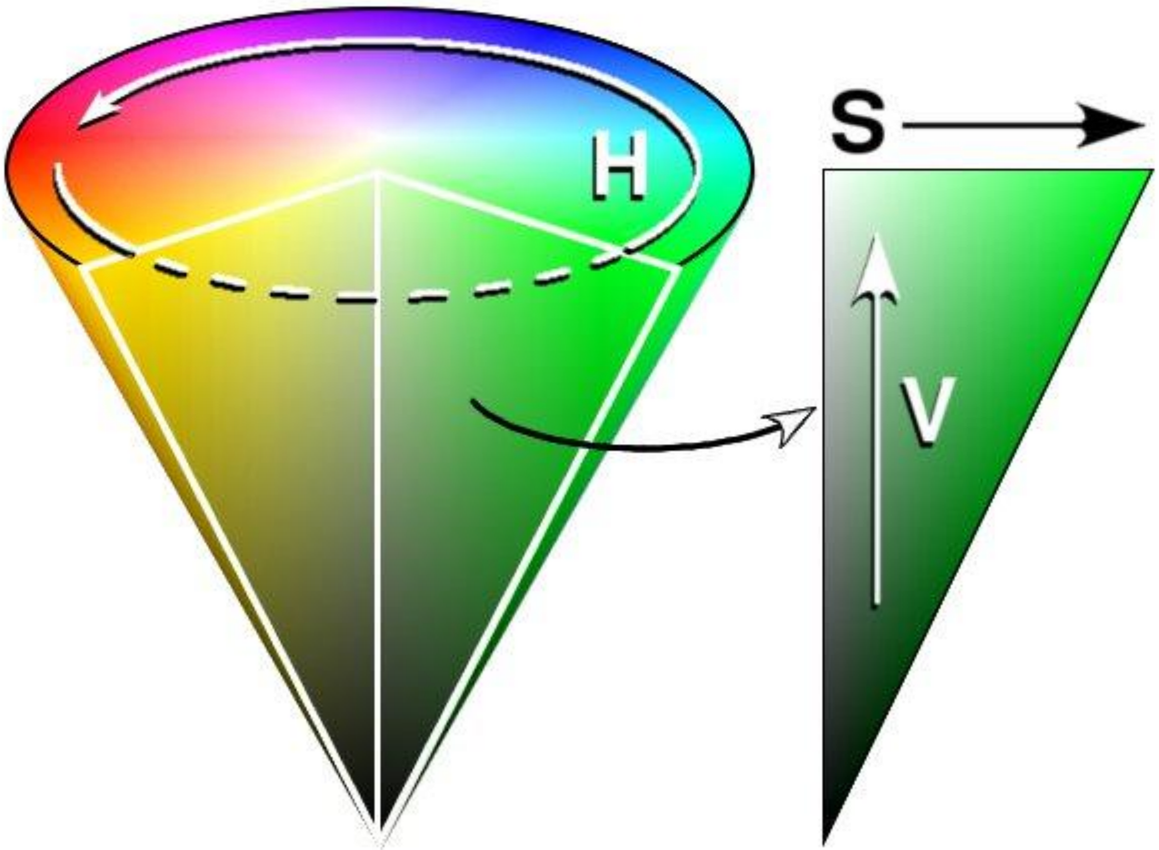


Example of L*a*b Colour Space



Example of HSV Colour Space

- Hue (colour angle), Saturation (colour amplitude), Visual Brightness



RGB Colour Space to L*a*b Colour Space Conversion

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.607 & 0.174 & 0.200 \\ 0.299 & 0.587 & 0.114 \\ 0.000 & 0.066 & 1.116 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{cases} L = 25.0 \bullet (100.0 \bullet Y / Y_{\max})^{1/3} - 16.0 \\ a = 500.0 \bullet [(X / X_{\max})^{1/3} - (Y / Y_{\max})^{1/3}] \\ b = 200.0 \bullet [(Y / Y_{\max})^{1/3} - (Z / Z_{\max})^{1/3}] \end{cases}$$

RGB Colour Space to HSV Colour Space Conversion

$$M = \max\{R/255, G/255, B/255\}$$

$$m = \min\{R/255, G/255, B/255\}$$

$$C = M - m$$

$$S = \frac{M - m}{M} \qquad V = M$$

$$H = \begin{cases} \dfrac{G/255 - B/255}{6C} & \text{if M = R/255} \\ \dfrac{B/255 - R/255}{6C} + \dfrac{1}{3} & \text{if M = G/255} \\ \dfrac{R/255 - G/255}{6C} + \dfrac{2}{3} & \text{if M = B/255} \end{cases}$$

Outline of Lecture 1

- Basics of Sensing and Measurement
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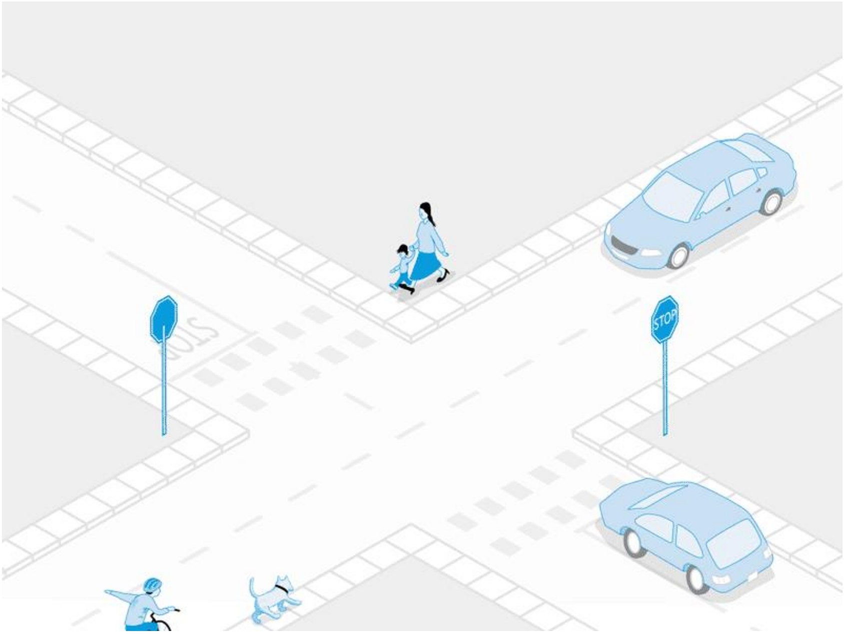
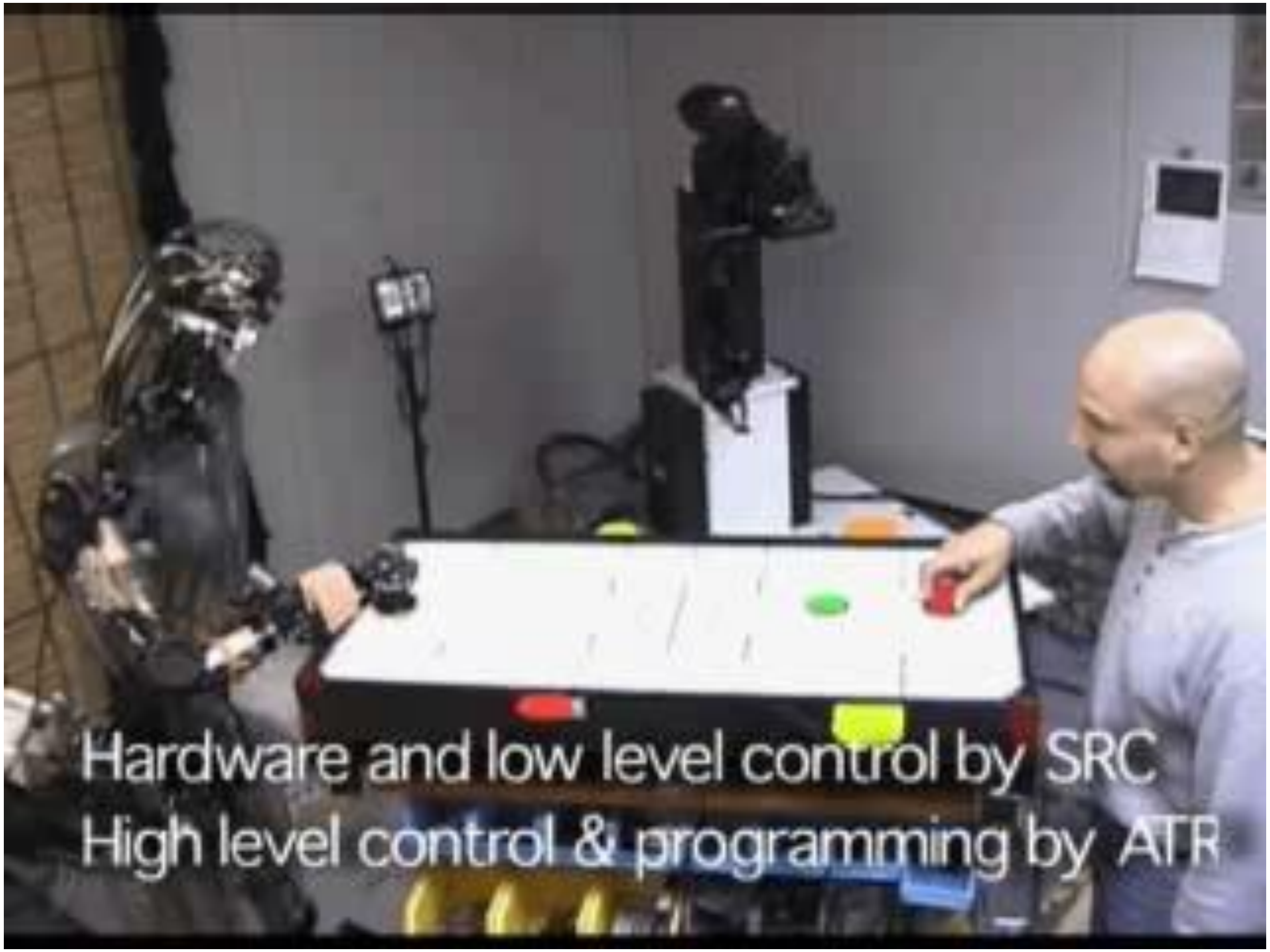


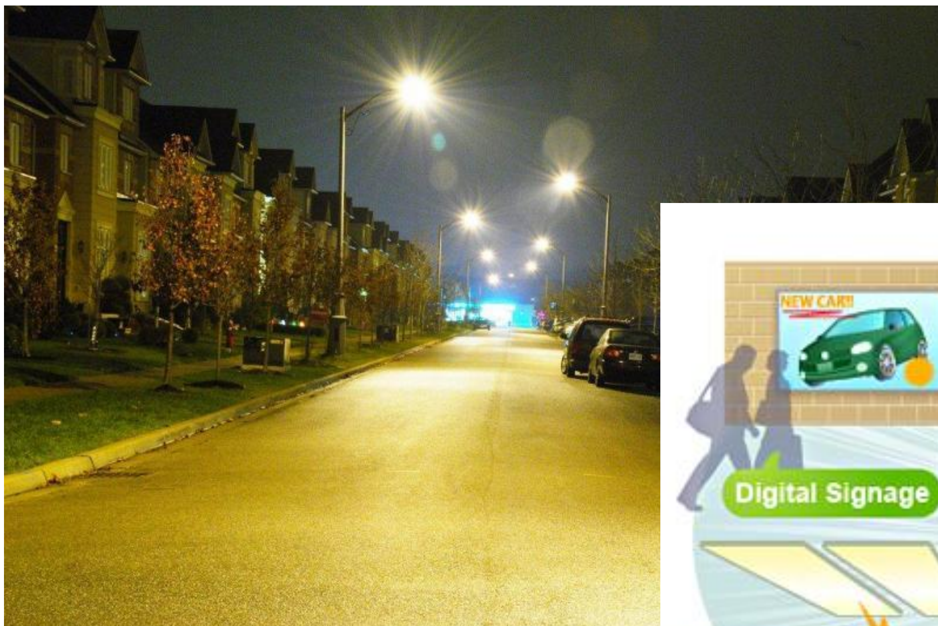
Visual Signals Enable Interaction …



Visual Signals Enable Automation …

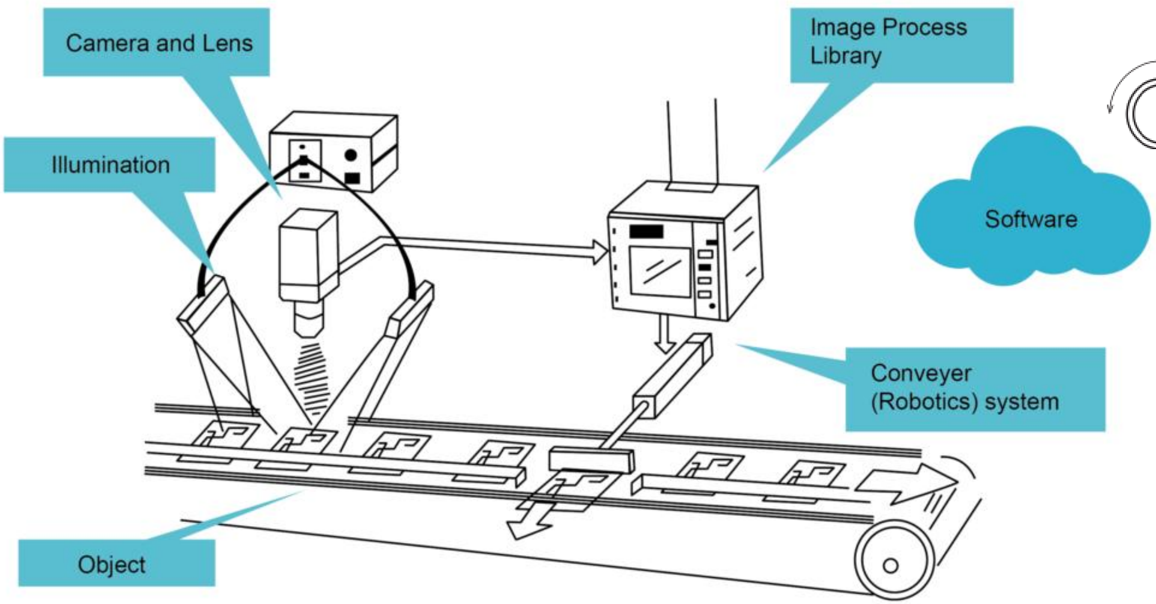
Applications (1)

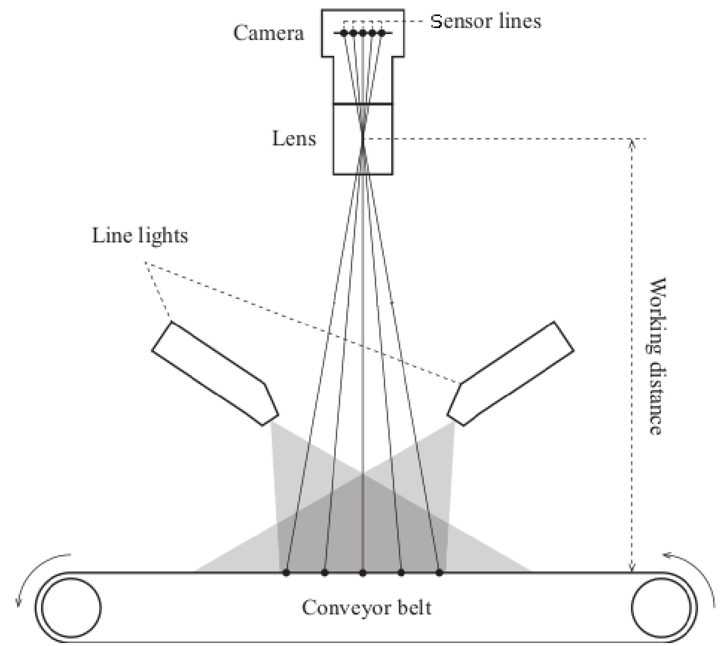
- Illuminations and communications



Applications (2)

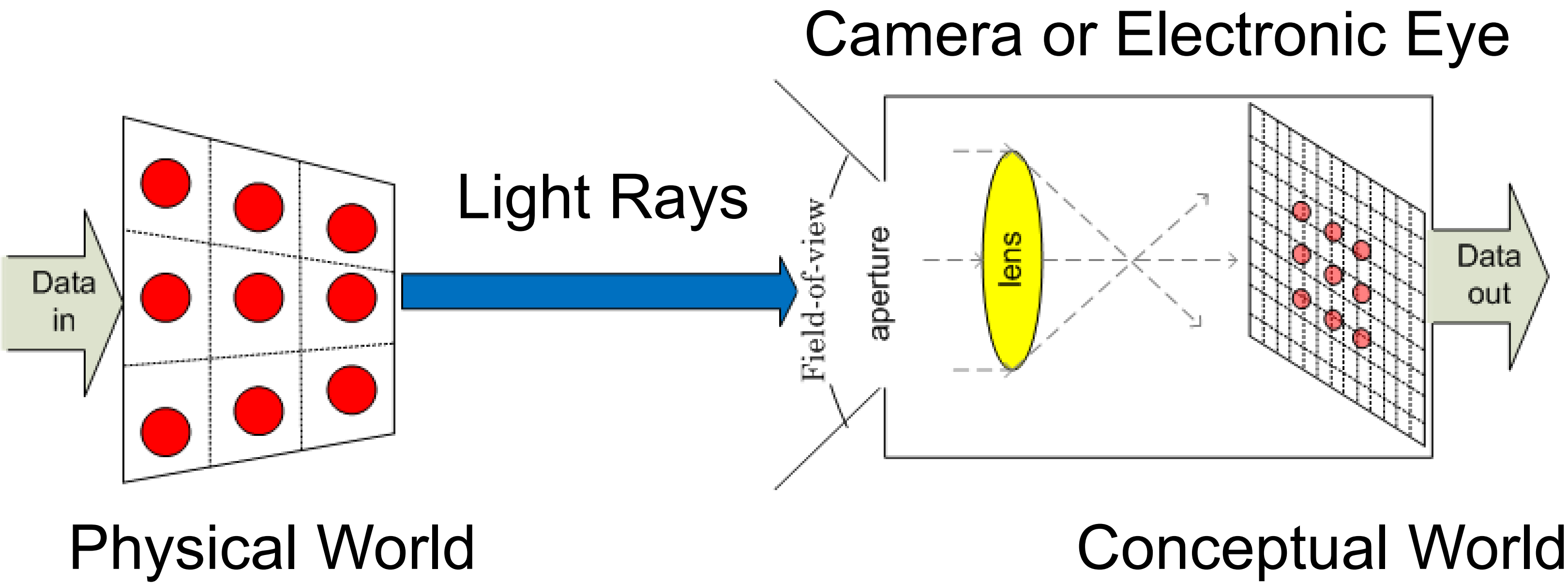
- Visual Inspection





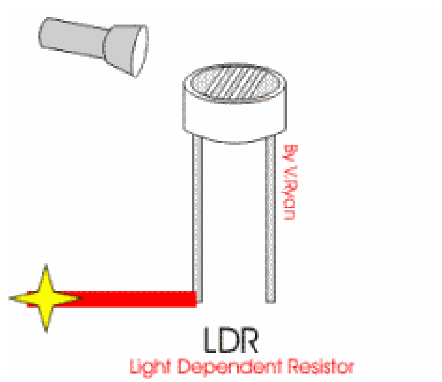
Applications (3)

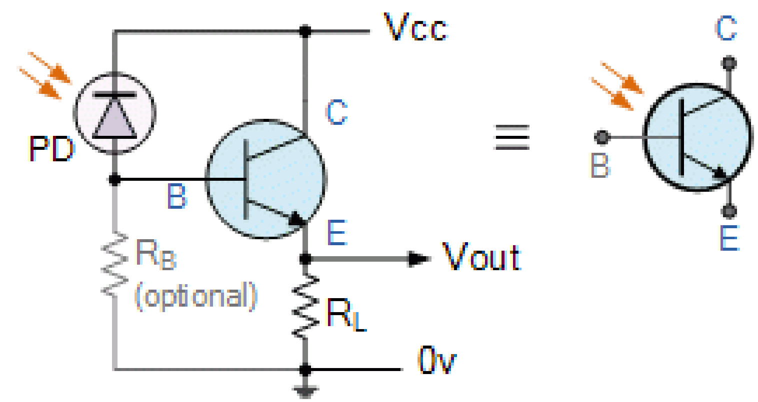
- Vision for Measurement, Inspection, Surveillance, Guidance, Recording, Learning, and Interaction, etc.

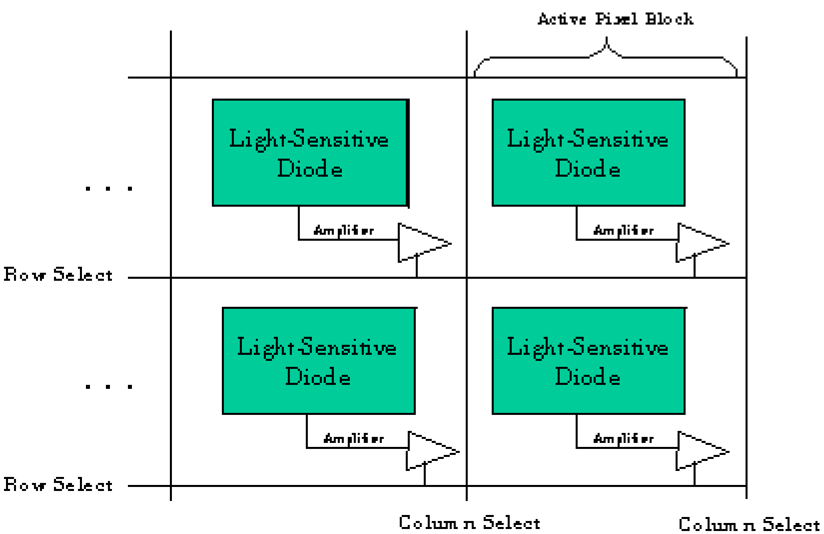


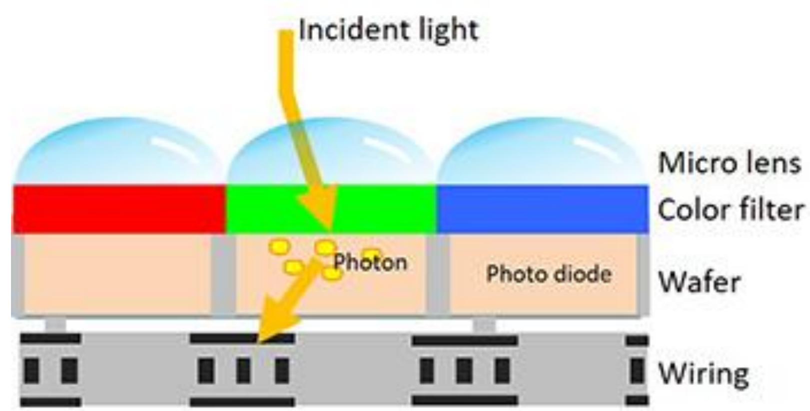
How to measure photometry?

- Use of Photoelectric Device to Sense:
  - Intensities
  - Colors



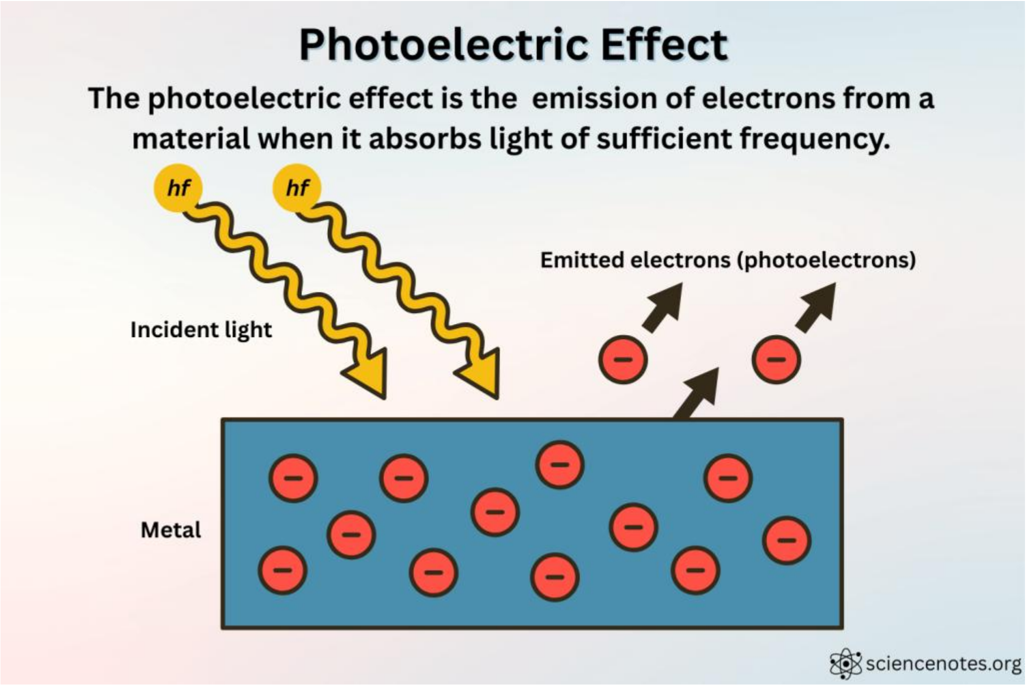


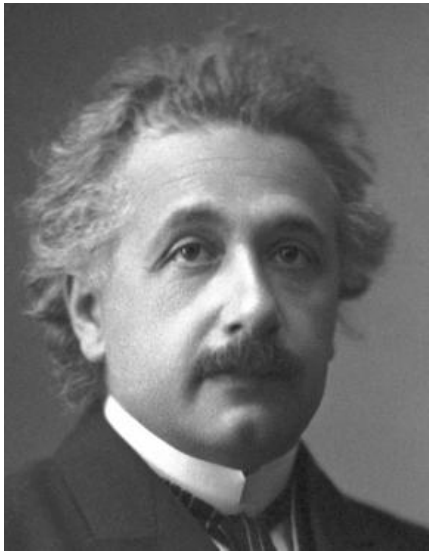




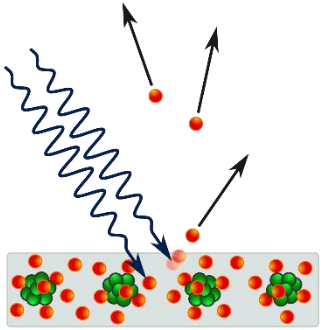
A Person to Remember: Albert Einstein

- He has discovered the photoelectric effect which is the emission of electrons from a material caused by electromagnetic radiation such as ultraviolet light. Electrons emitted in this manner are called photoelectrons.



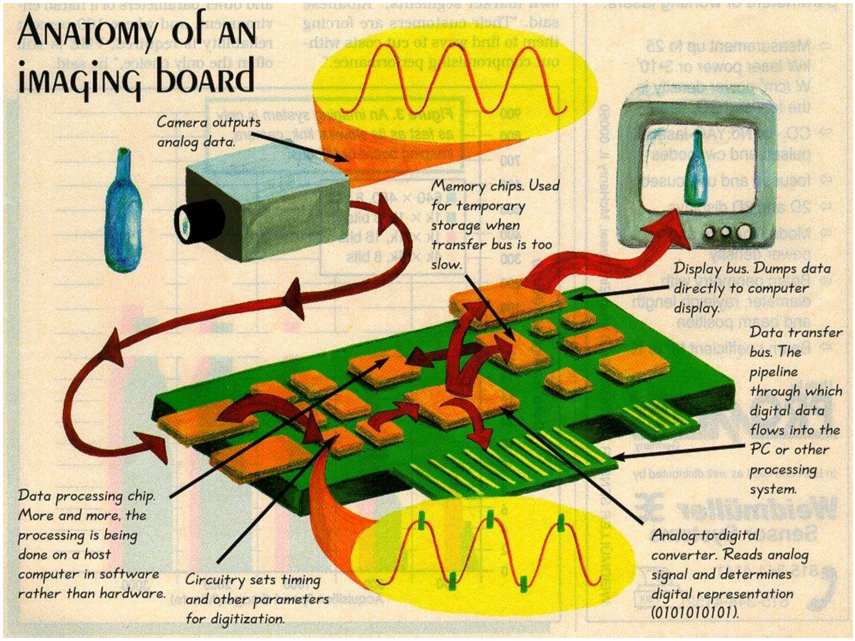


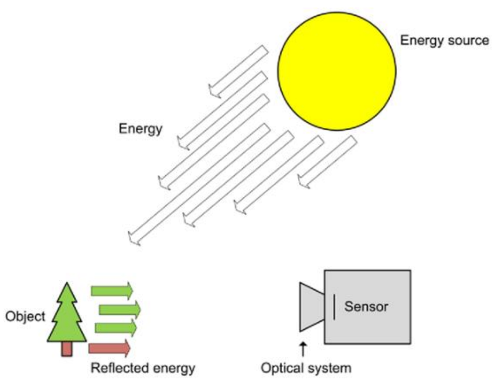
Nobel Prize in Physics 1921

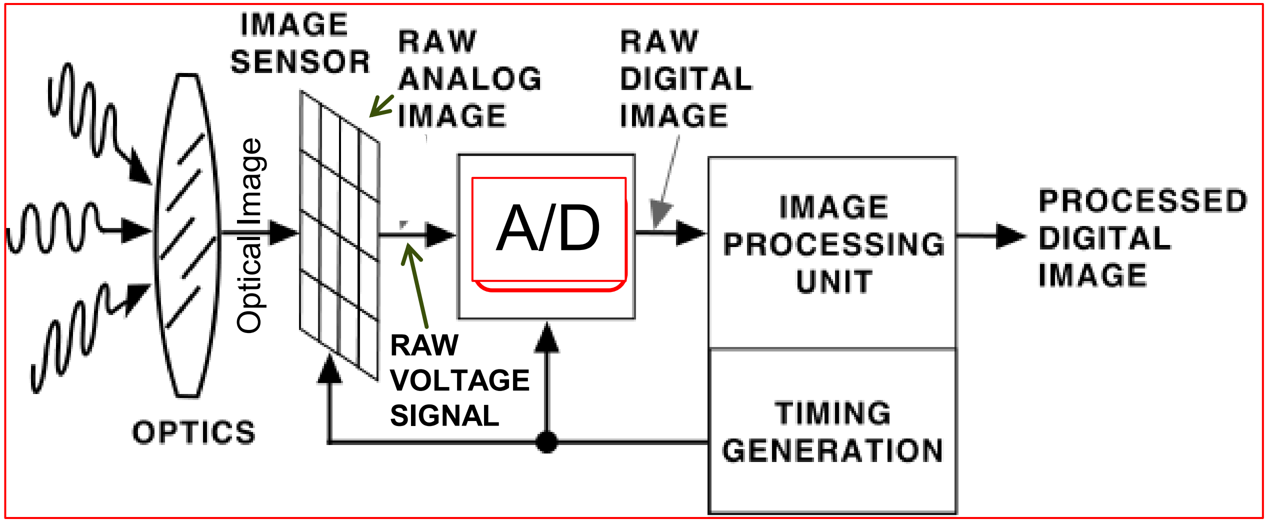


Principle of Measurement

- The principle consists of converting lights into optical images, which are then converted into analogue images. The analogue images could be formatted into time functions of voltages which could be automatically measured by microcontrollers or other advanced electronic hardware.



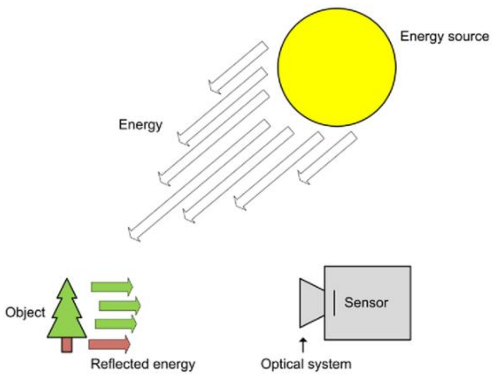


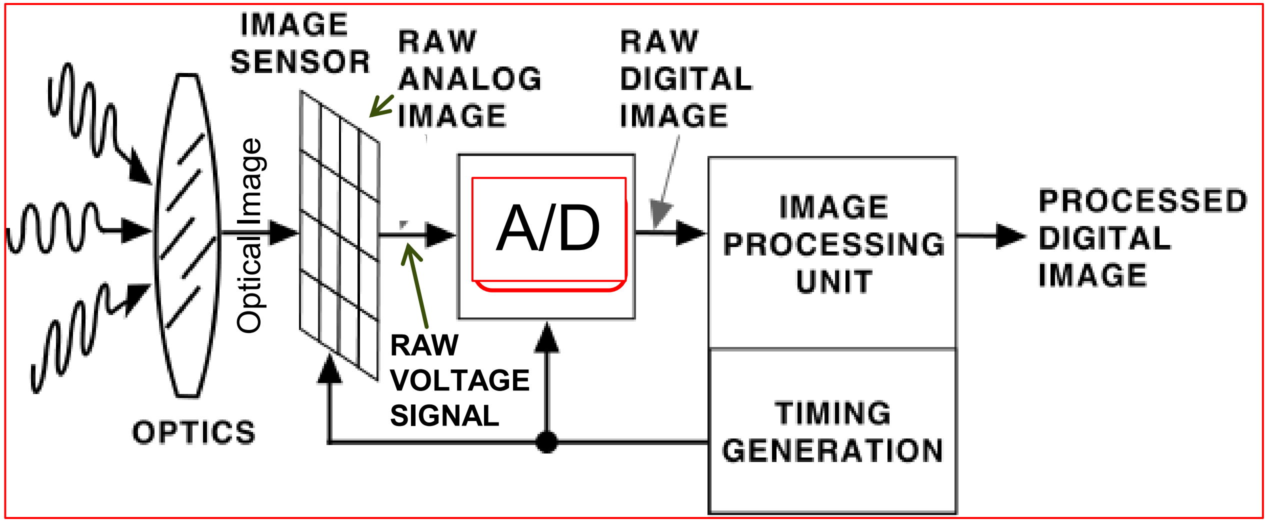


The Pipeline of Signal Flows

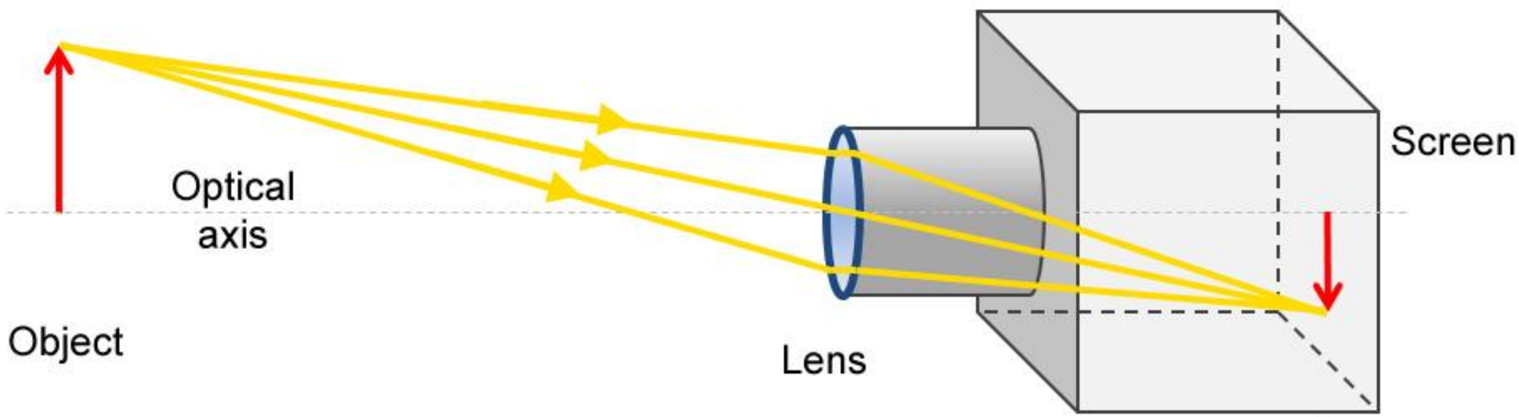
- 1. From Light Rays to Optical Image: MIMO
- 2. From Optical Image to Analogue Image: MIMO
- 3. From Analogue Image to Voltage Signals: MISO
- 4. From Voltage Signals to Digital Image: SIMO
- 5. From Digital Image to Display
- 6. From Digital Image to Image Processing …

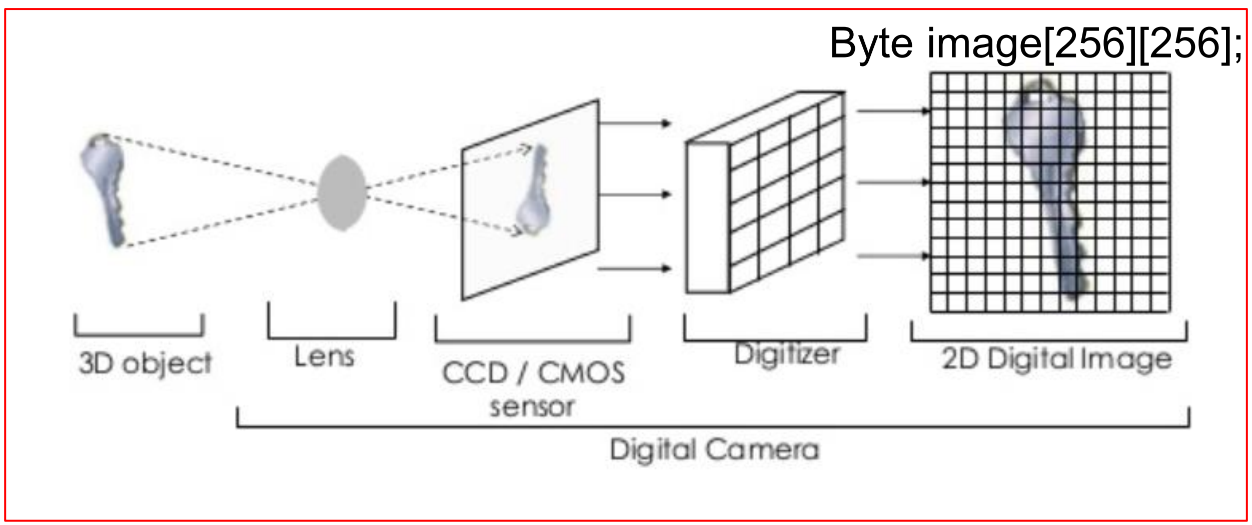
(Items 1–4: MIMO)



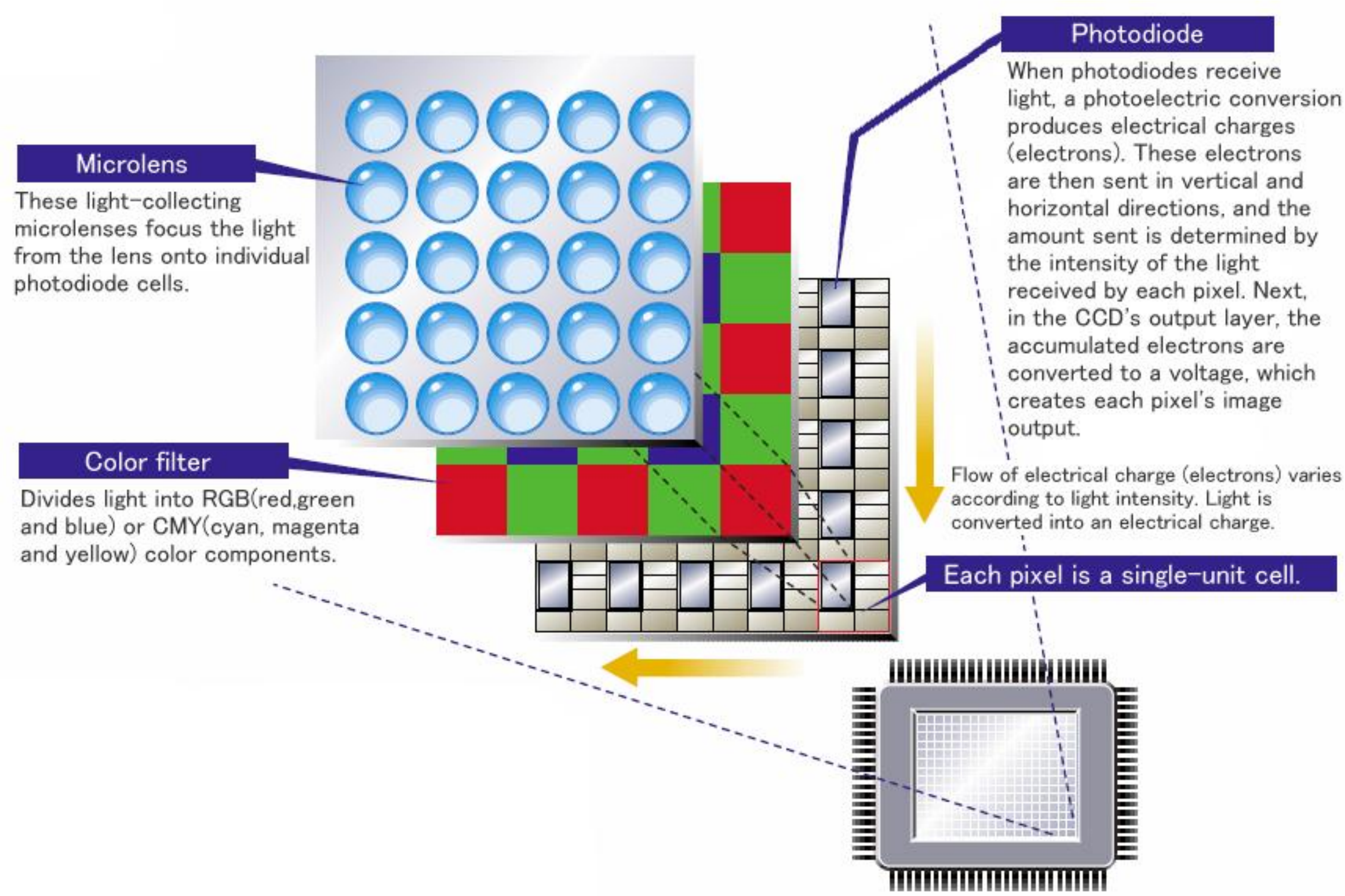


Step 1: From Light Rays to Optical Image



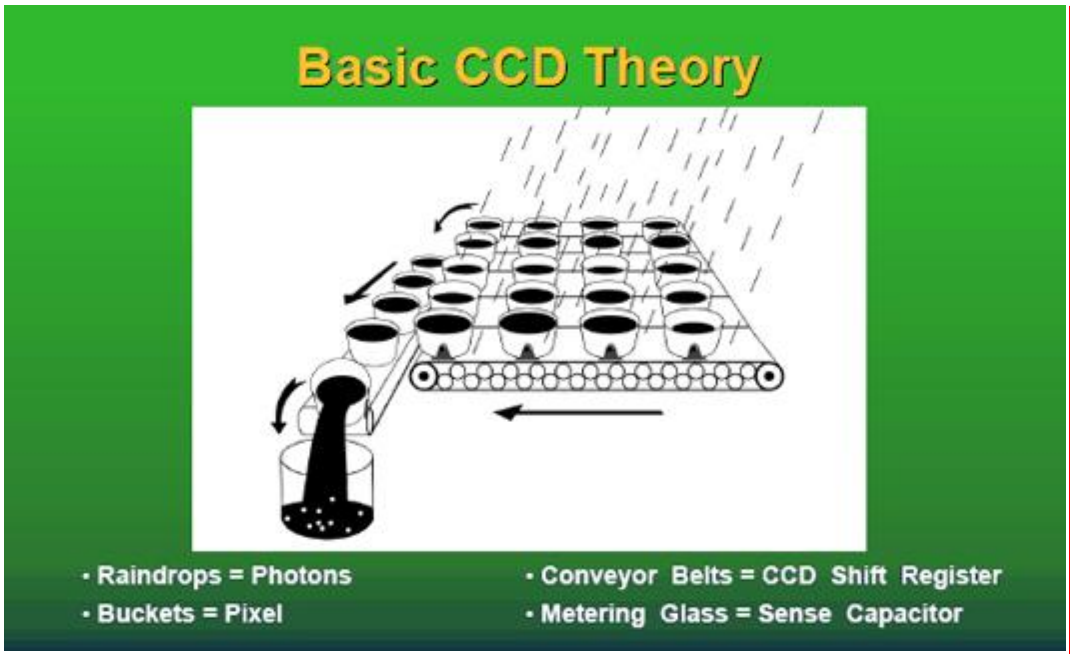


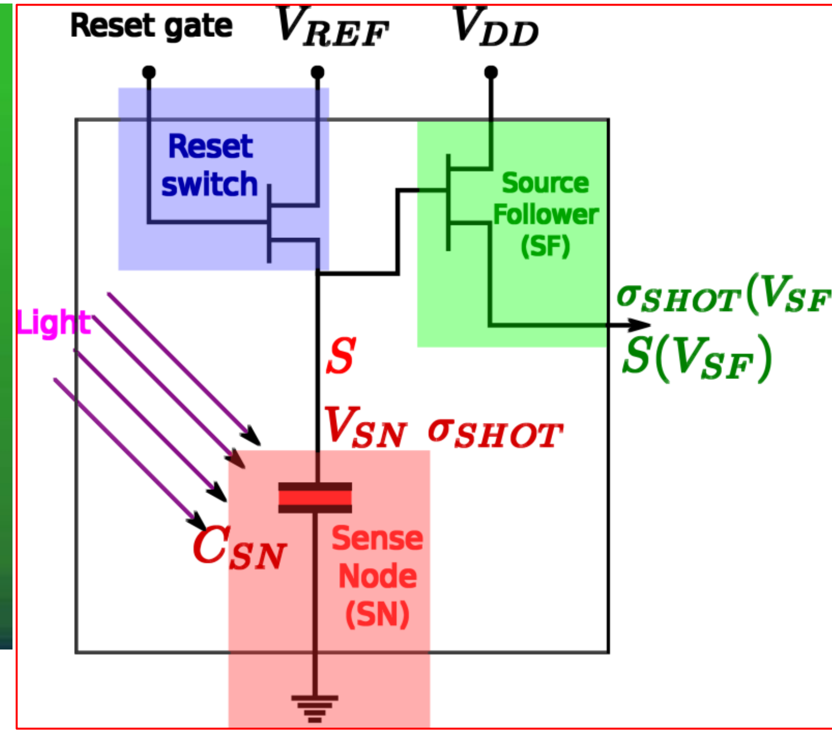
Step 2: From Optical Image to Analogue Image

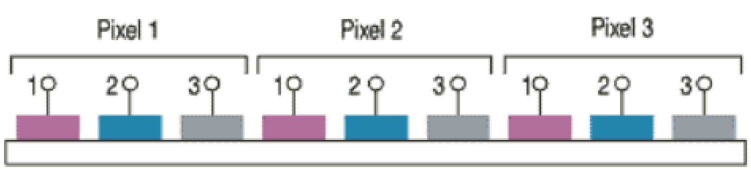


Example of Imaging Sensor

Step 3: From Analogue Image to Voltage Signals

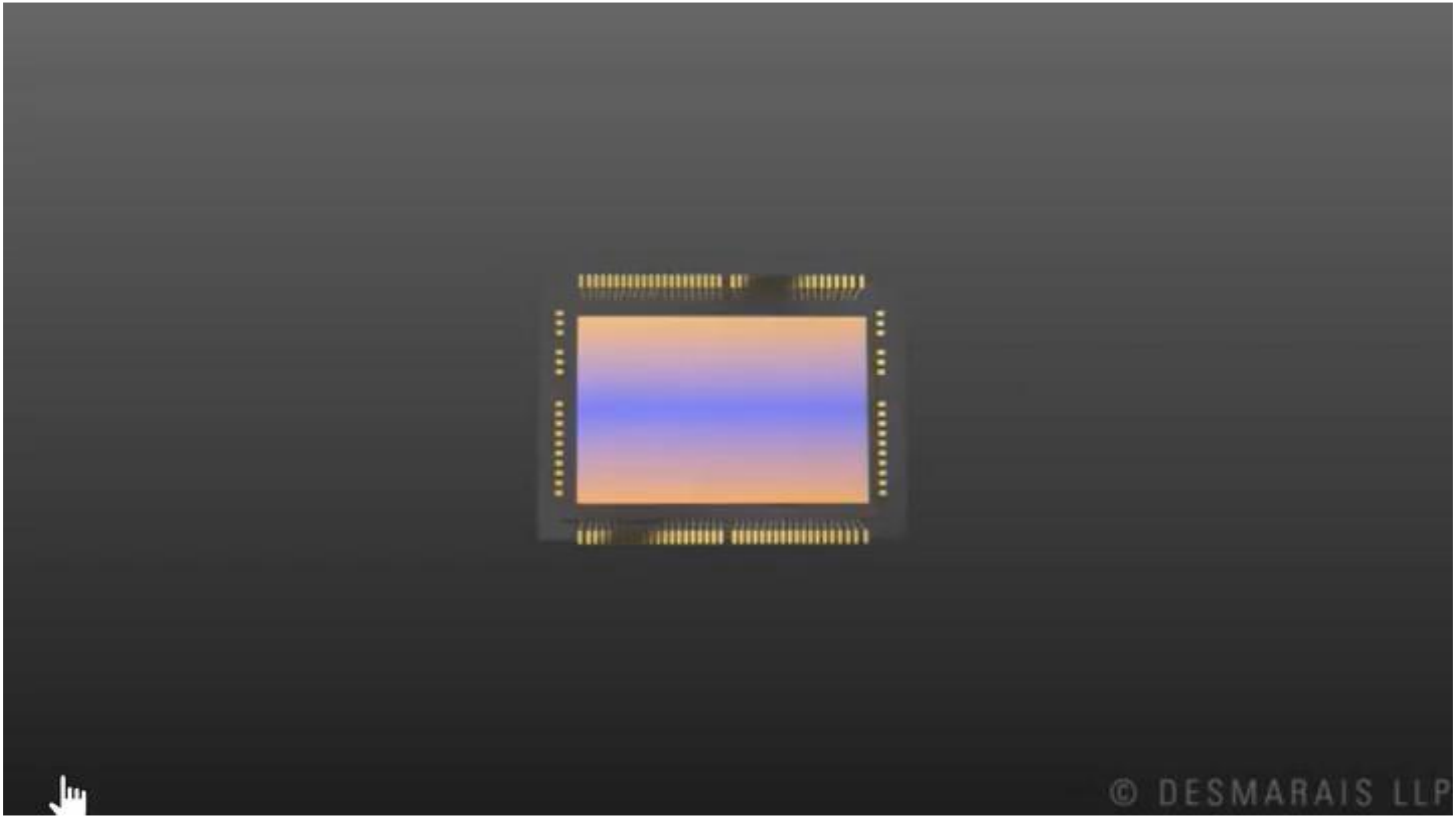






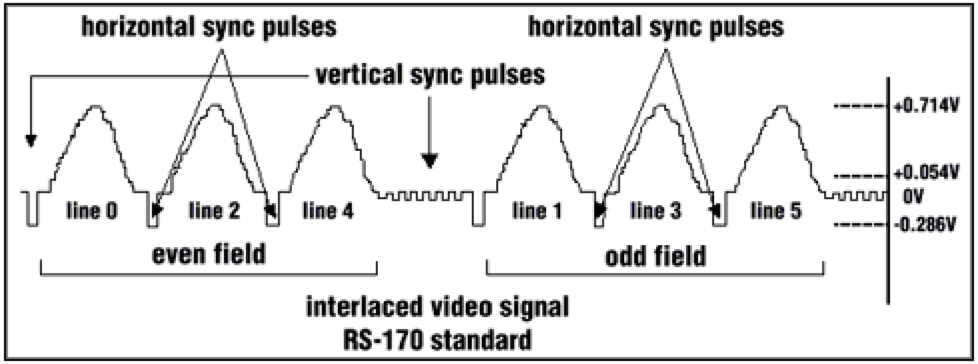
Pixel = Picture Element

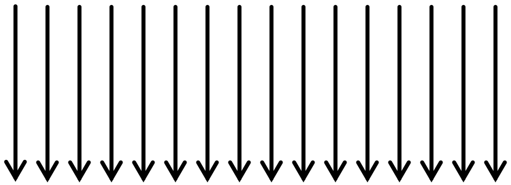
Example of Image Acquisition



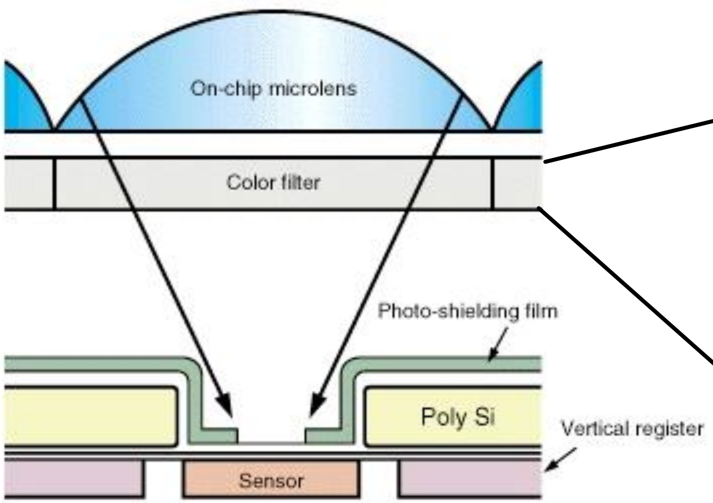
Further Illustration …

One Dimensional Time Signals
Which Consists of a Series of Line Signals

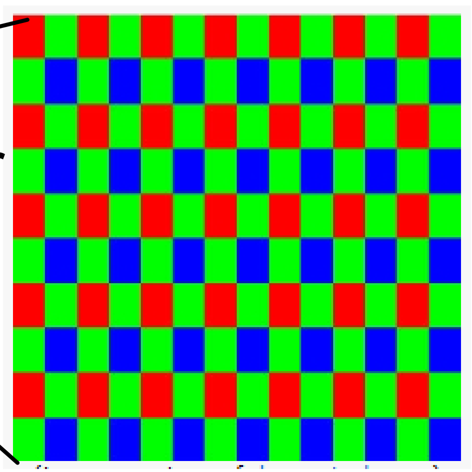




Light Rays

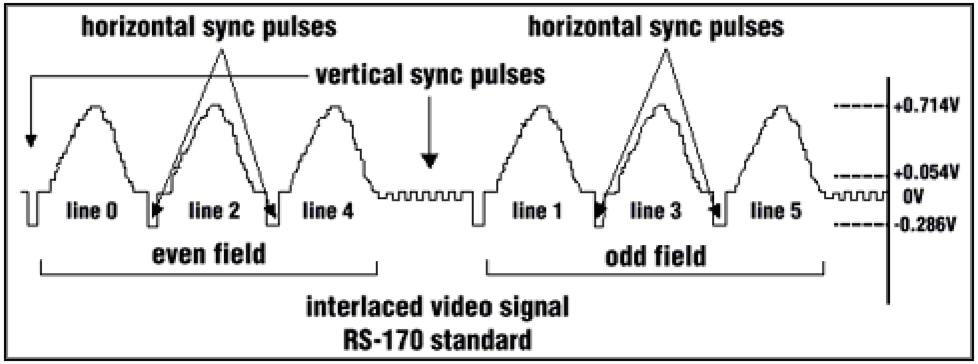


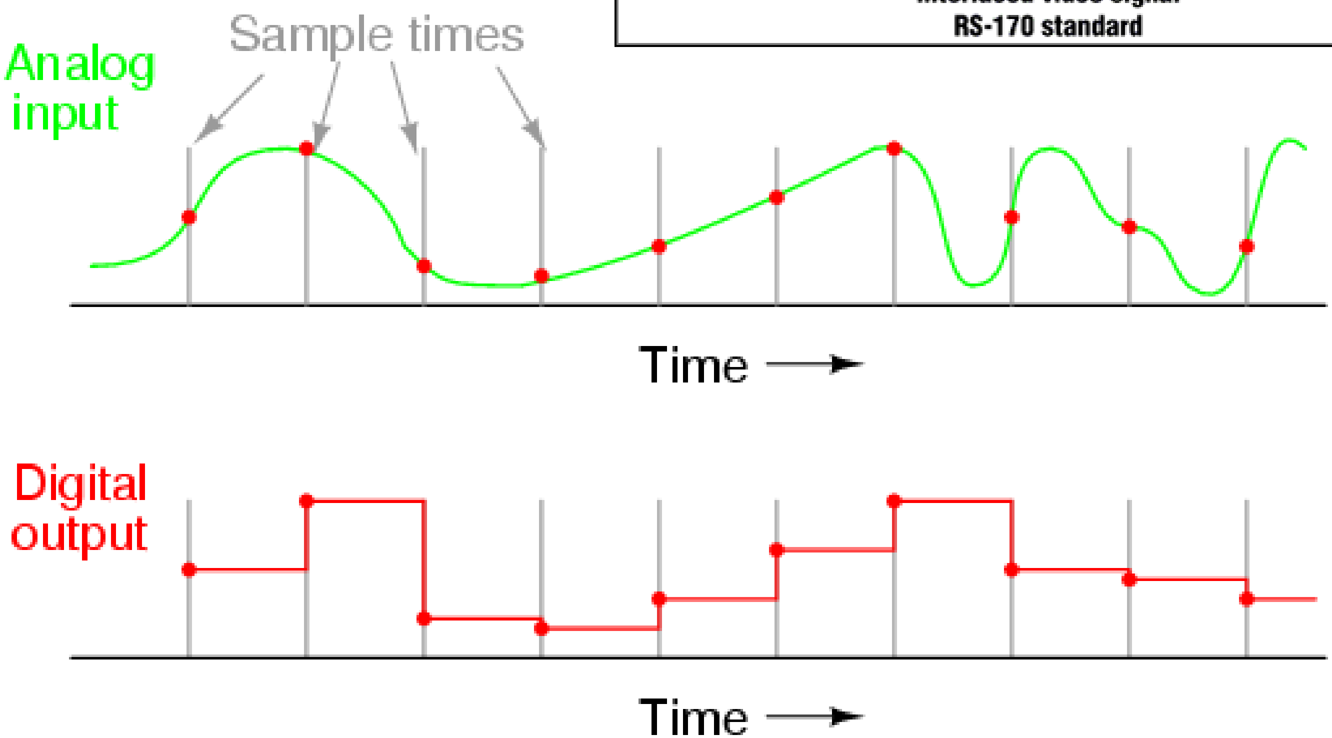
Color Filter



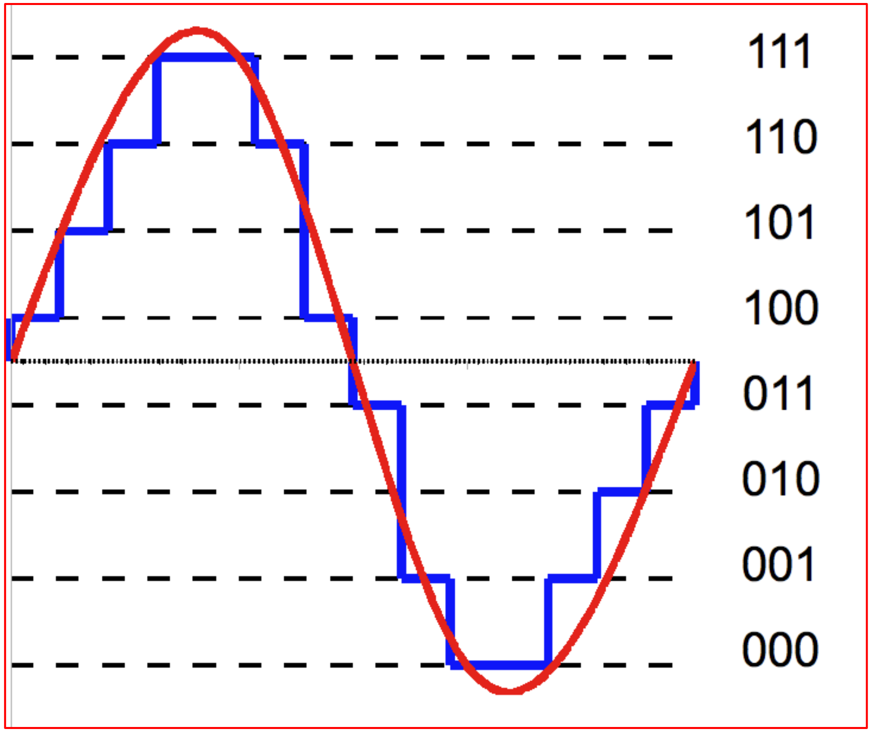
Two Dimensional Spatial Data (Matrix of Image)

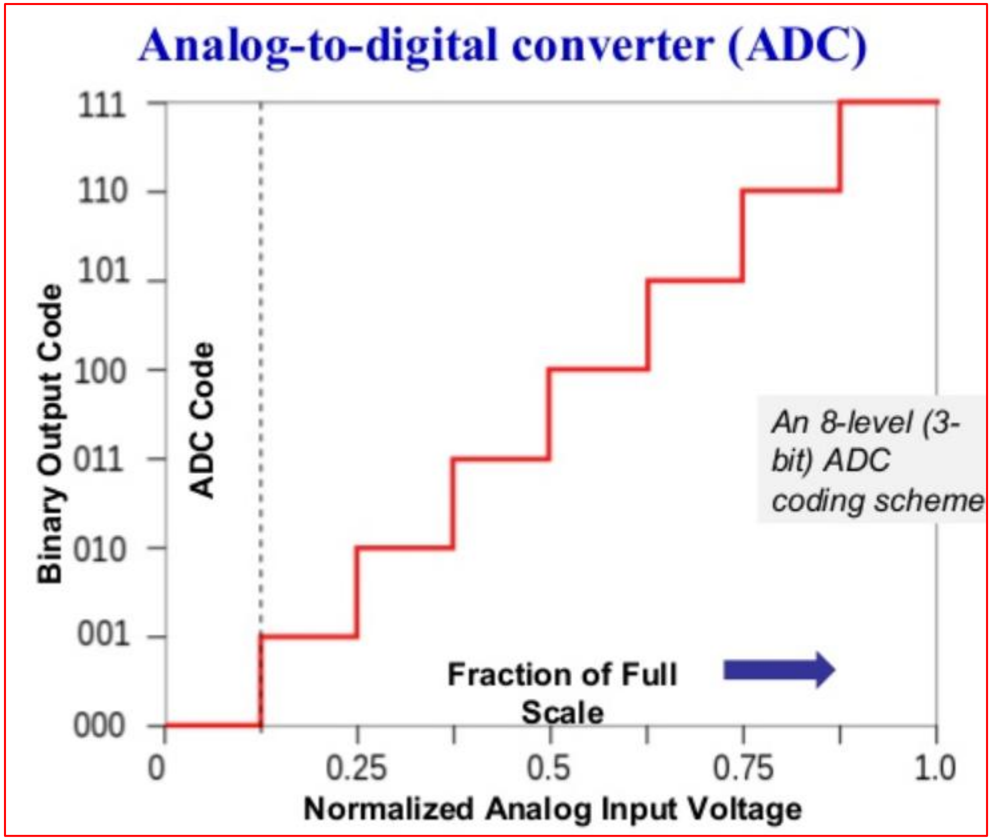
Step 4: From Voltage Signals to Digital Image



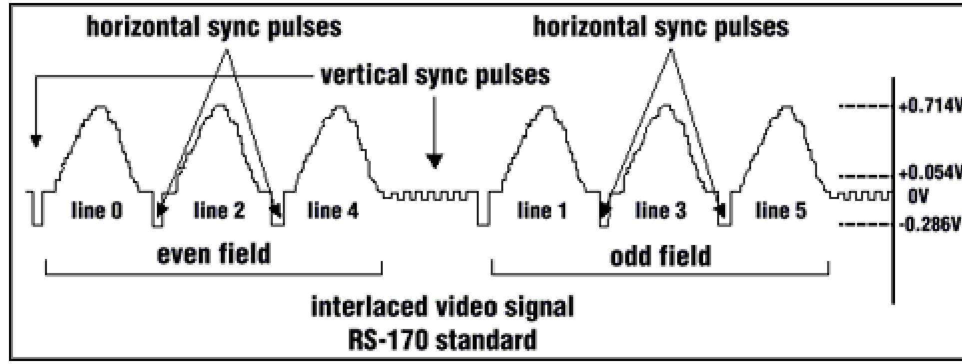


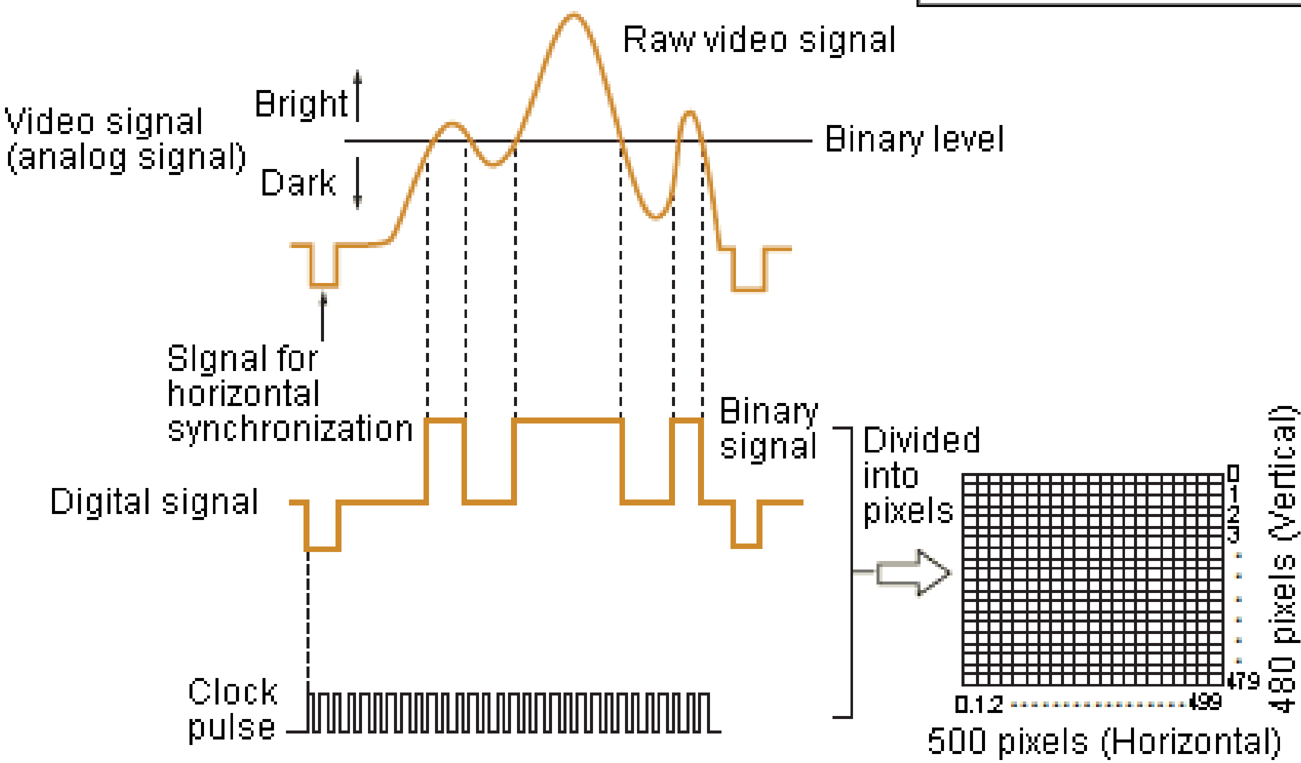
Step 4: (continued)





Example of Quantization

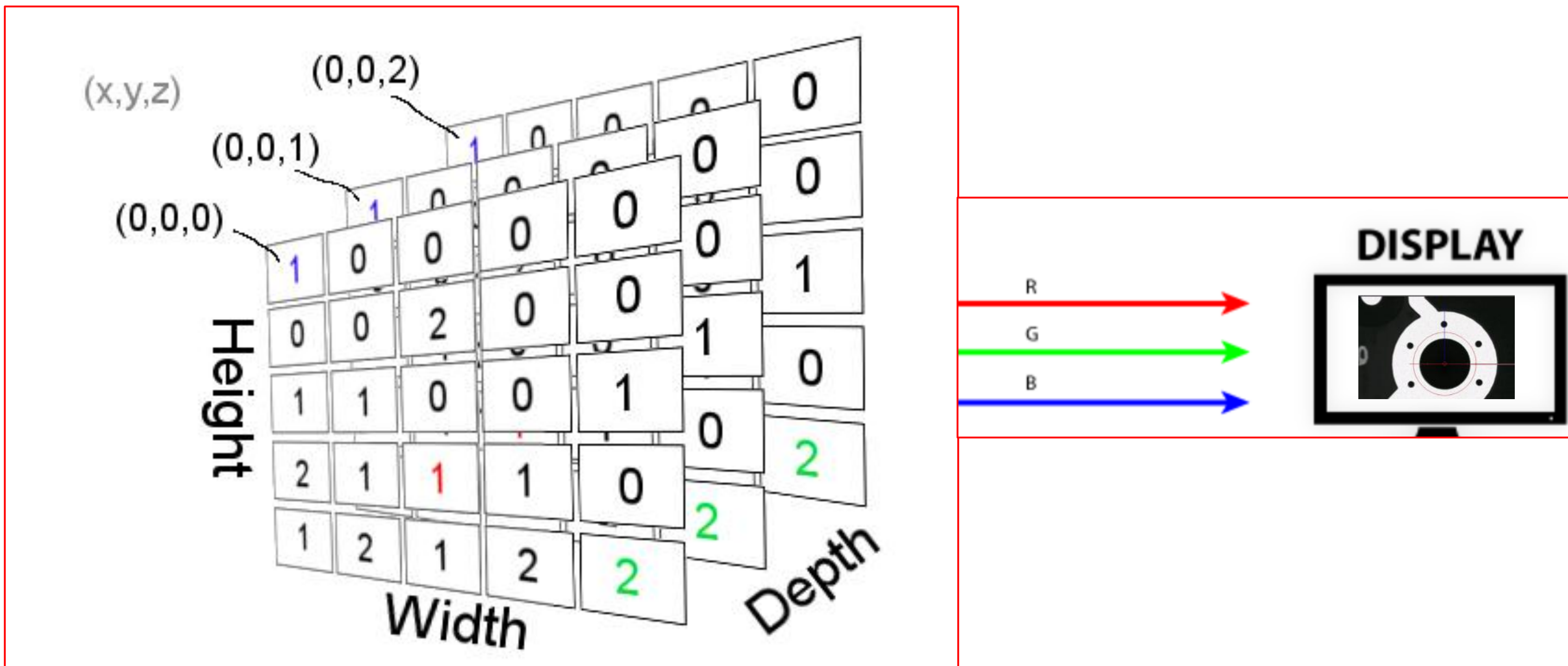




Byte image[480][500];

Step 5: From Digital Image to Display

- Output from digitization of visual signals is a series of three images: red component image, green component image and blue component image. These images can be displayed.



Example of Pixel Values of Specific Colour Effects

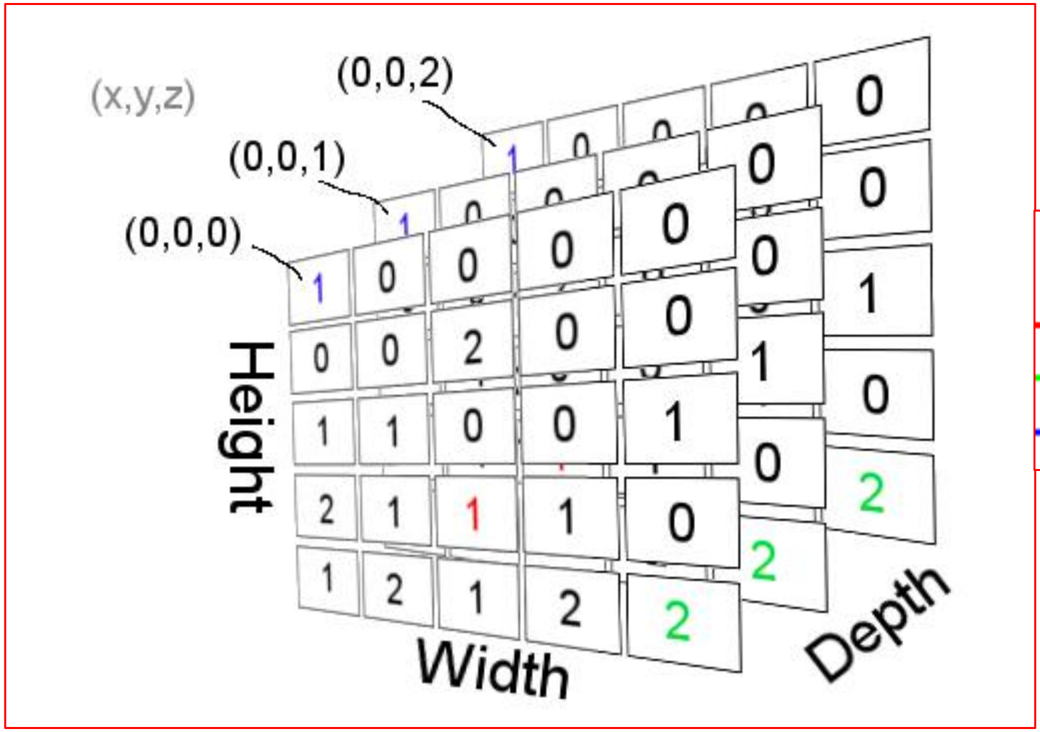
R	G	B	
255	0	0	FF0000
255	127	0	FF7F00
255	255	0	FFFF00
127	255	0	7FFF00
0	255	0	00FF00
0	255	127	00FF7F
0	255	255	00FFFF
0	127	255	007FFF
0	0	255	0000FF
127	0	255	7F00FF
255	0	255	FF00FF
255	0	127	FF007F
0	0	0	000000

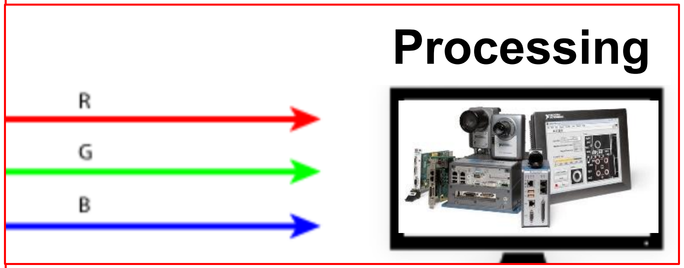
R	G	B	
240	160	160	F0A0A0
220	180	140	DCB48C
200	200	120	C8C878
180	220	140	B4DC8C
160	240	160	A0F0A0
140	220	180	8CDCB4
120	200	200	78C8C8
140	180	220	8CB4DC
160	160	240	A0A0F0
180	140	220	B48CDC
200	120	200	C878C8
220	140	180	DC8CB4
160	160	160	A0A0A0

R	G	B	
238	170	170	EEAAAA
221	187	153	DDBB99
204	204	136	CCCC88
187	221	153	BBDD99
170	238	170	AAEEAA
153	221	187	99DDBB
136	204	204	88CCCC
153	187	221	99BBDD
170	170	238	AAAAEE
187	153	221	BB99DD
204	136	204	CC88CC
221	153	187	DD99BB
170	170	170	AAAAAA

Step 6: From Digital Image to Image Processing

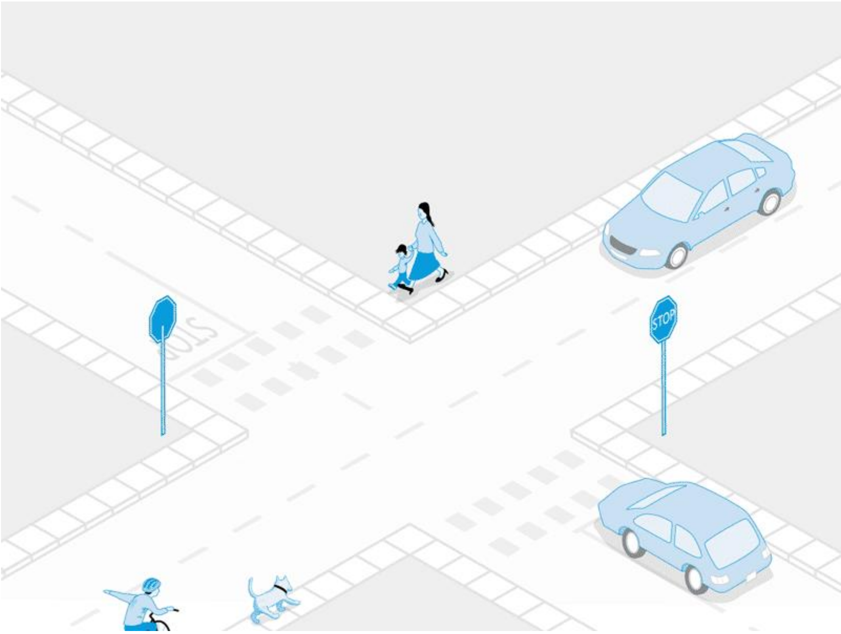
- Output from digitization of visual signals is a series of three images: red component image, green component image and blue component image. These images can be processed.



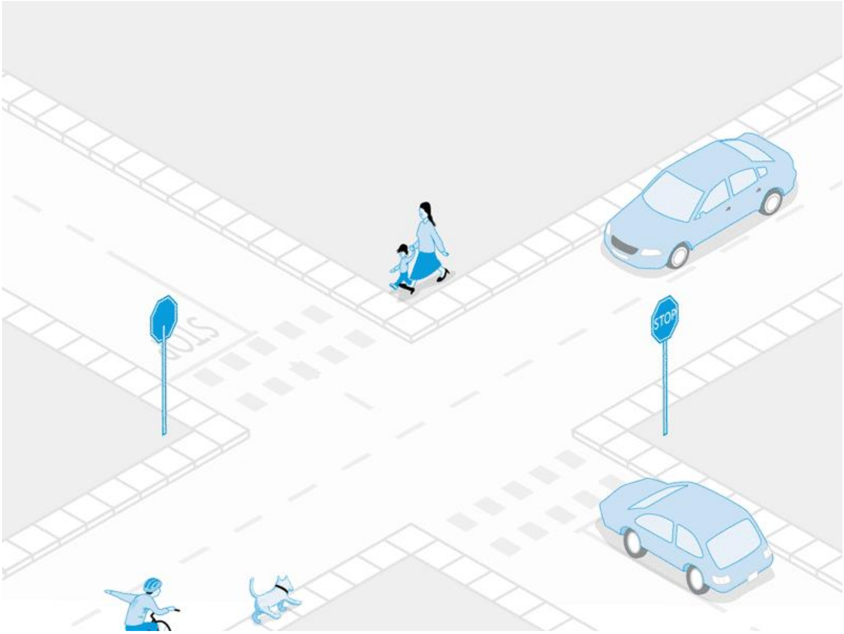


Summary of Lecture 1

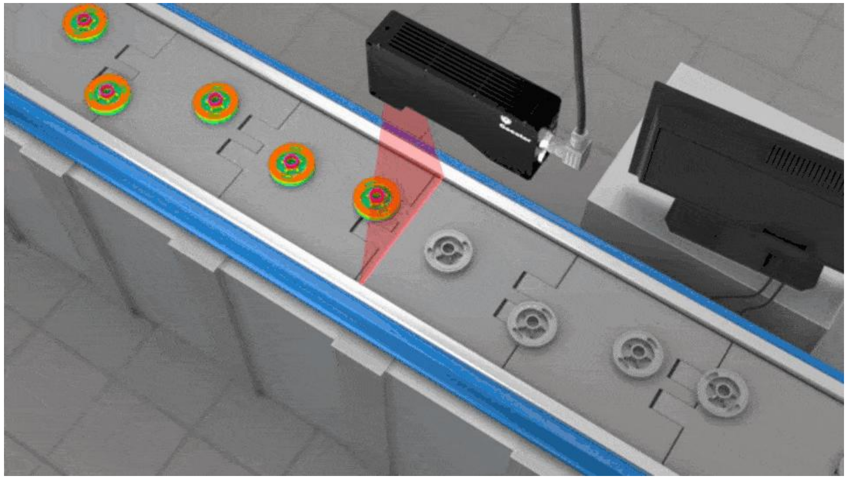
- Basics of Sensing and Measurement
- Basics of Visual Signals
- Parameter(s) of Visual Signals
- Measurement of Photometry



Outline of Module 2



- Perception of Photometry
- Perception of 2D Geometry
- Perception of 3D Geometry



School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 2

MA4825 Robotics

Lecture 2

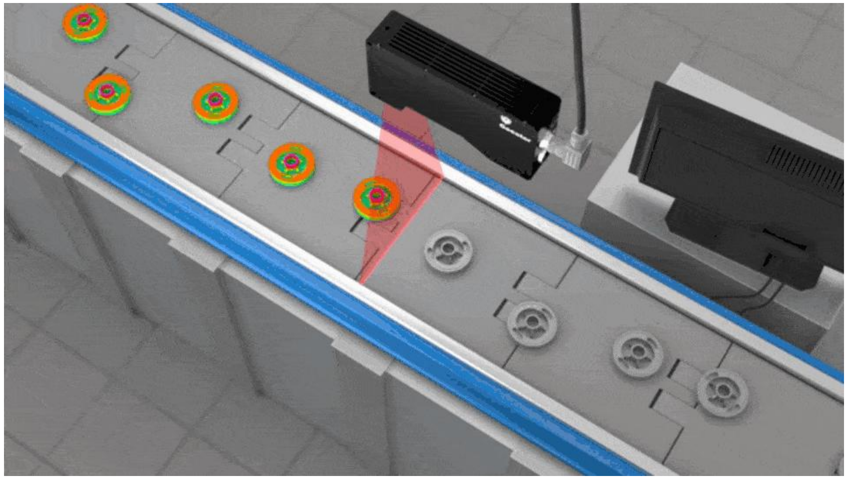
Perception of 2D Geometry

Xie Ming, PhD (France)

http://personal.ntu.edu.sg/mmxie

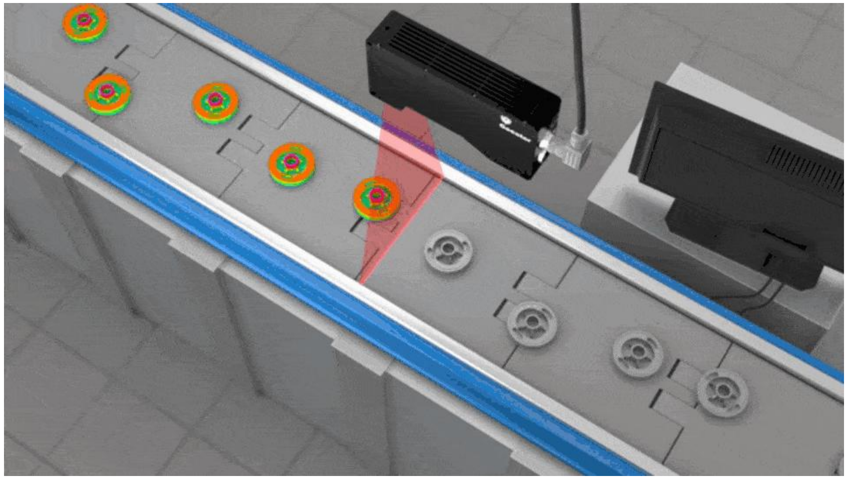
Outline of Lecture 2

- Basics of 2D Geometry
- Parameters of 2D Geometry
- Measurement of 2D Geometry



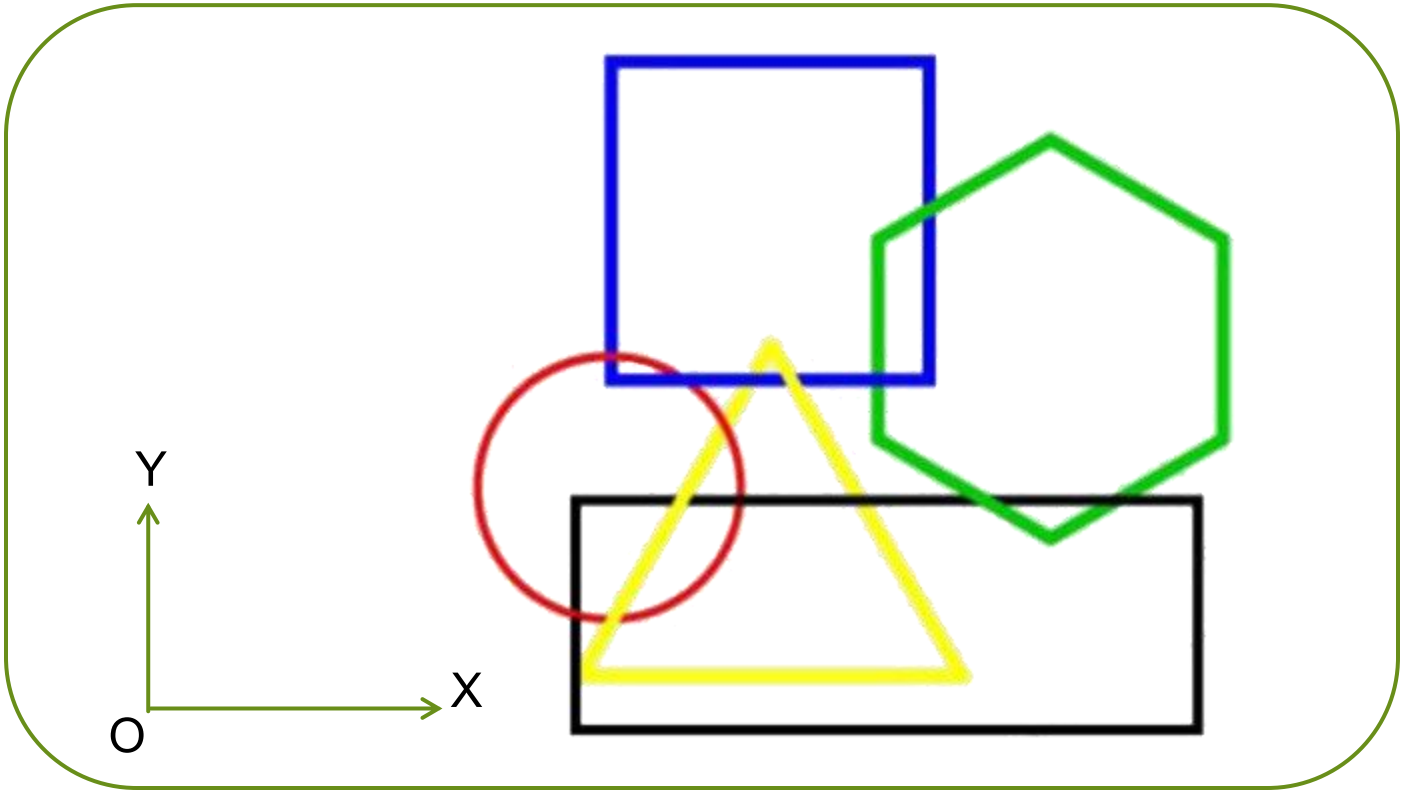
Outline of Lecture 2

- Basics of 2D Geometry
- Parameters of 2D Geometry
- Measurement of 2D Geometry



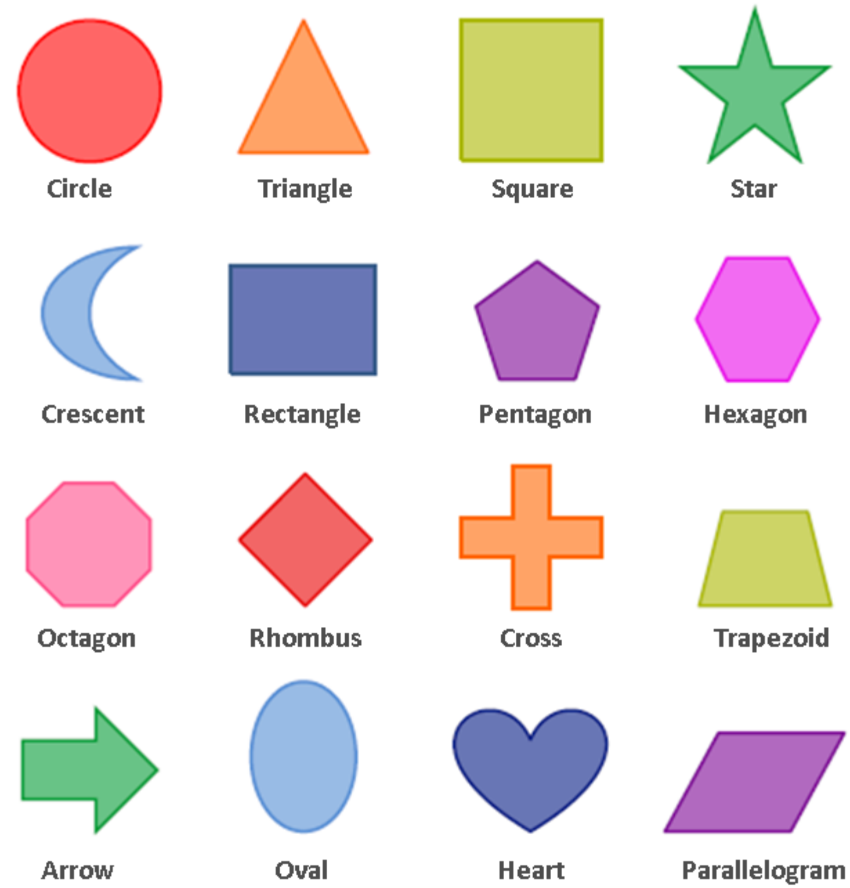
Understanding 2D Geometry (1)

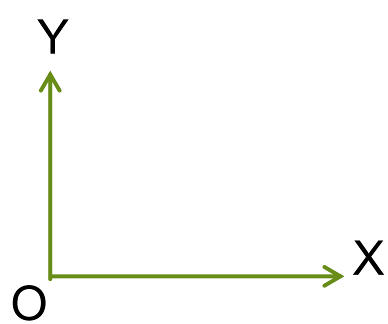
- 2D geometry refers to the appearance of physical entities in a two-dimensional space.
- 2D space consists of a set of positions which are fully determined with two coordinates.



Understanding 2D Geometry (2)

- The appearance of physical entities in a 2D space is manifested in the form of shapes.



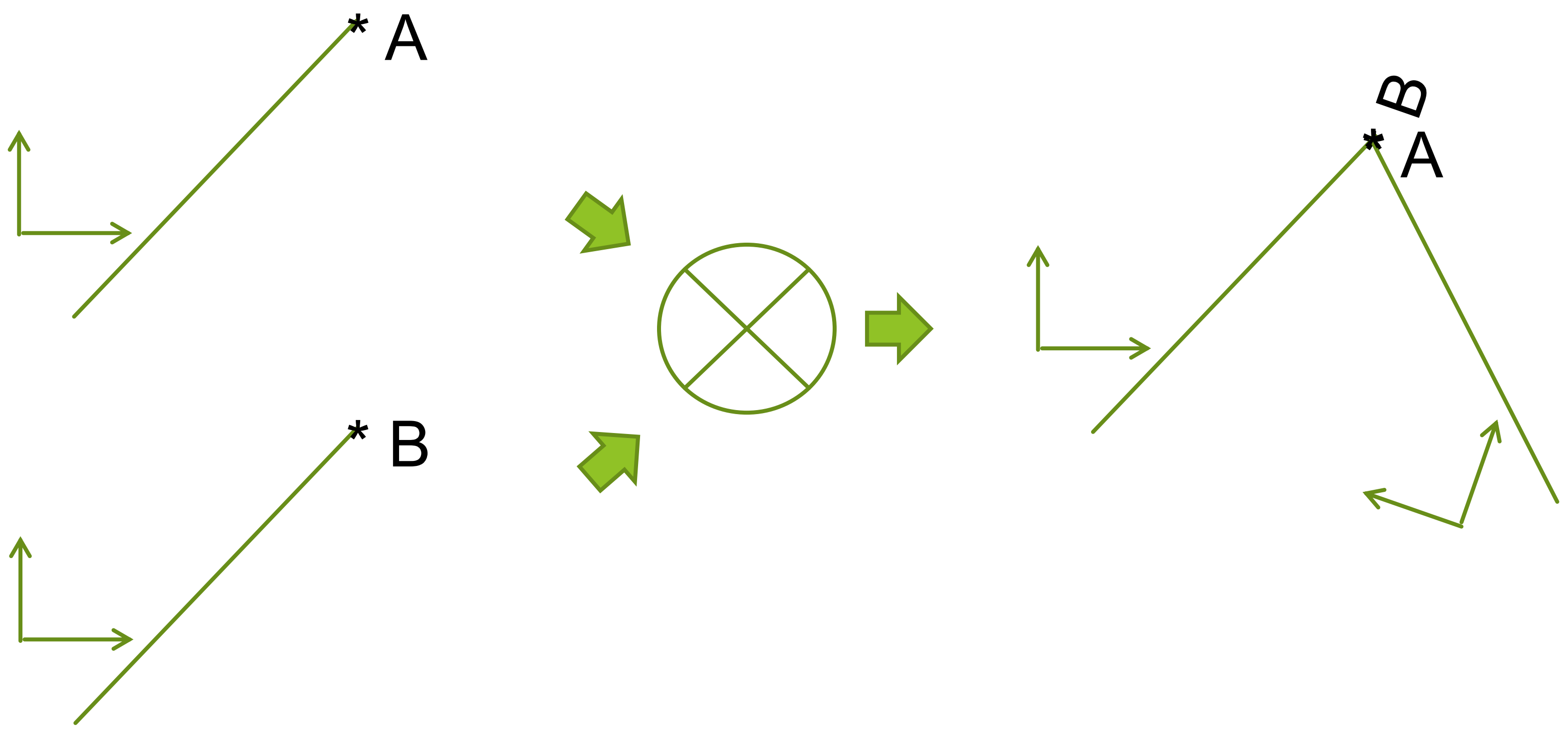


Understanding 2D Geometry (3)

- Complex shapes in a 2D space are the results of compositional rules such as:
  - Connect Between Points(point of shape 1, point of shape 2) at Angle(angle between shape 1 and shape 2):
    - ConnectPointsAtAngle(point, point, angle)
  - Connect Between Curves(curve of shape 1, curve of shape 2) with Offset(distance between the endpoints of two curves)
    - ConnectCurvesAtOffset(curve, curve, offset)

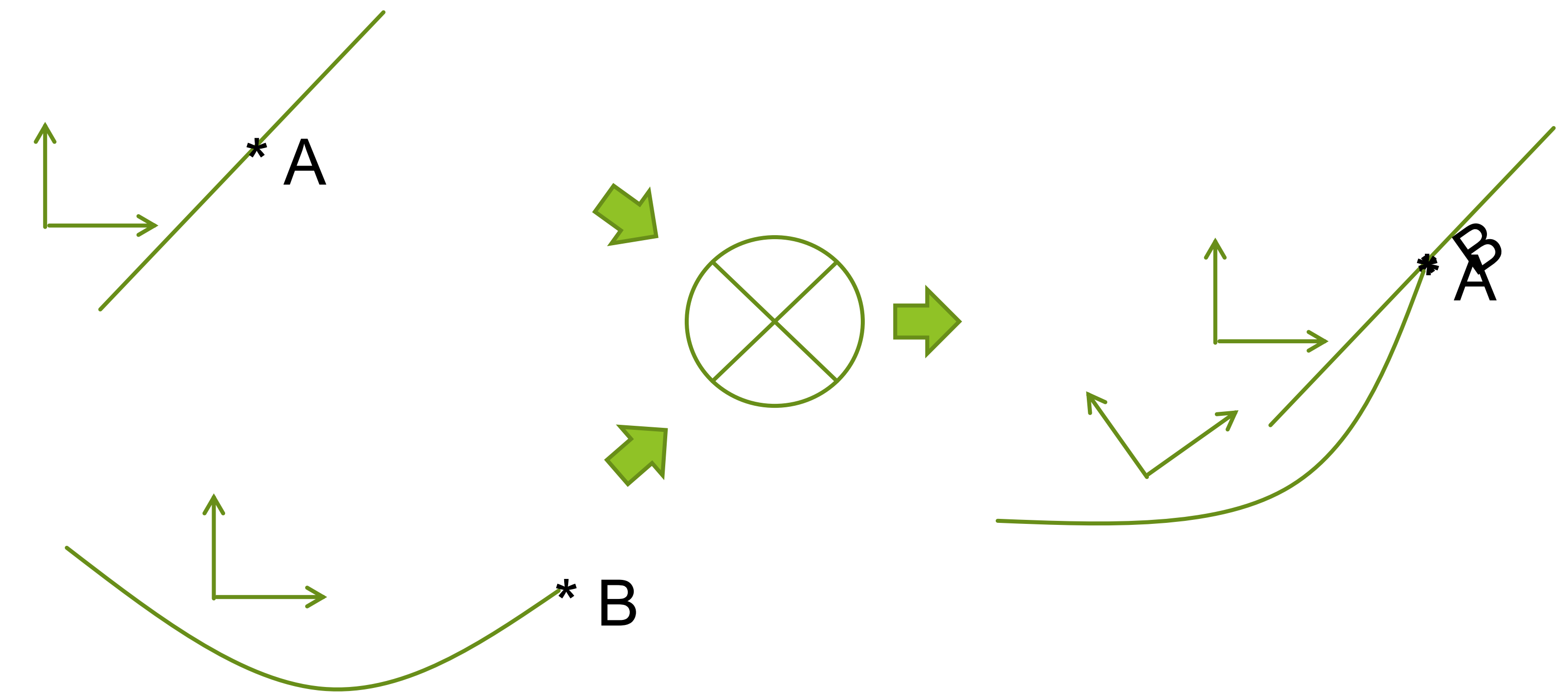
Example

- Connect Between Points(A, B) with Angle(60°)



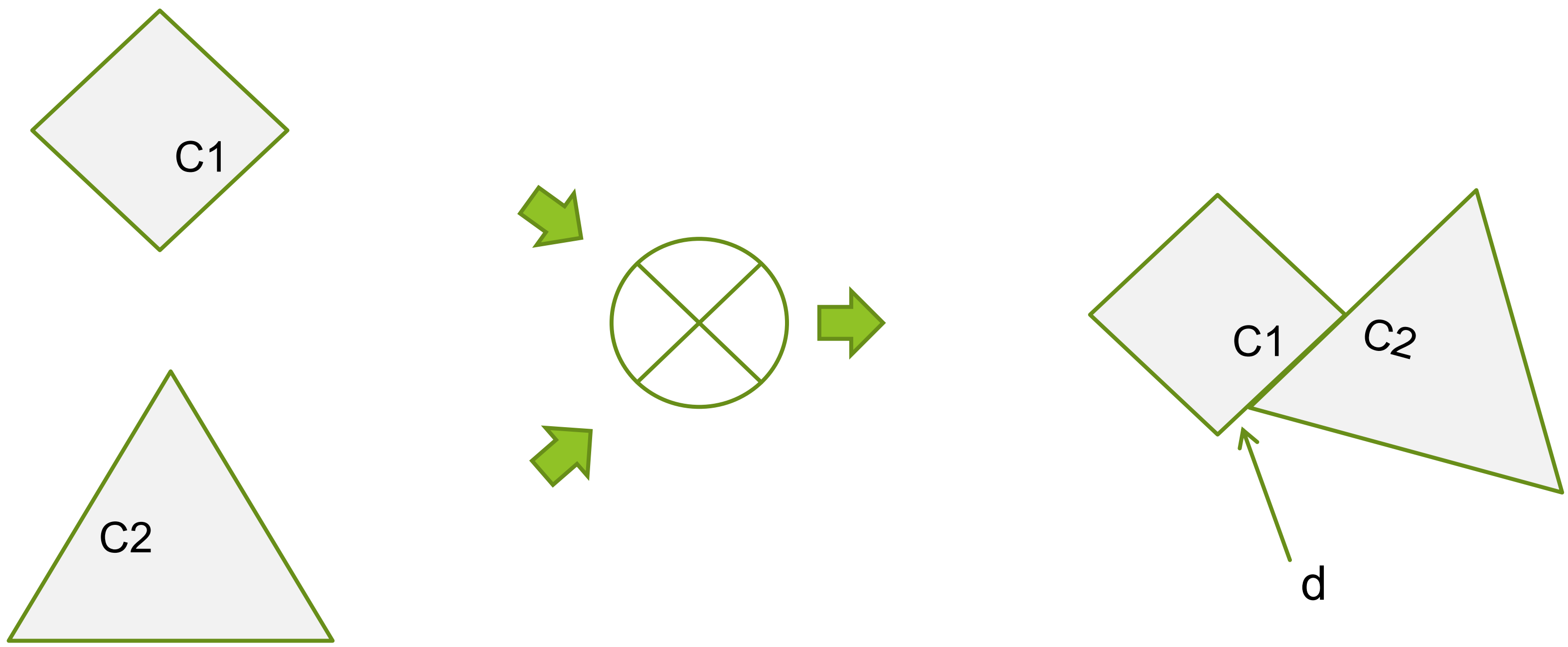
Example

- Connect Between Points(A, B) with Angle(45°)



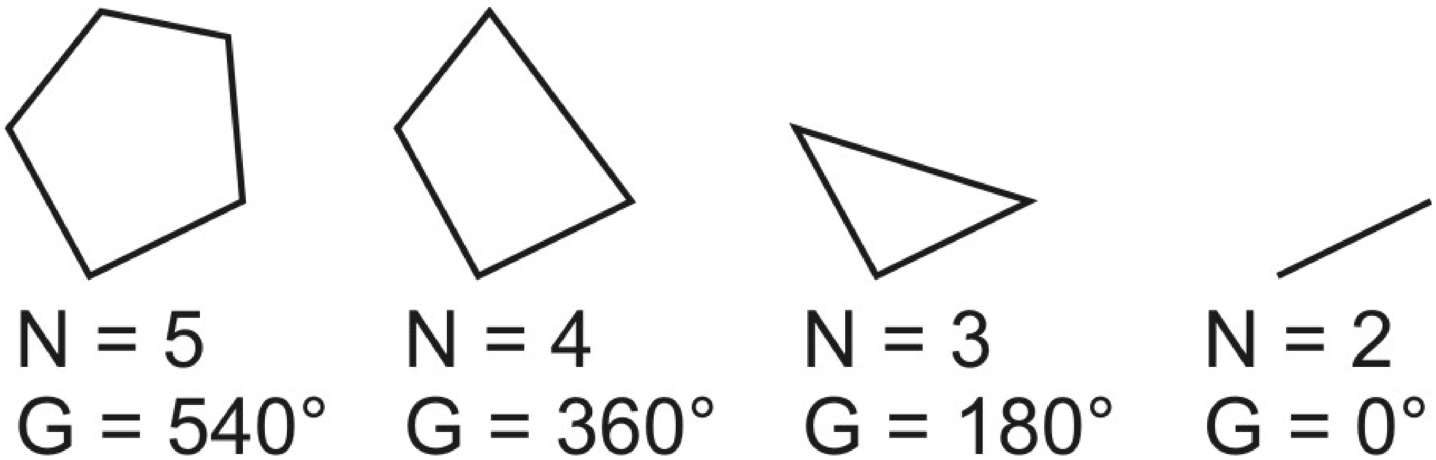
Example

- Connect Between Lines(C1, C2) with Offset(d)



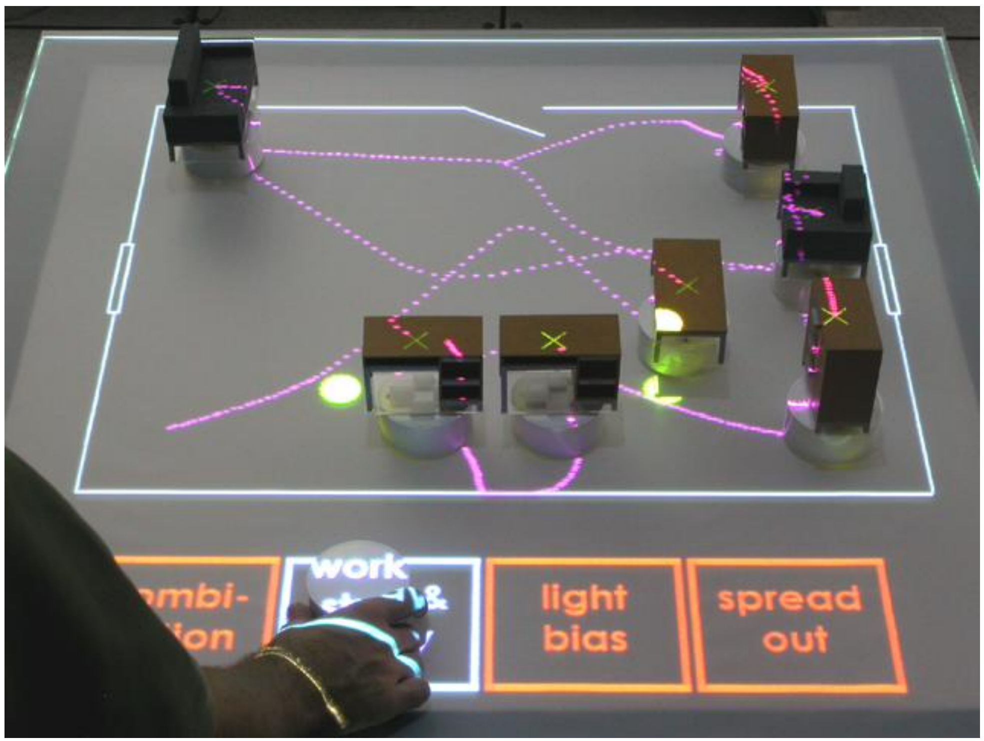
Understanding 2D Geometry (4)

- The sum of inter angles of a 2D polygon is equal to (N-2) x 180 degrees.



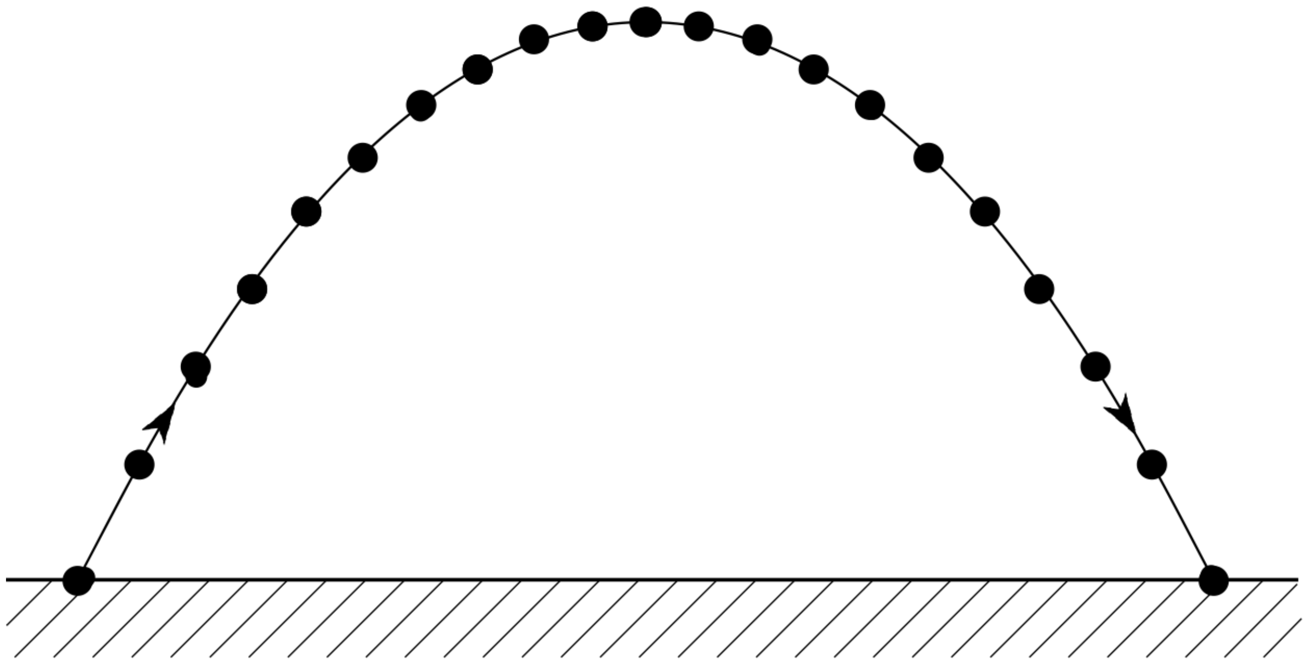
Understanding 2D Geometry (5)

- The appearance of physical entities in a 2D space also includes the travelled locations.

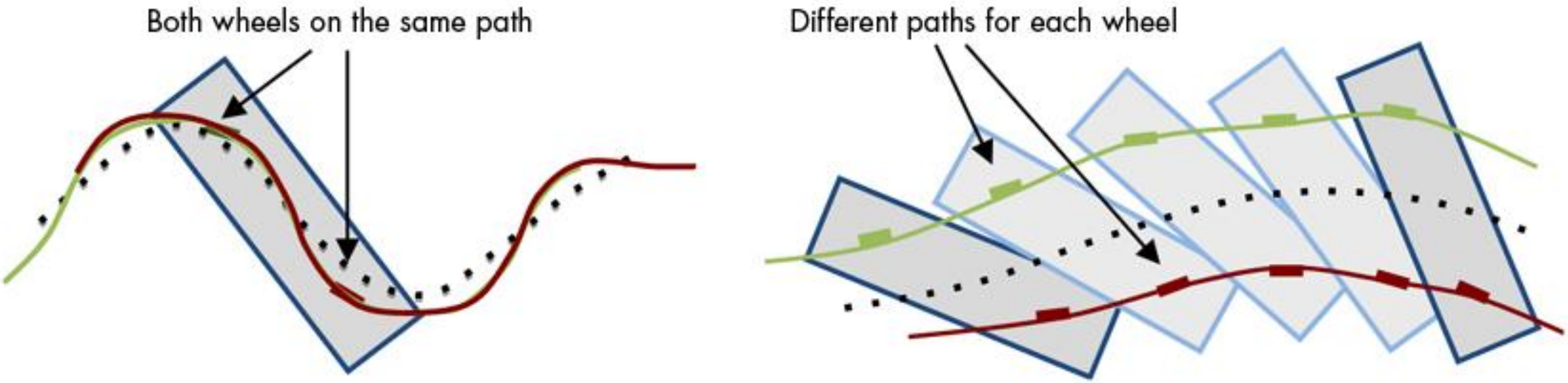


Understanding 2D Geometry (6)

- The spatial locations travelled or to be travelled are called paths.

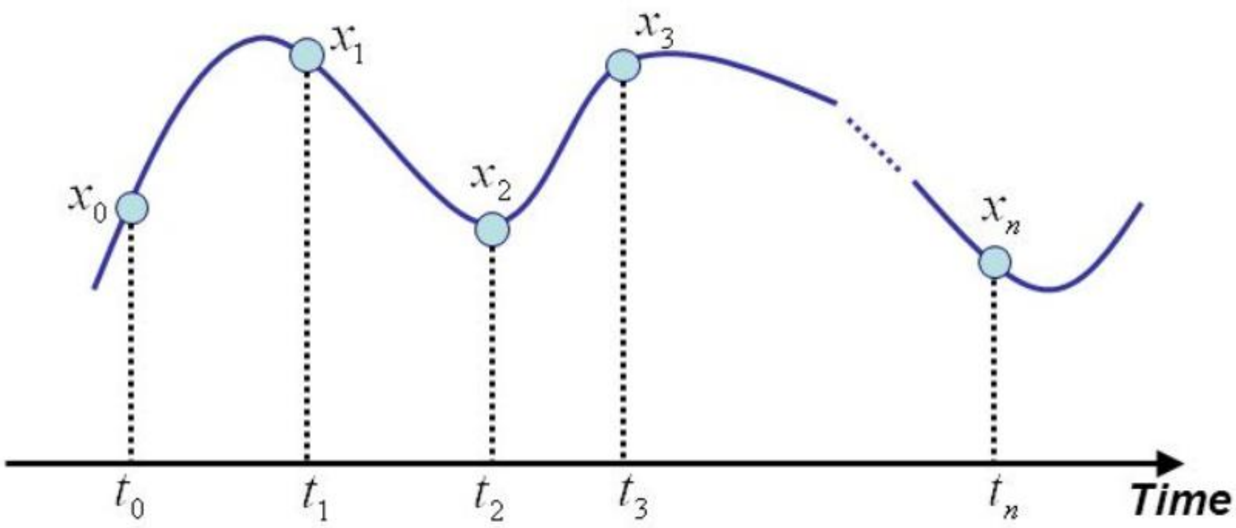


Example of Path

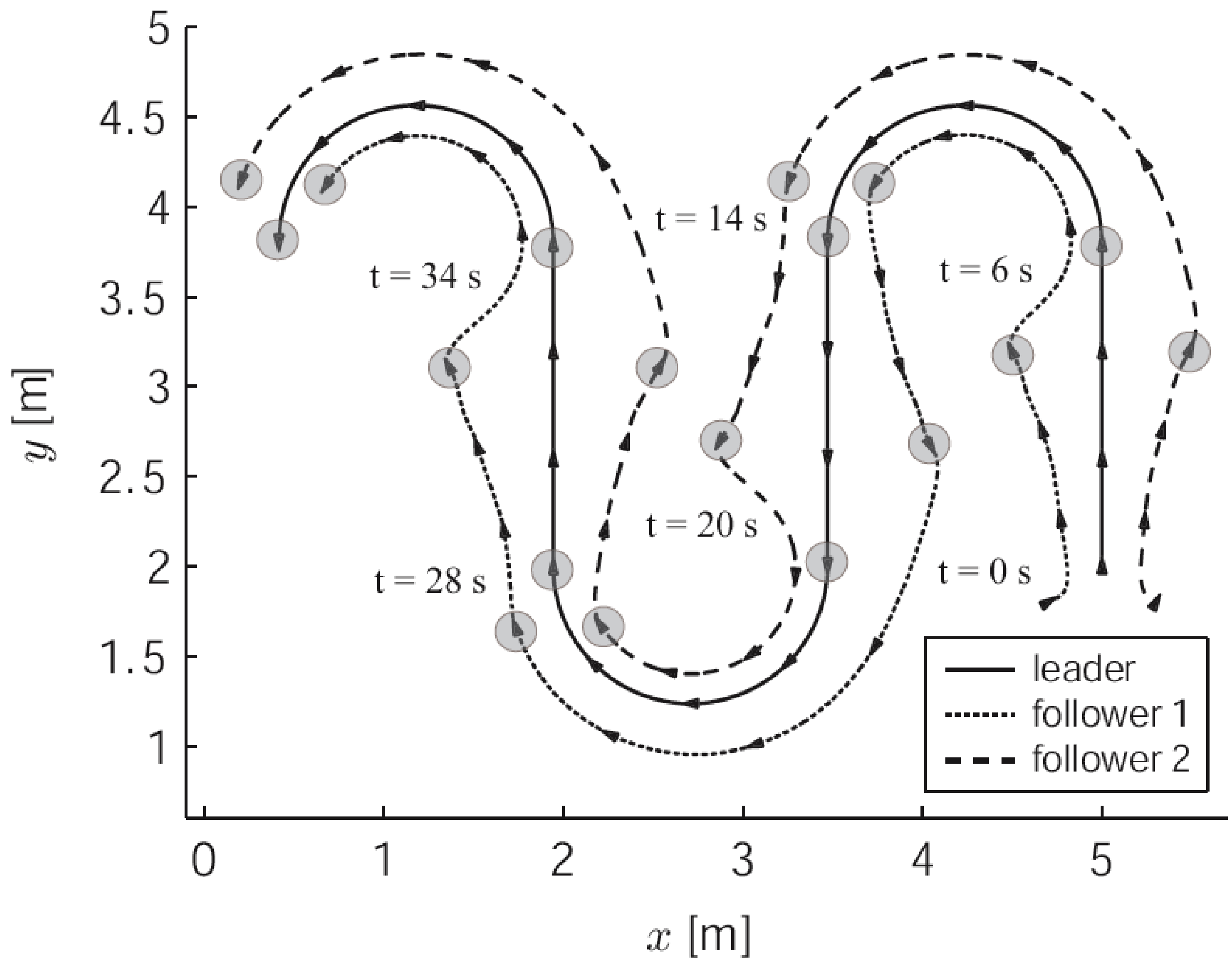


Understanding 2D Geometry (7)

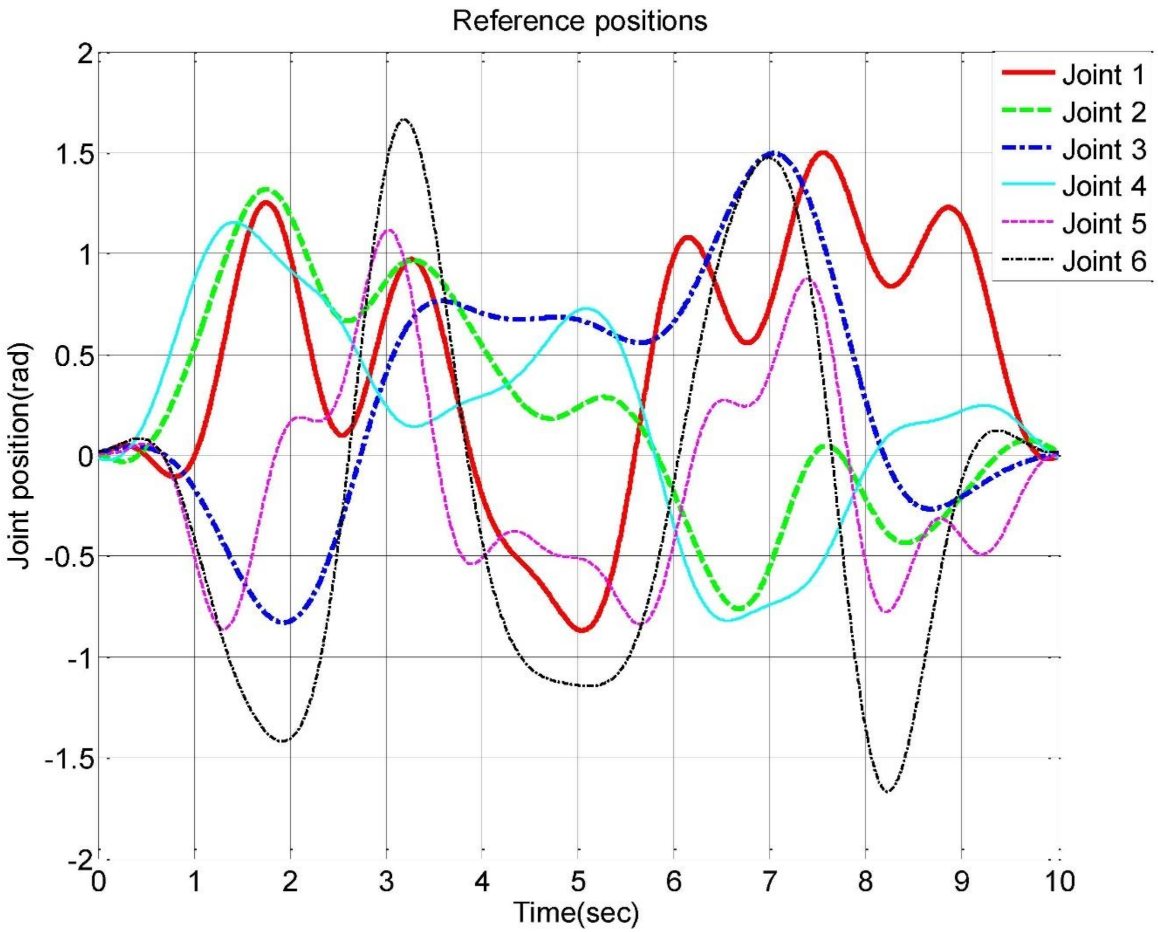
- The spatial locations with time constraint are called trajectories.



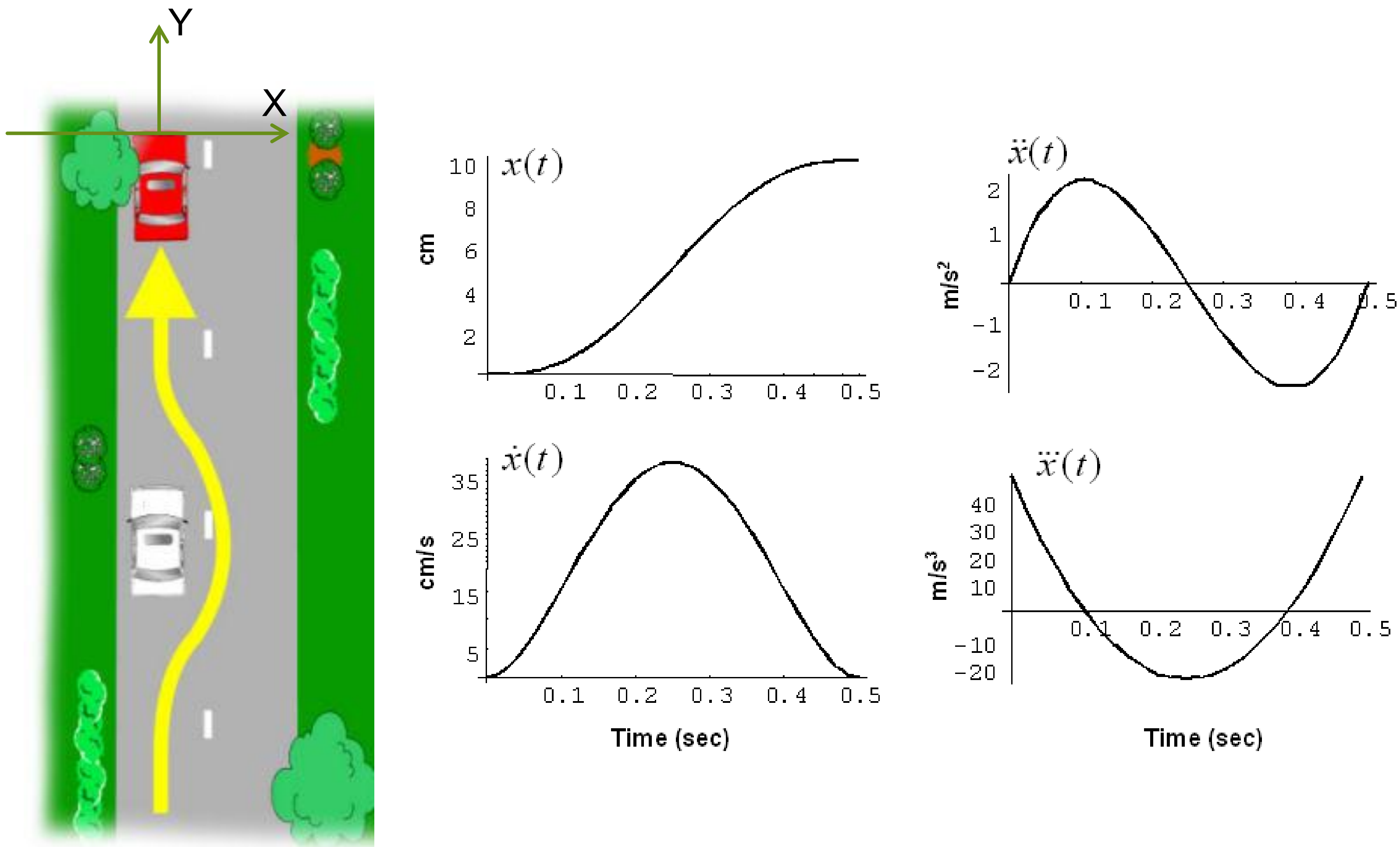
Example of Trajectories



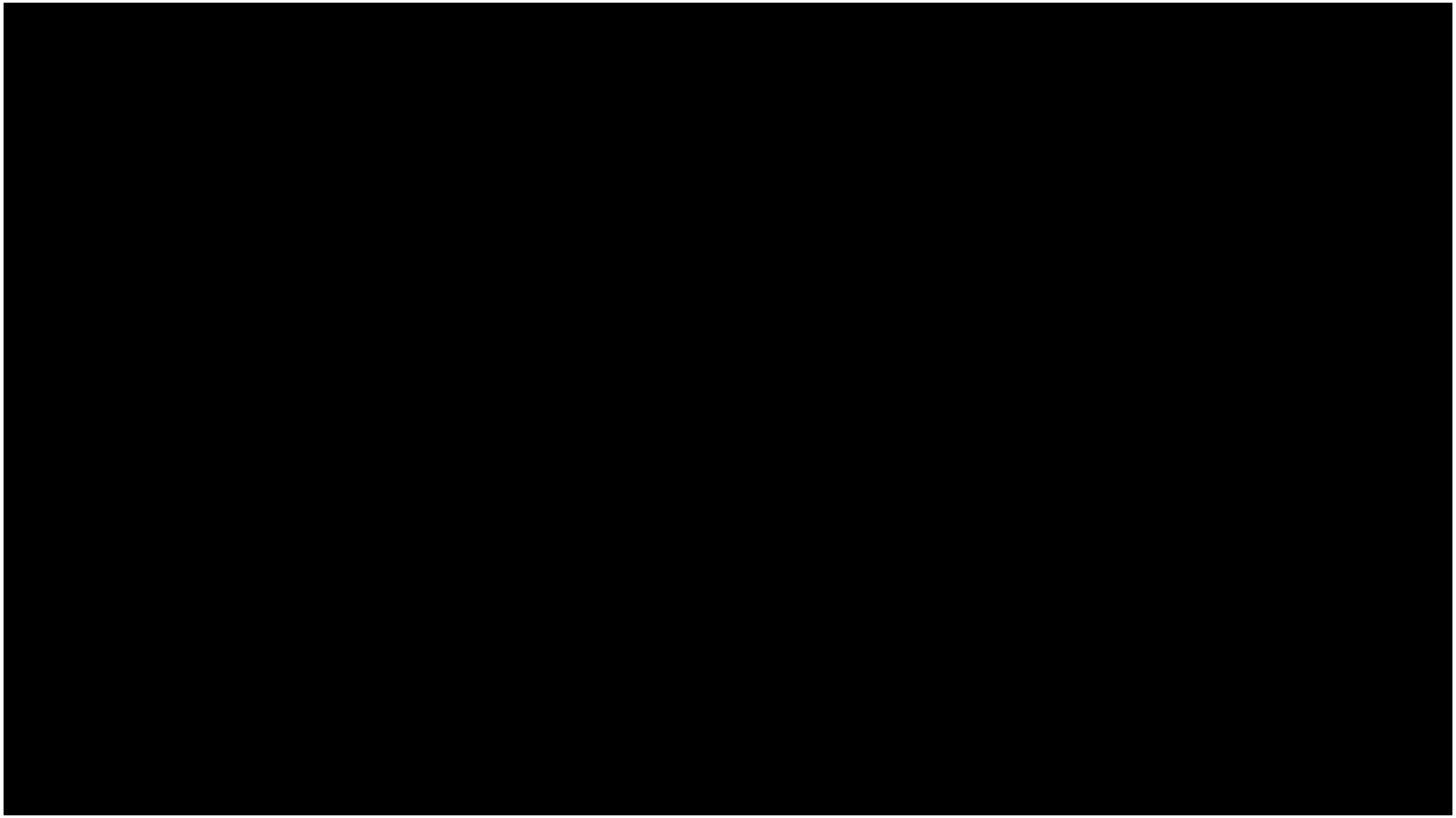
Example of Trajectories of Angular Positions



Example of Motion Planning and Control



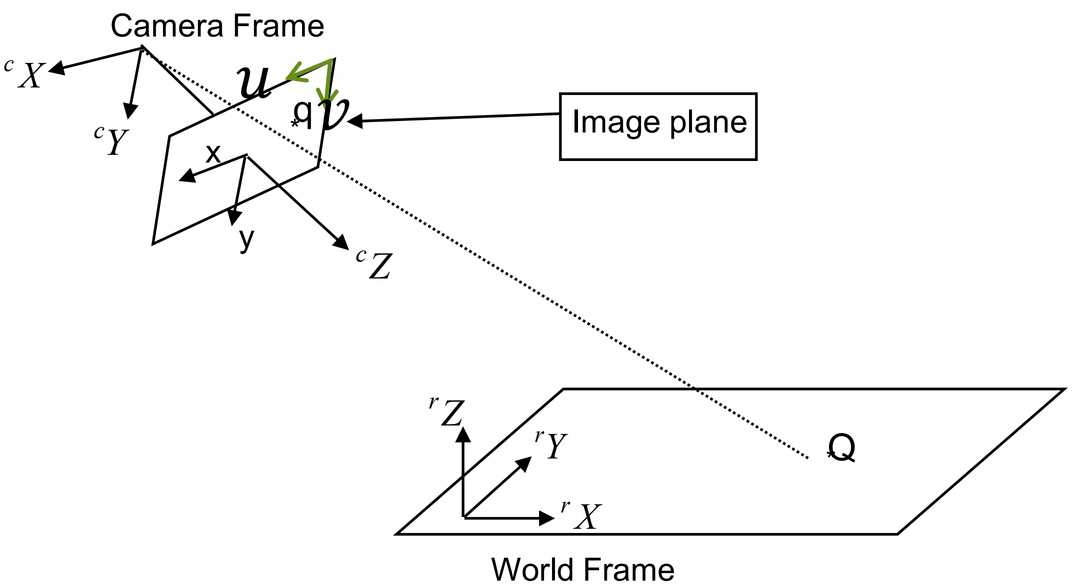
What is the scenario of coordinate transformations in general?



Basics of Homogeneous Transformation

$$H_{camera} = \begin{bmatrix} R_{camera} & T_{camera} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

$$H_{world} = \begin{bmatrix} R_{camera}^{-1} & -R_{camera}^{-1} \times T_{camera} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$



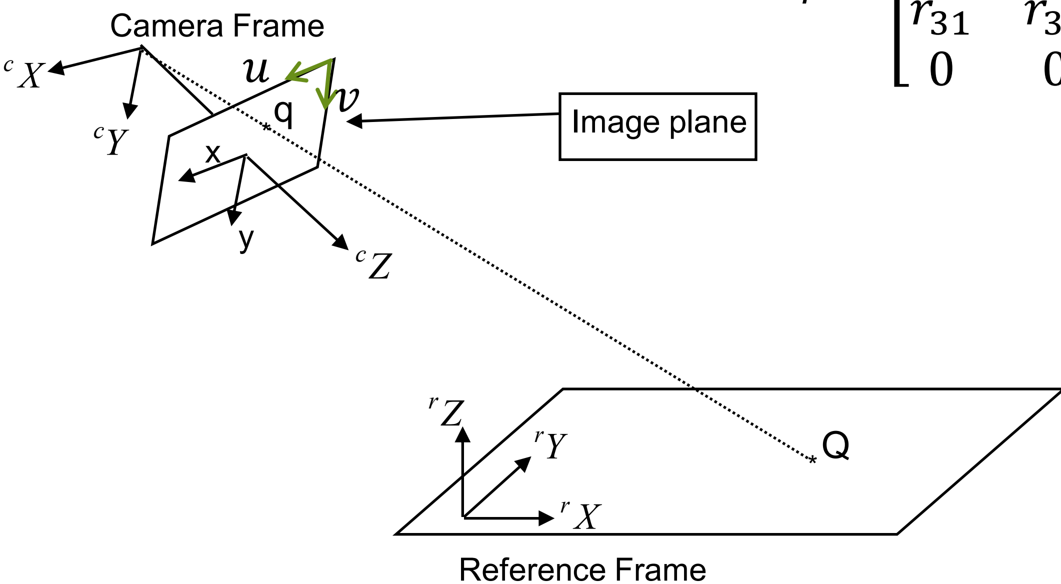
$$H_{camera} \times H_{world} = I_{4\times4}$$

$$H_{camera} = H_{world}^{-1}$$

Transformation of Coordinates in 3D Space

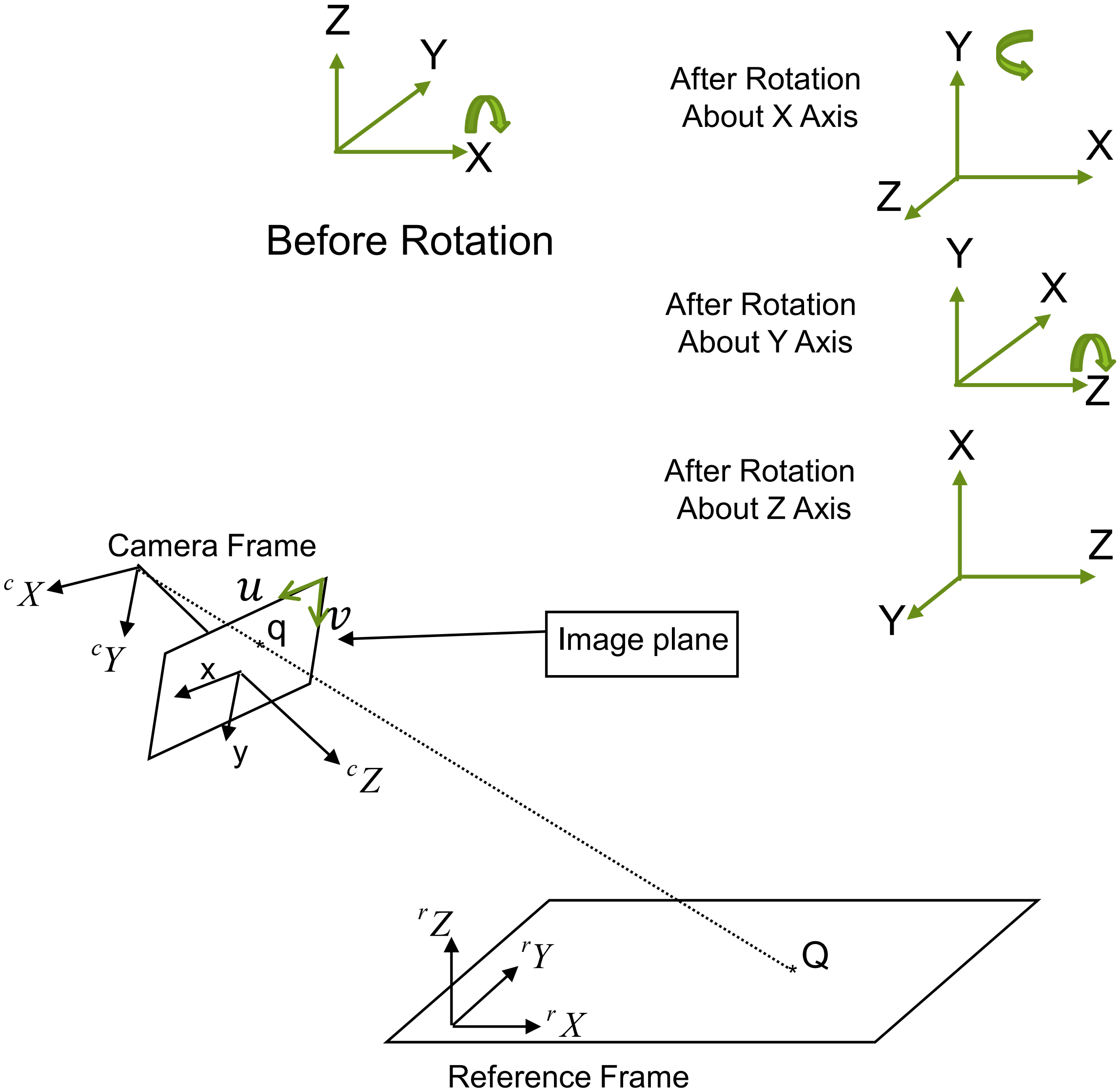
- The coordinates in the reference frame can be transformed into the coordinates in the camera frame.

$$ {}^{c}H_{r} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



$$ \begin{bmatrix} {}^{c}X \\ {}^{c}Y \\ {}^{c}Z \\ 1 \end{bmatrix} = {}^{c}H_{r} \bullet \begin{bmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{bmatrix} $$

Practices with Rotational Transformation




```matlab
ax = 90*pi/180;
rx = [1    0          0 ;
      0  cos(ax)   -sin(ax);
      0  sin(ax)    cos(ax)];

ay = 90*pi/180;
ry = [cos(ay)    0     sin(ay);
        0        1     0 ;
      -sin(ay)   0     cos(ay)];

az = 90*pi/180;
rz = [cos(az)    -sin(az)   0;
      sin(az)     cos(az)   0;
      0           0         1];

r = rx*ry*rz
```

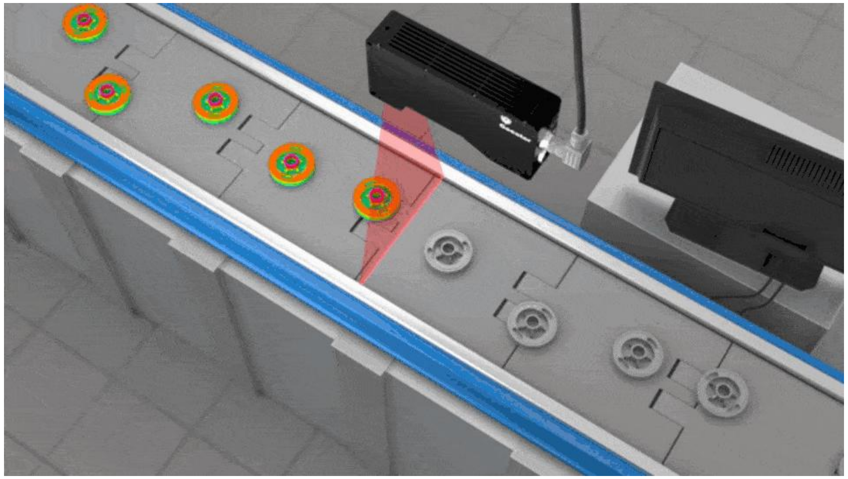
Command Window

New to MATLAB? See resources for Getting Started.

```
r =

    0.0000   -0.0000    1.0000
    0.0000   -1.0000   -0.0000
    1.0000    0.0000    0.0000
```

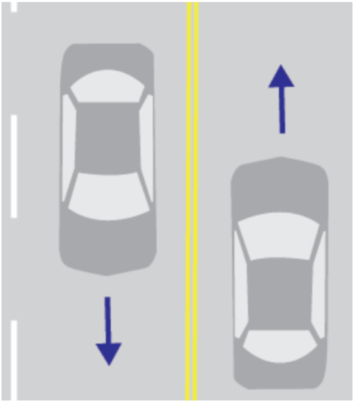
Outline of Lecture 2

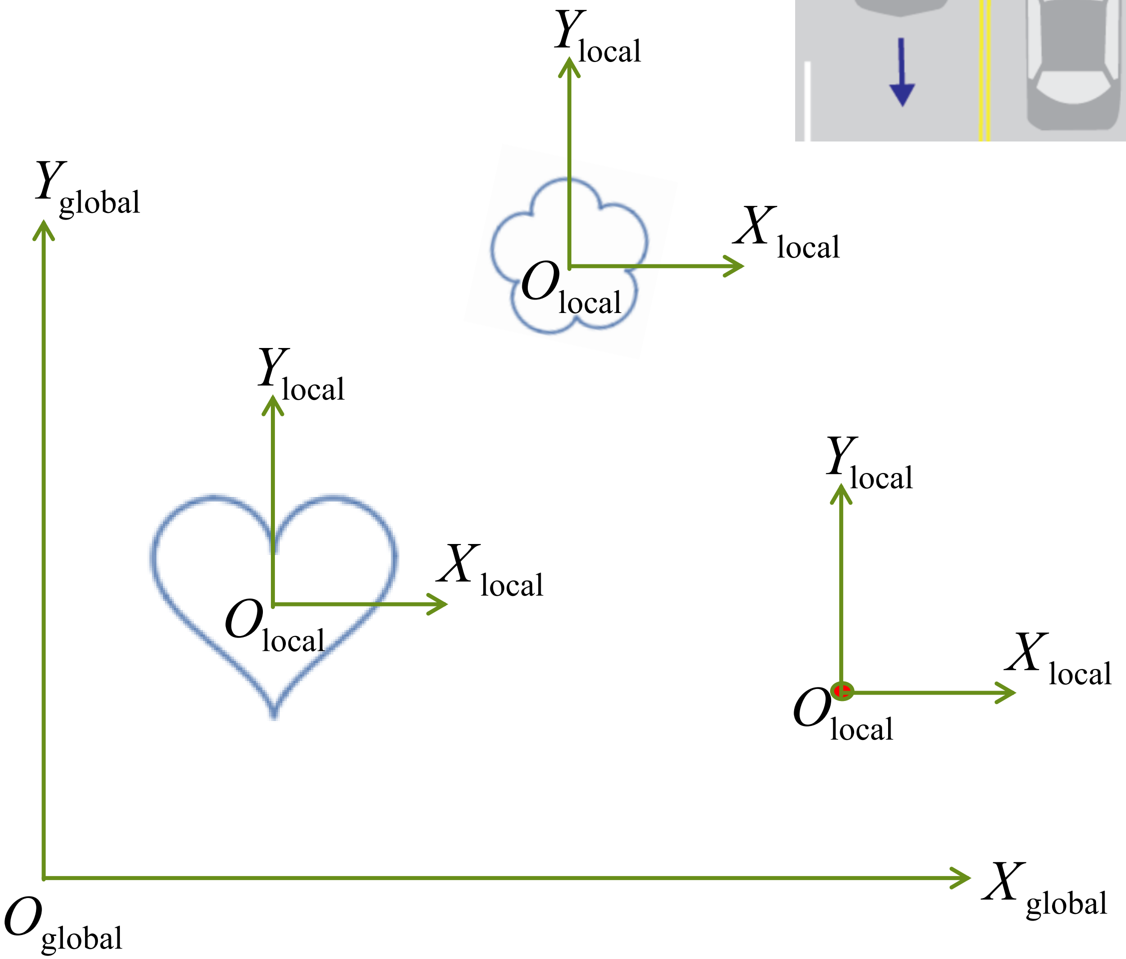
- Basics of 2D Geometry
- Parameters of 2D Geometry
- Measurement of 2D Geometry



There Are Two Types of Parameters

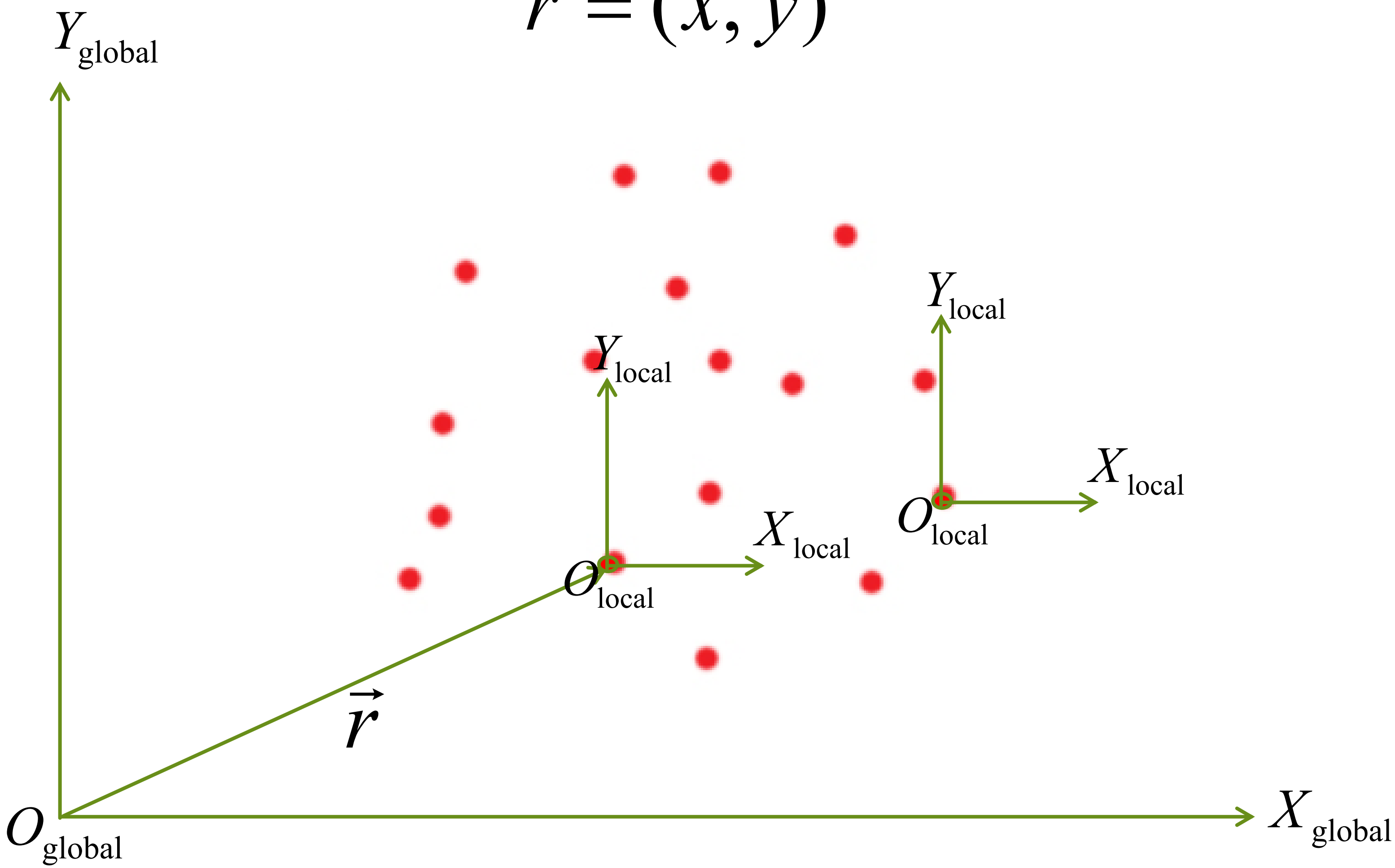
- Pose:
  - It refers to positions and orientations of entities with respect to a Global Coordinate System.
- Shape:
  - It refers to outlines of entities with respect to Local Coordinate Systems assigned to these shapes, respectively.





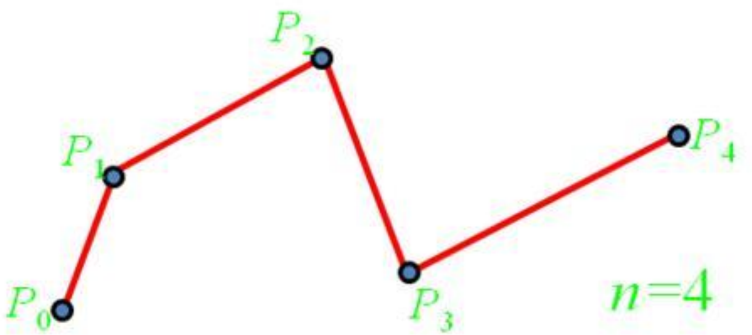
Parameters of Positions in 2D Space (1)

$$\vec{r} = (x, y)$$

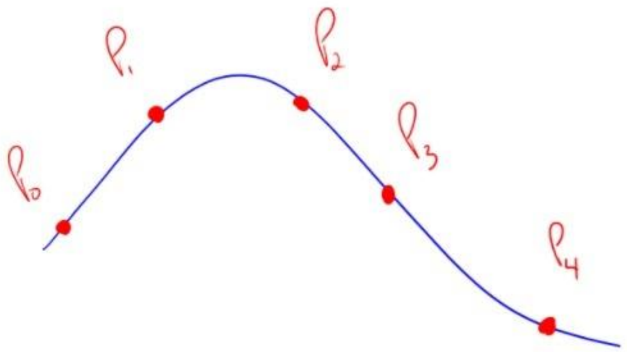


Parameters of Positions in 2D Space (2)

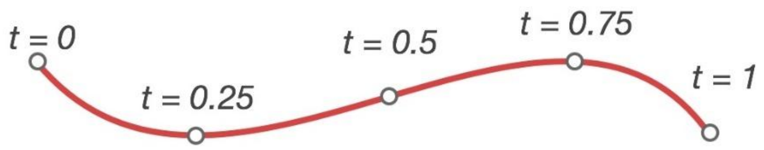
$$\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} a_x t + b_x \\ a_y t + b_y \end{pmatrix}$$



$$\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} a_x t^2 + b_x t + c_x \\ a_y t^2 + b_y t + c_y \end{pmatrix}$$



$$\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} a_x t^3 + b_x t^2 + c_x t + d_x \\ a_y t^3 + b_y t^2 + c_y t + d_y \end{pmatrix}$$



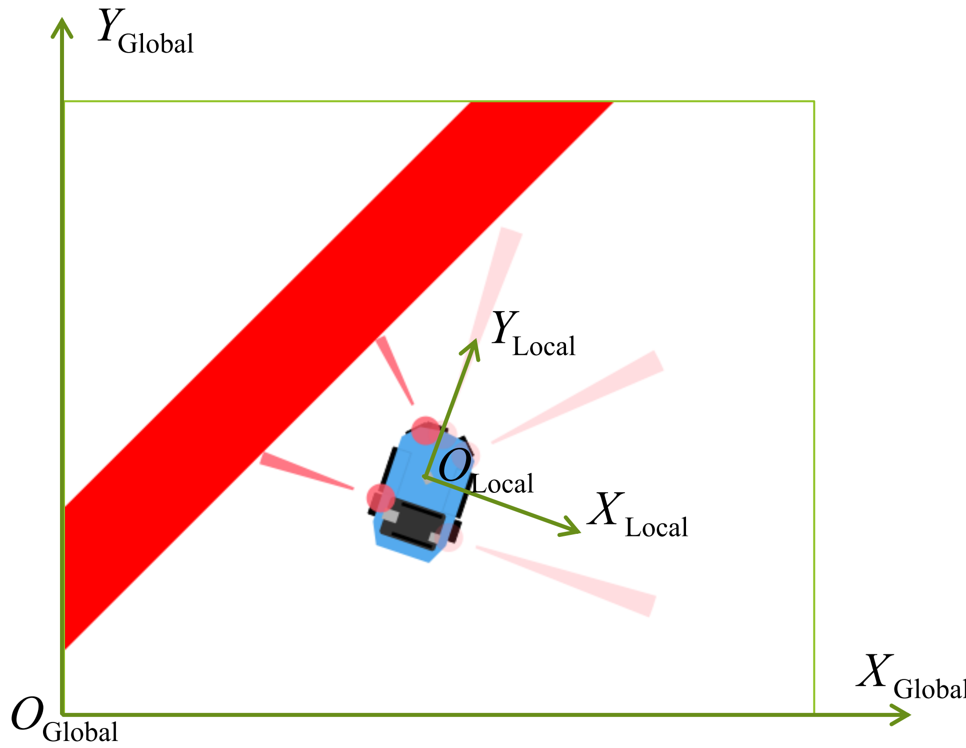
Example

- A mobile robot moves on a floor by following a straight line. It passes point A(2.0, 2.0) (cm) at time t1=1.0 s, and point B(4.0, 7.0) (cm) at t2=3.0 s. What are the coordinates of the origin of the local coordinate system assigned to the mobile robot?
- Answer:

$$a_x \times 1.0 + b_x = 2.0, \quad a_x \times 3.0 + b_x = 4.0 \Rightarrow a_x = 1.0, \quad b_x = 1.0$$

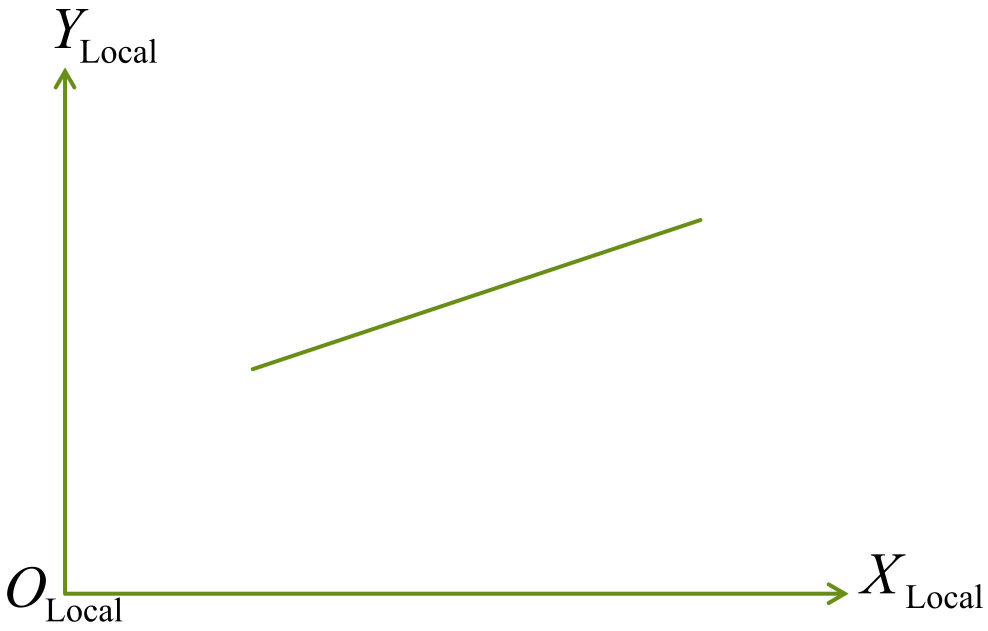
$$a_y \times 1.0 + b_y = 2.0, \quad a_y \times 3.0 + b_y = 7.0 \Rightarrow a_y = 2.5, \quad b_y = -0.5$$

$$\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} t + 1.0 \\ 2.5t - 0.5 \end{pmatrix}$$



Parameters of Lines in 2D Space

$$y = ax + b$$



Example

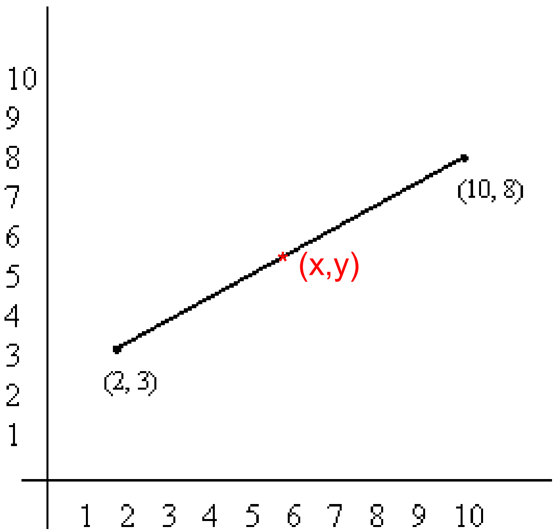
- A line in a 2D space passes through two points: (2,3) and (10,8). What are the parameters of the line?

- Answer:

$$\frac{y-3}{x-2} = \frac{8-3}{10-2}$$

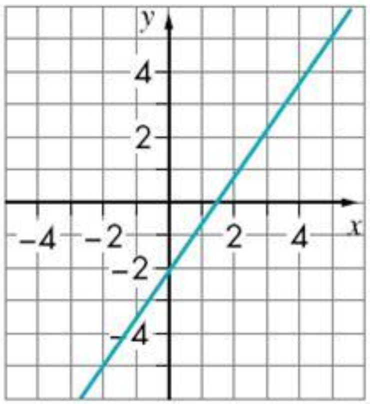
$$y = \frac{5}{8}(x-2)+3$$

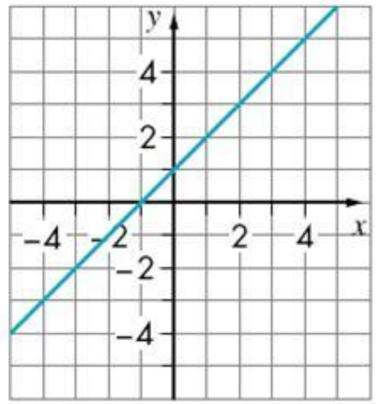
$$y = \frac{5}{8}x + \frac{7}{4} \quad \Rightarrow \quad (a,b) = (\frac{5}{8}, \frac{7}{4})$$

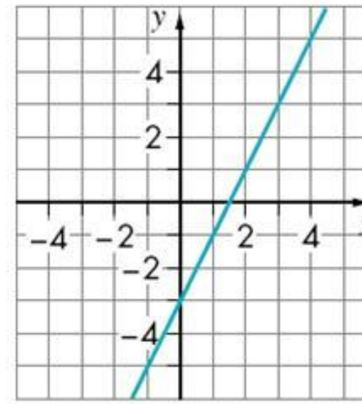


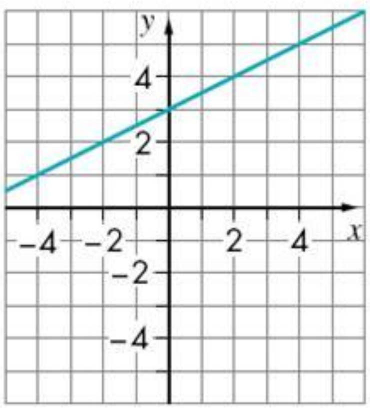
Exercises

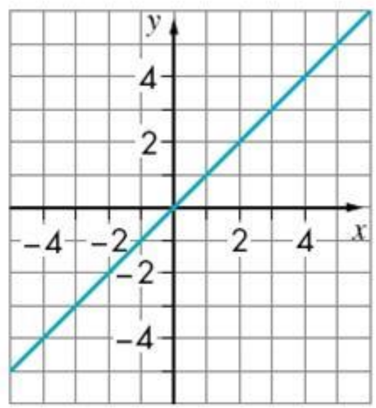
Find the equations of these straight lines.

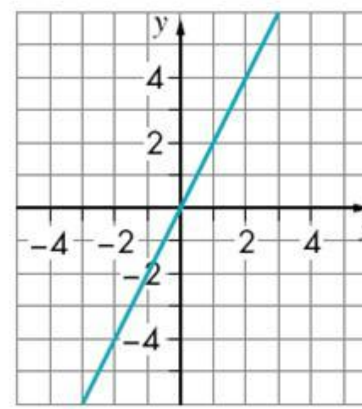
a


b


c


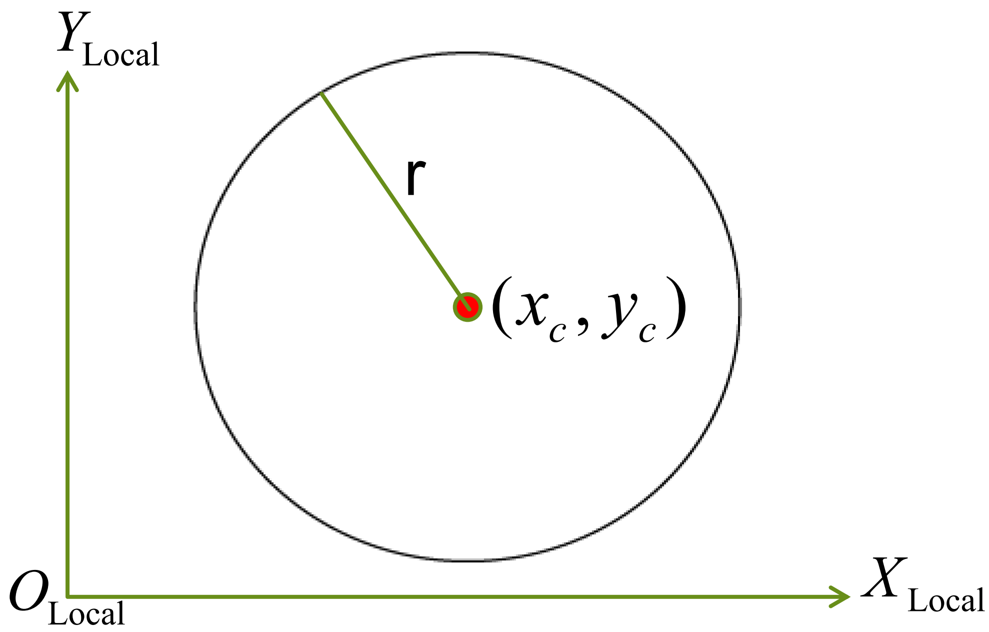
d


e


f


Parameters of Circles in 2D Space

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

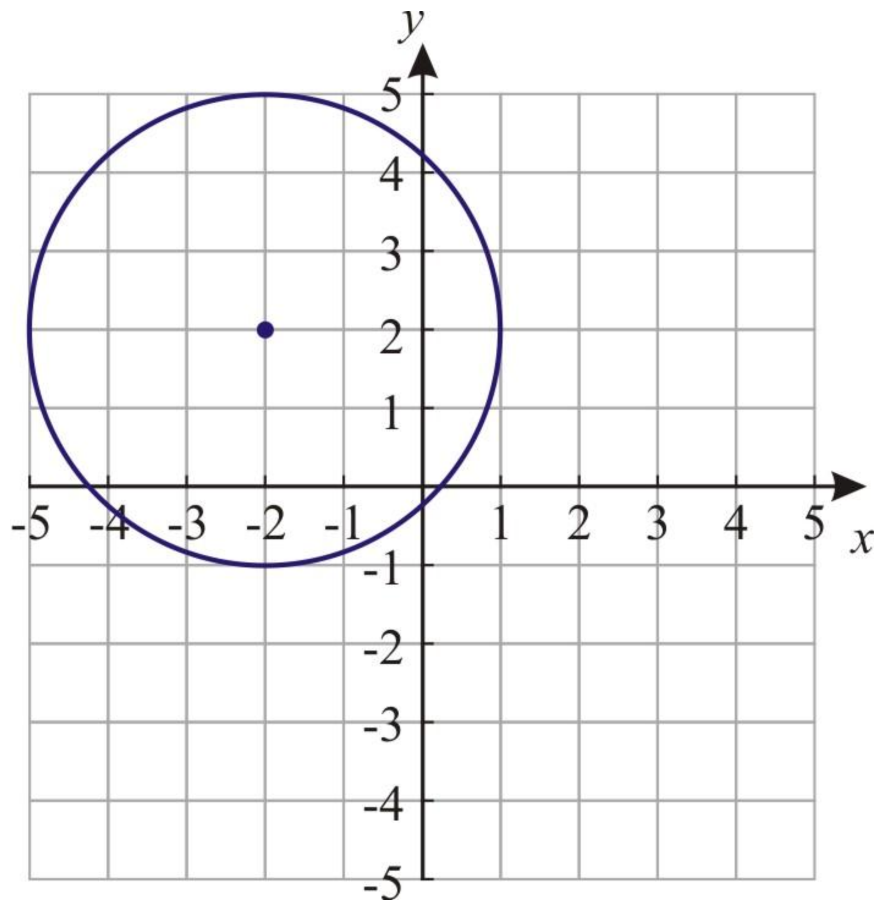


Example

- What are the parameters of the circle shown in the figure?

- Answer:

$$\begin{pmatrix} x_c \\ y_c \\ r \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}$$



Example

- A mechanical gear is under visual inspection. If the tips of the gear's three teeth are at the positions (x1, y1), (x2, y2) and (x3, y3), what are the parameters of the circle which envelopes the tips of all the teeth of the gear?

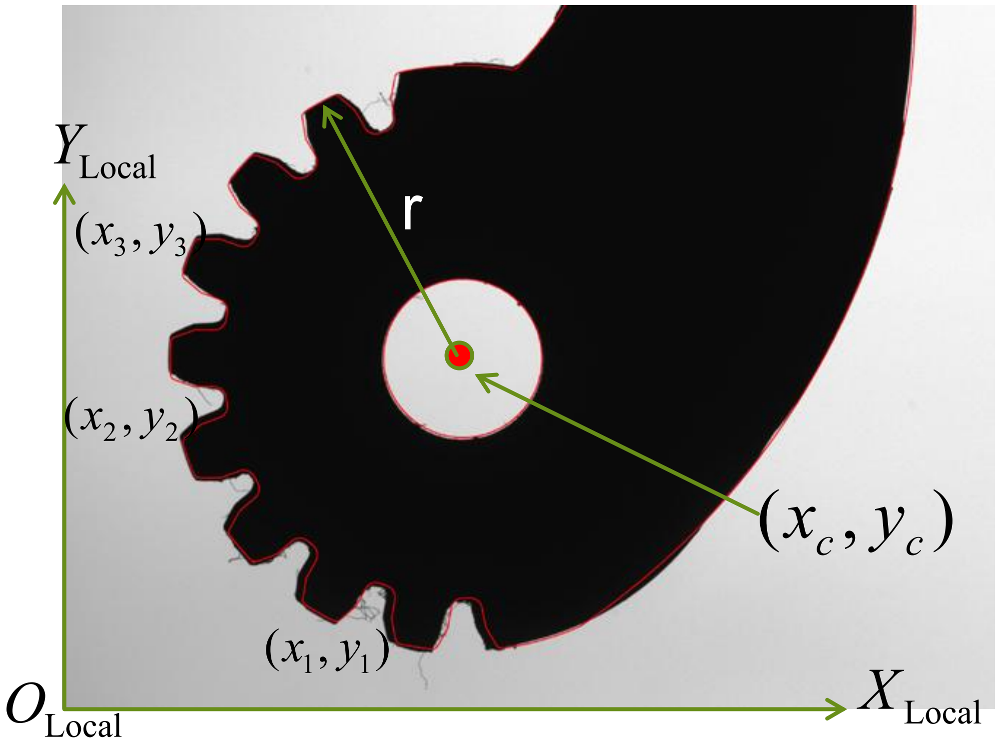
- Answer:

$$(x_1 - x_c)^2 + (y_1 - y_c)^2 = r^2$$

$$(x_2 - x_c)^2 + (y_2 - y_c)^2 = r^2$$

$$(x_3 - x_c)^2 + (y_3 - y_c)^2 = r^2$$

(to continue)



$$x_1^2 - 2x_1x_c + x_c^2 + y_1^2 - 2y_1y_c + y_c^2 = r^2$$

$$x_2^2 - 2x_2x_c + x_c^2 + y_2^2 - 2y_2y_c + y_c^2 = r^2$$

$$x_3^2 - 2x_3x_c + x_c^2 + y_3^2 - 2y_3y_c + y_c^2 = r^2$$

$$2(x_1 - x_2)x_c + 2(y_1 - y_2)y_c = x_1^2 - x_2^2 + y_1^2 - y_2^2$$

$$2(x_1 - x_3)x_c + 2(y_1 - y_3)y_c = x_1^2 - x_3^2 + y_1^2 - y_3^2$$

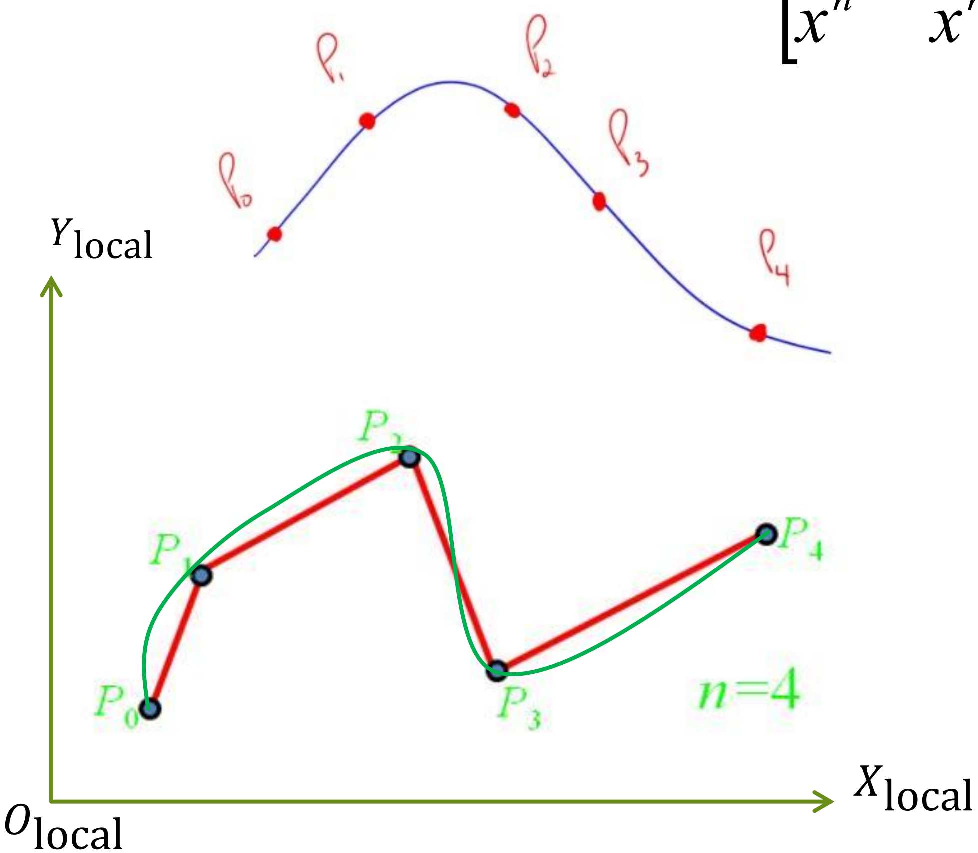
$$\begin{bmatrix} 2(x_1 - x_2) & 2(y_1 - y_2) \\ 2(x_1 - x_3) & 2(y_1 - y_3) \end{bmatrix} \bullet \begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} x_1^2 - x_2^2 + y_1^2 - y_2^2 \\ x_1^2 - x_3^2 + y_1^2 - y_3^2 \end{bmatrix}$$

$$\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} 2(x_1 - x_2) & 2(y_1 - y_2) \\ 2(x_1 - x_3) & 2(y_1 - y_3) \end{bmatrix}^{-1} \bullet \begin{bmatrix} x_1^2 - x_2^2 + y_1^2 - y_2^2 \\ x_1^2 - x_3^2 + y_1^2 - y_3^2 \end{bmatrix}$$

Representation of Curve-type Shape in 2D Space

$$\begin{bmatrix} x^n & x^{n-1} & \dots & 1 \end{bmatrix} P_{n\times n} \begin{bmatrix} y^n \\ y^{n-1} \\ \dots \\ 1 \end{bmatrix} = 0$$

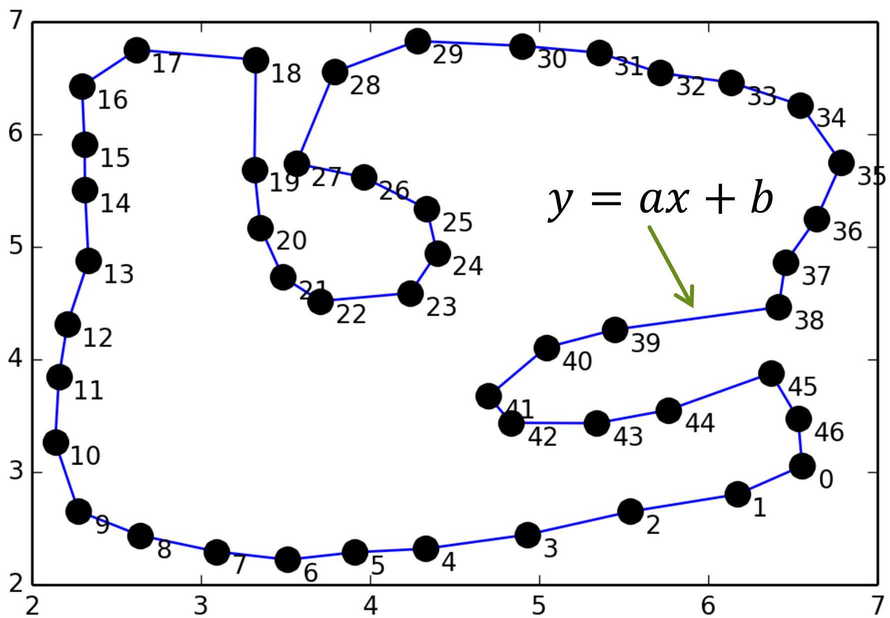
n=1,2,…

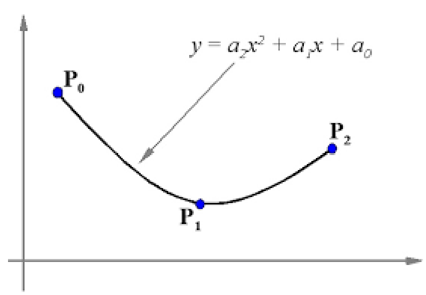


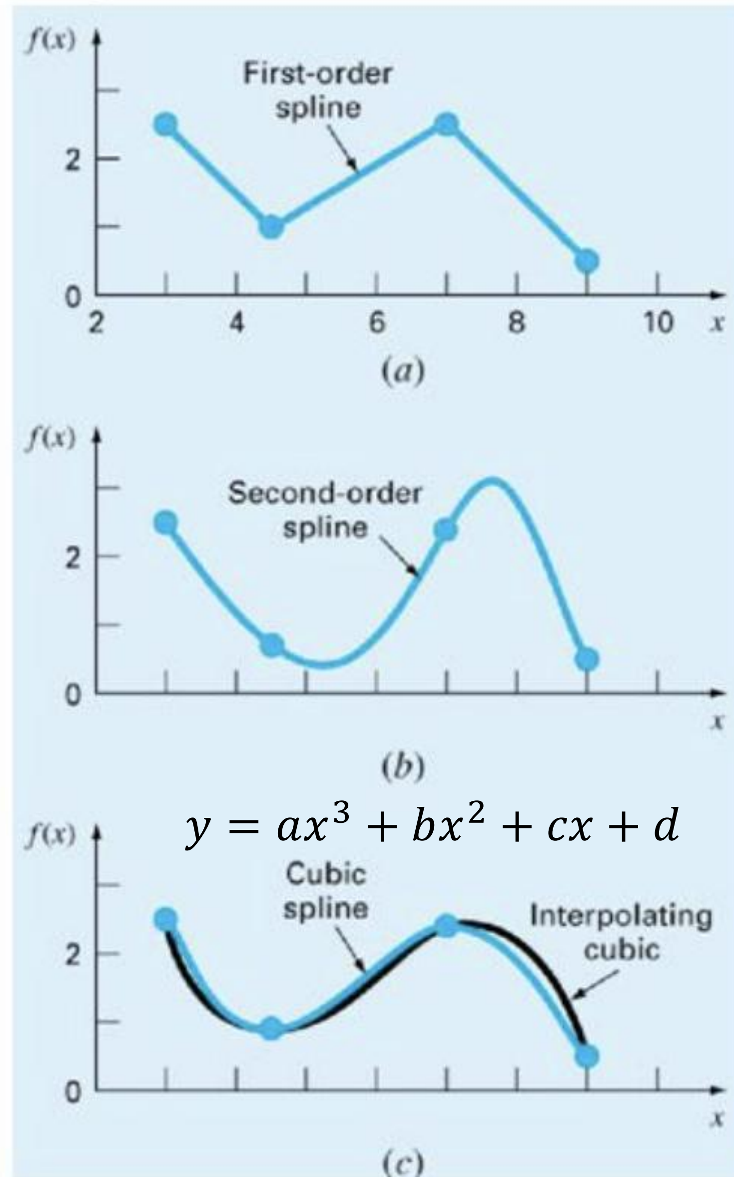
Any curve could be approximated by a set of line segments and arcs

Next Slide

Example of Using Splines …

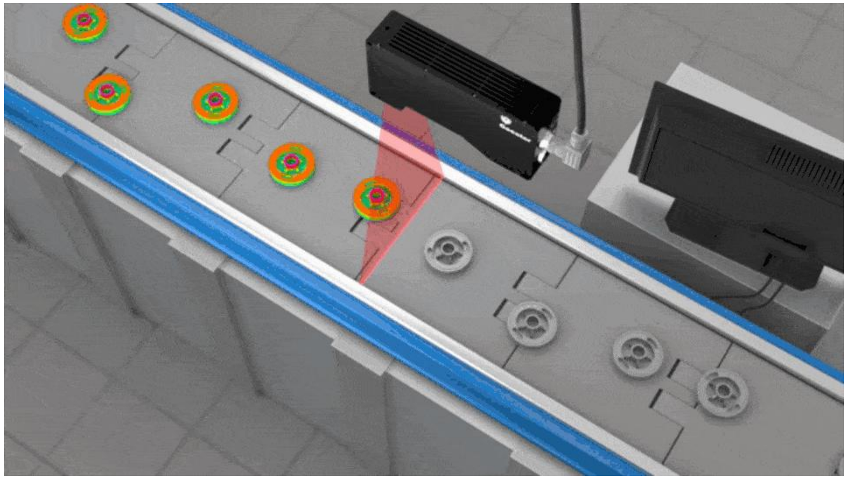






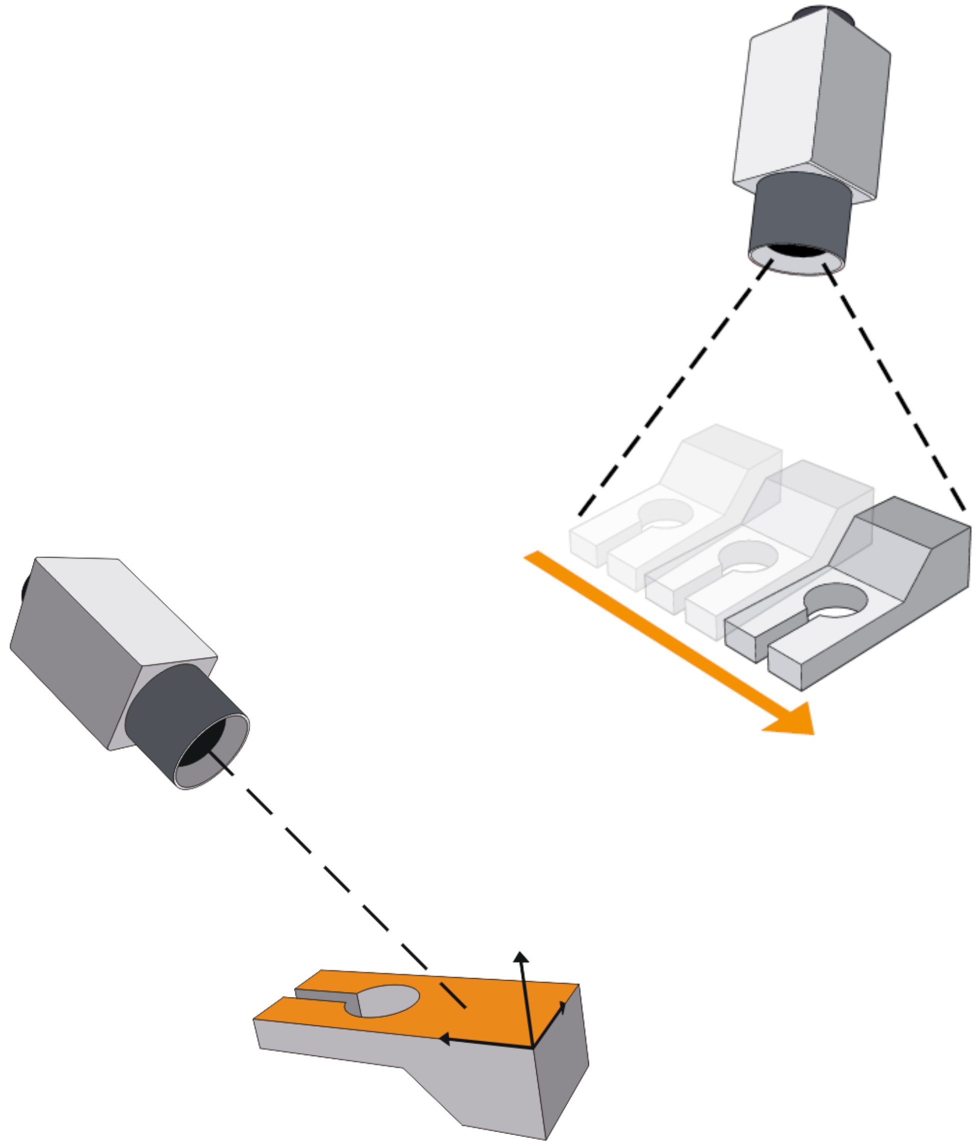
Outline of Lecture 2

- Basics of 2D Geometry
- Parameters of 2D Geometry
- Measurement of 2D Geometry

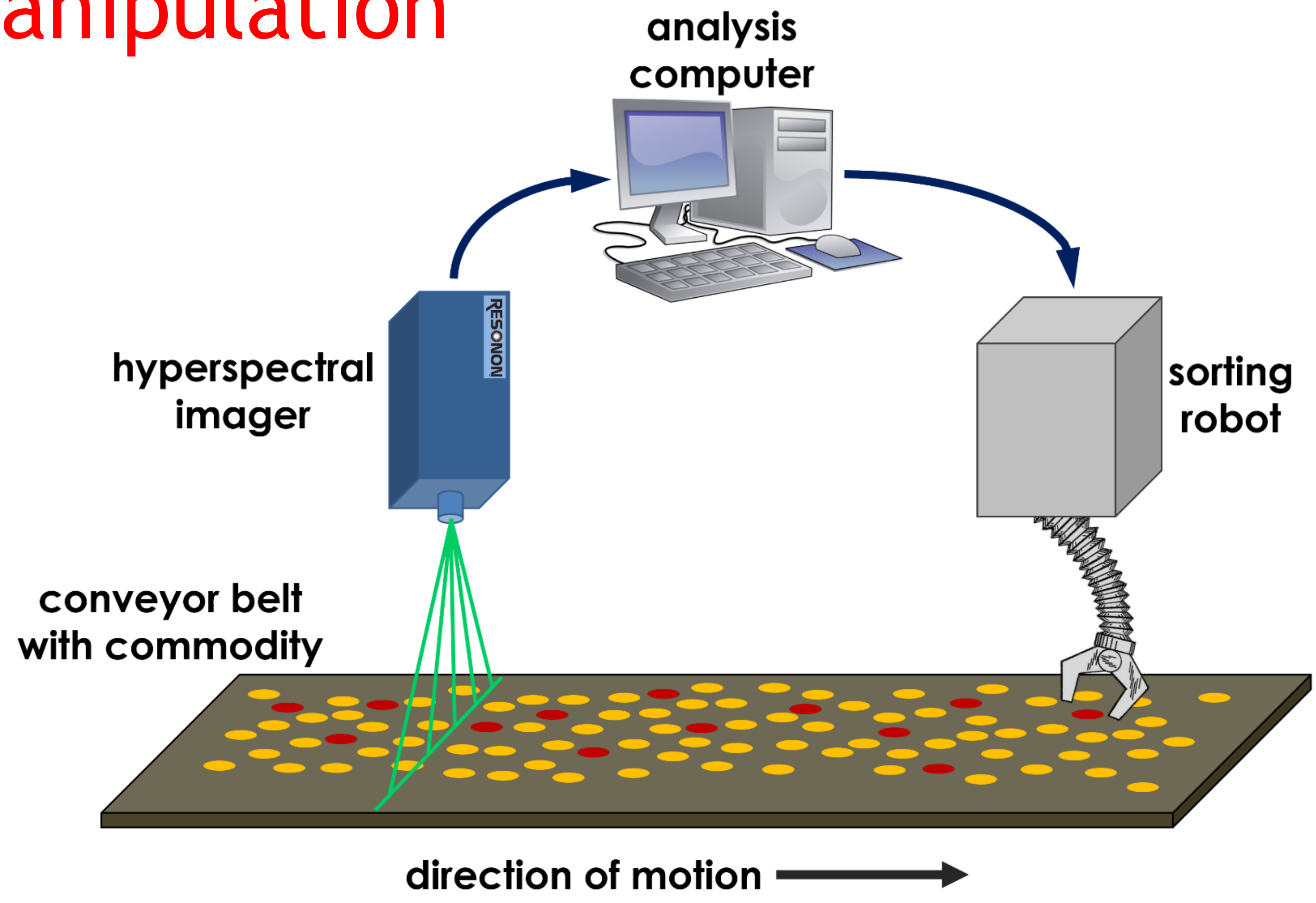


Applications

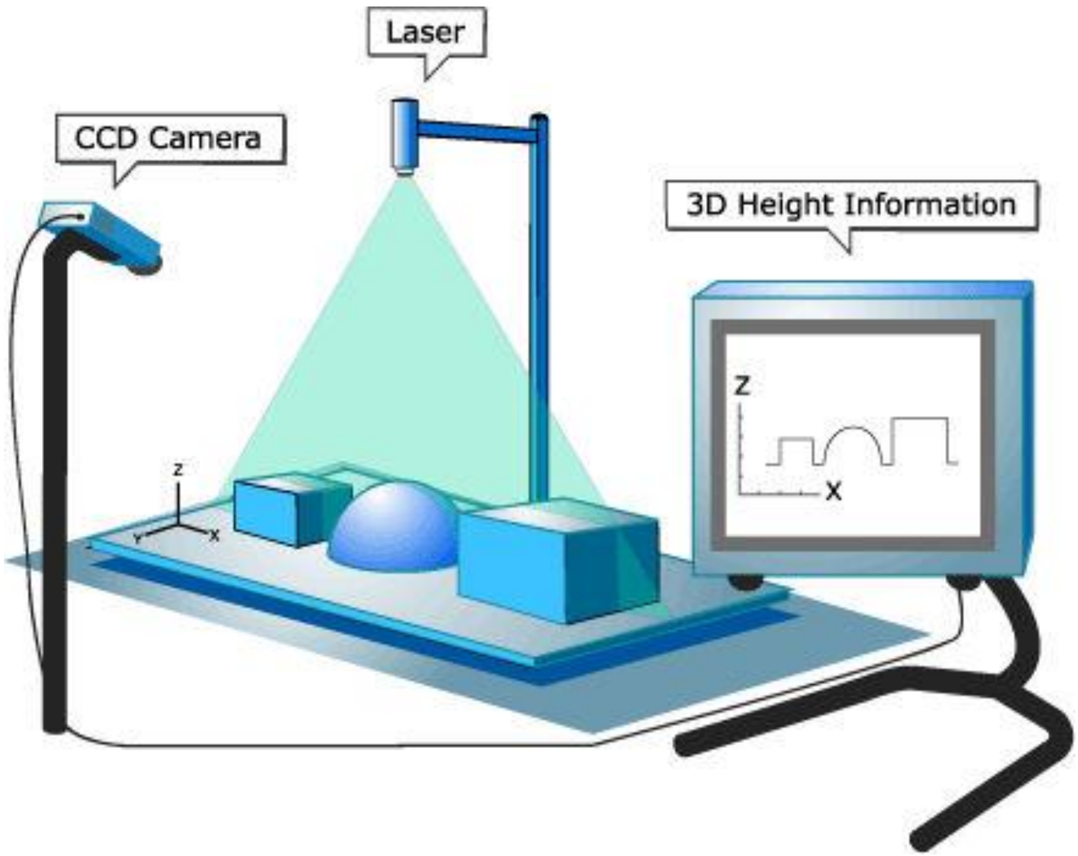
- Inspection
- Grasping
- Manipulation
- Guidance
- Cognition / Recognition



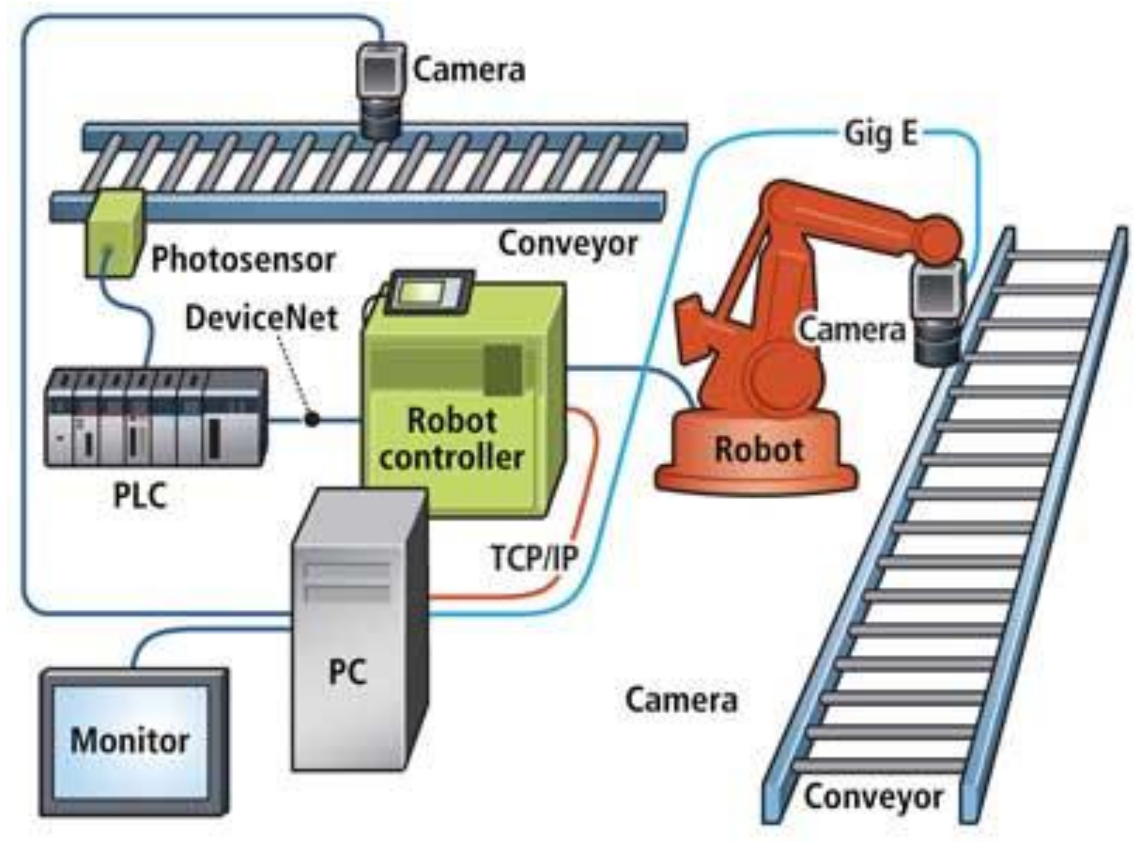
Example of Vision-Guided Manipulation



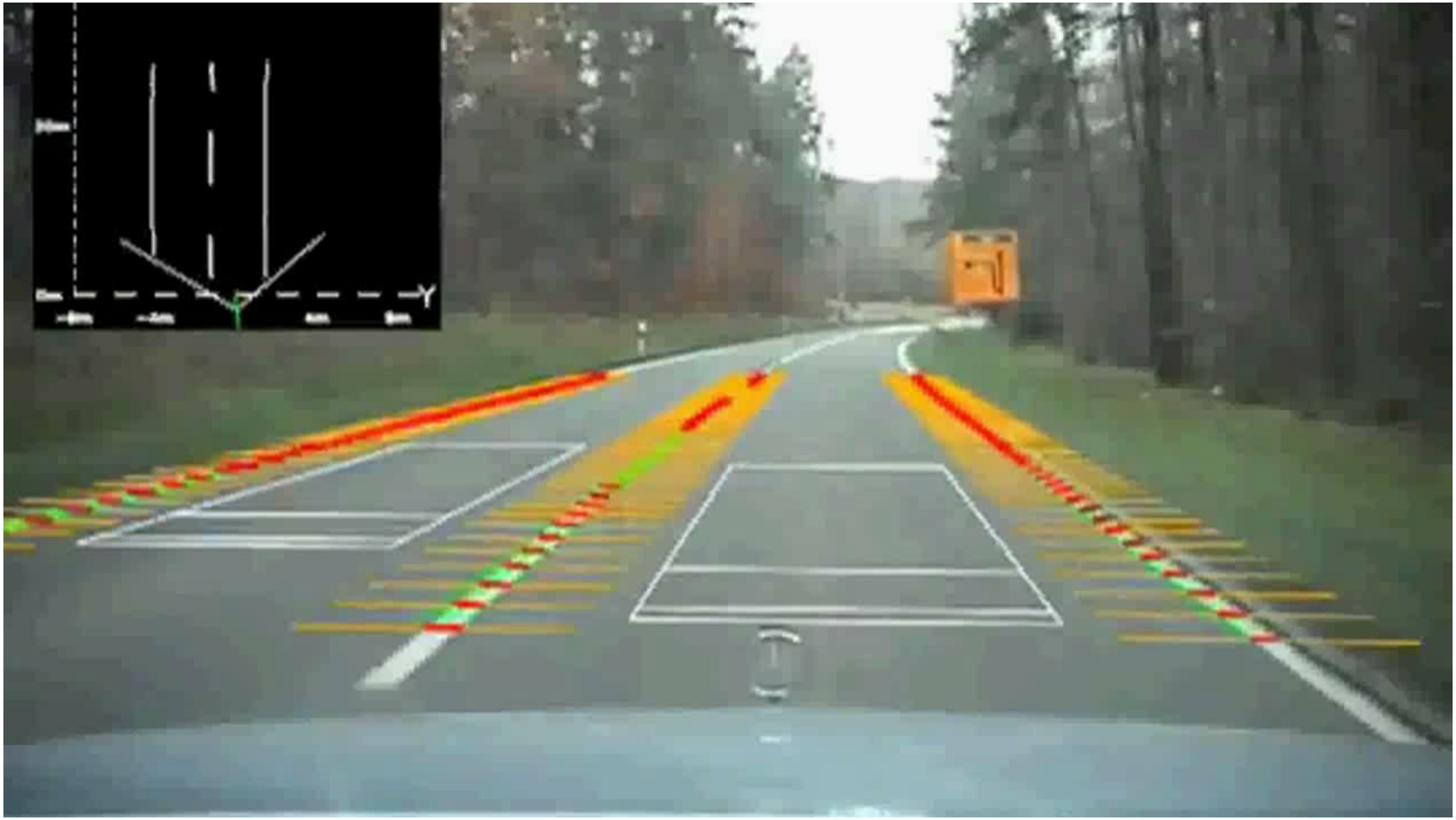
Example of Vision-Guided Quality Control in Manufacturing



Example of Vision-Guided Material Handling in Manufacturing

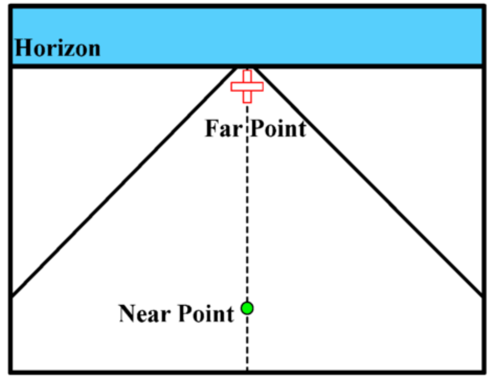


Example of Vision-Guided Locomotion

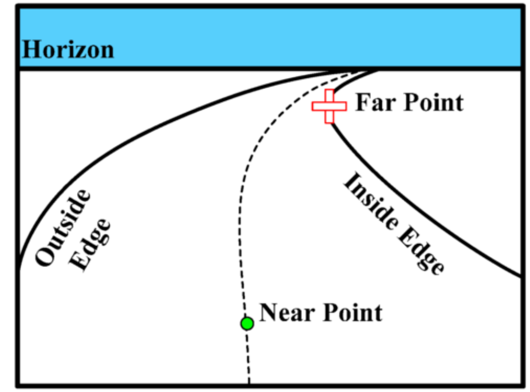


Example of Vision-Guided Locomotion

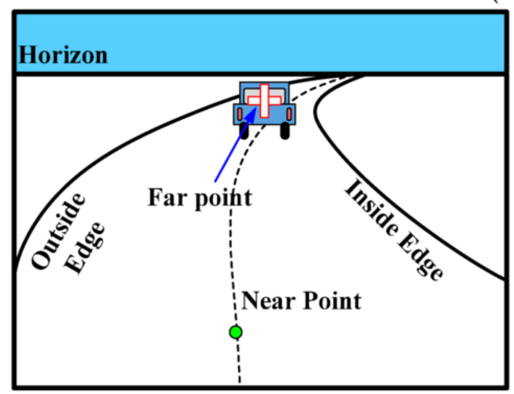
More Example



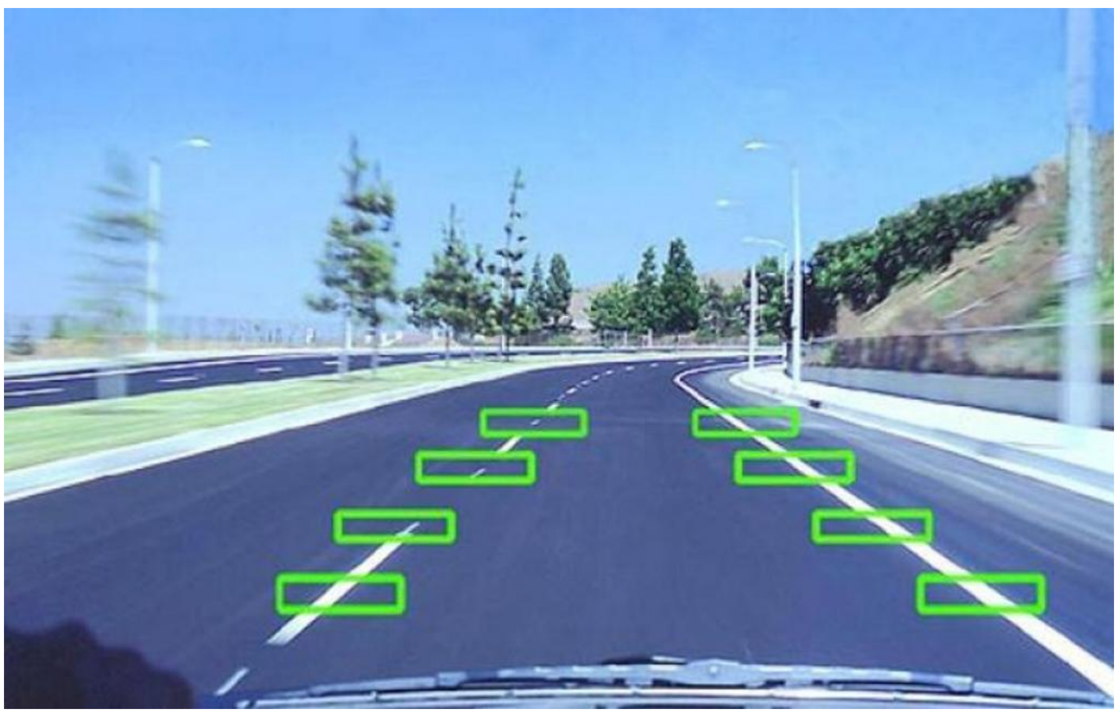
(a)



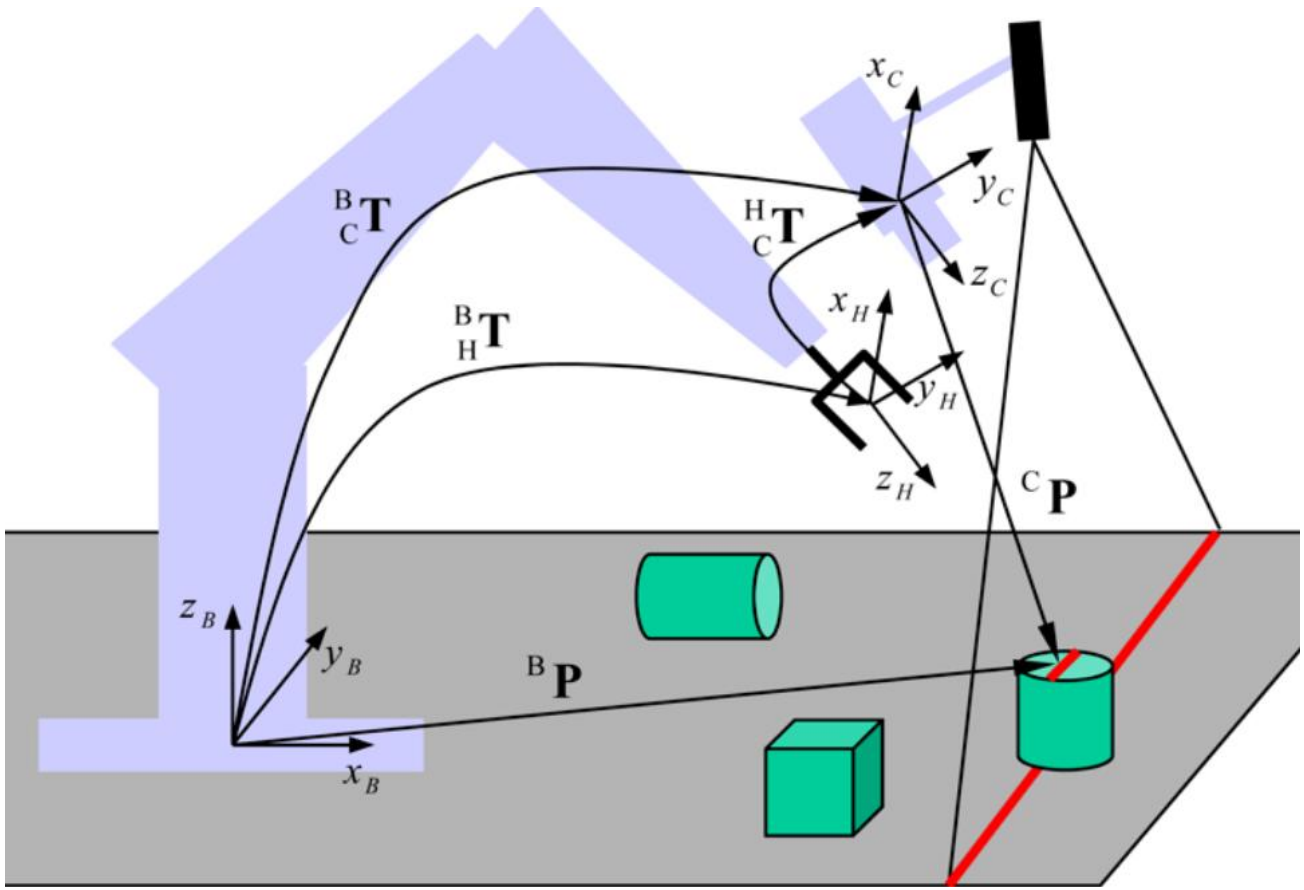
(b)



(c)

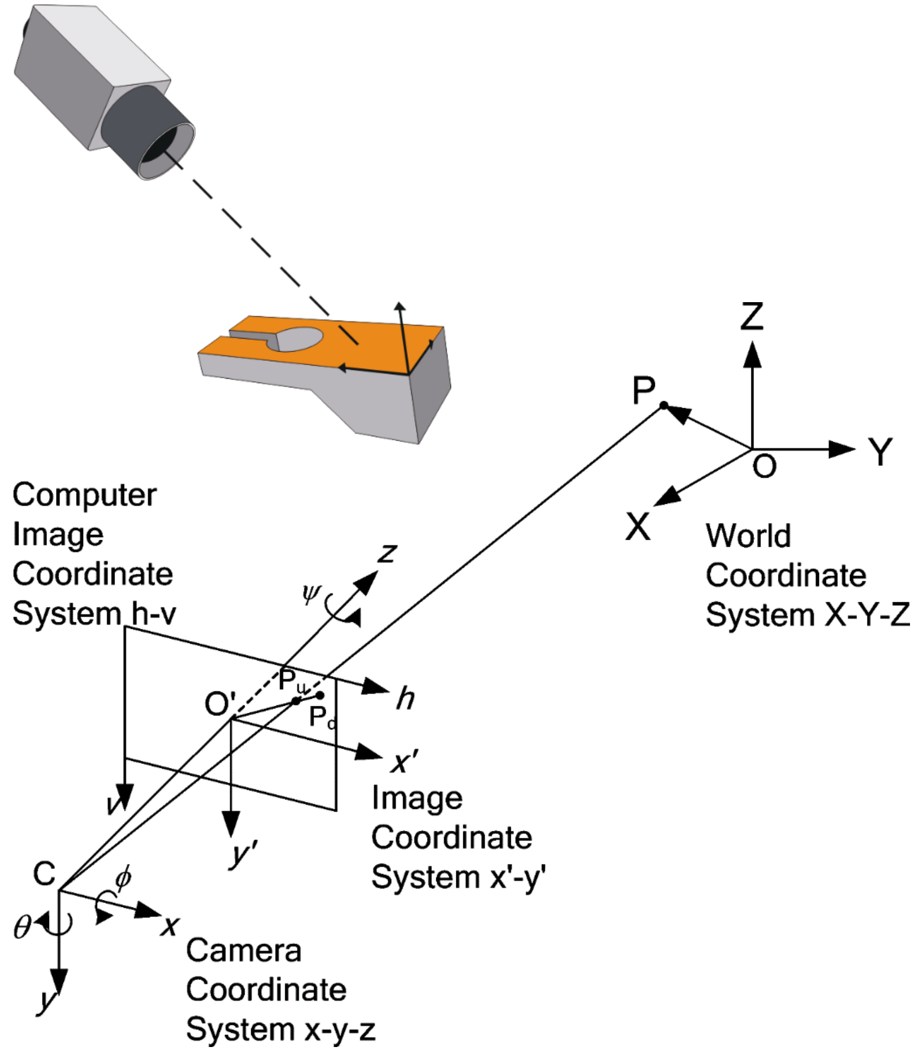


More Example



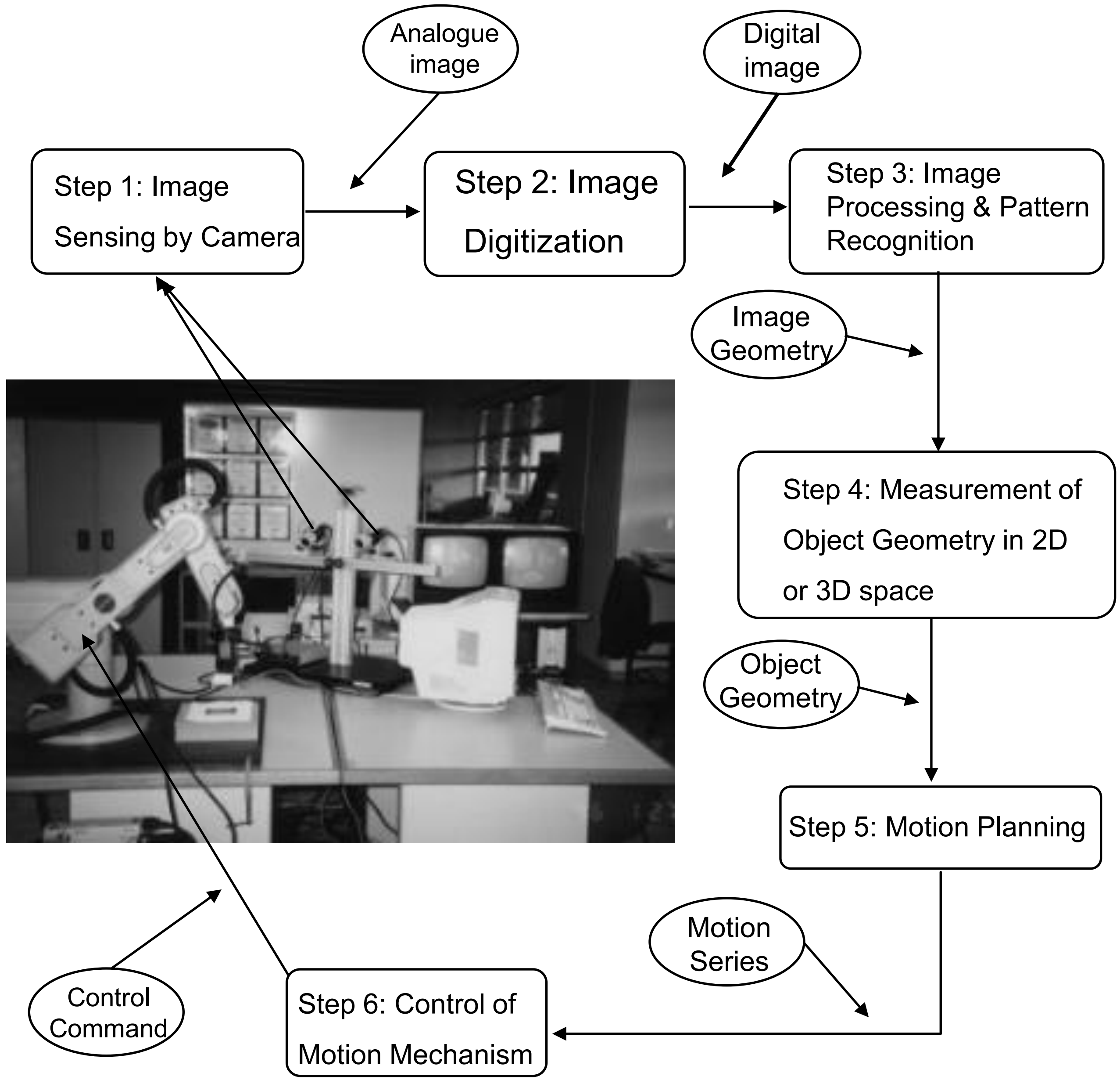
Principle of Using Single Camera

- The geometry of physical entities in a 2D space depends on the coordinates of points.
- The measurement of coordinates of points in a 2D space can be achieved with a single camera or monocular vision system.
- A monocular vision system relates the coordinates of points in 2D space to the coordinates in image space in a unique way.

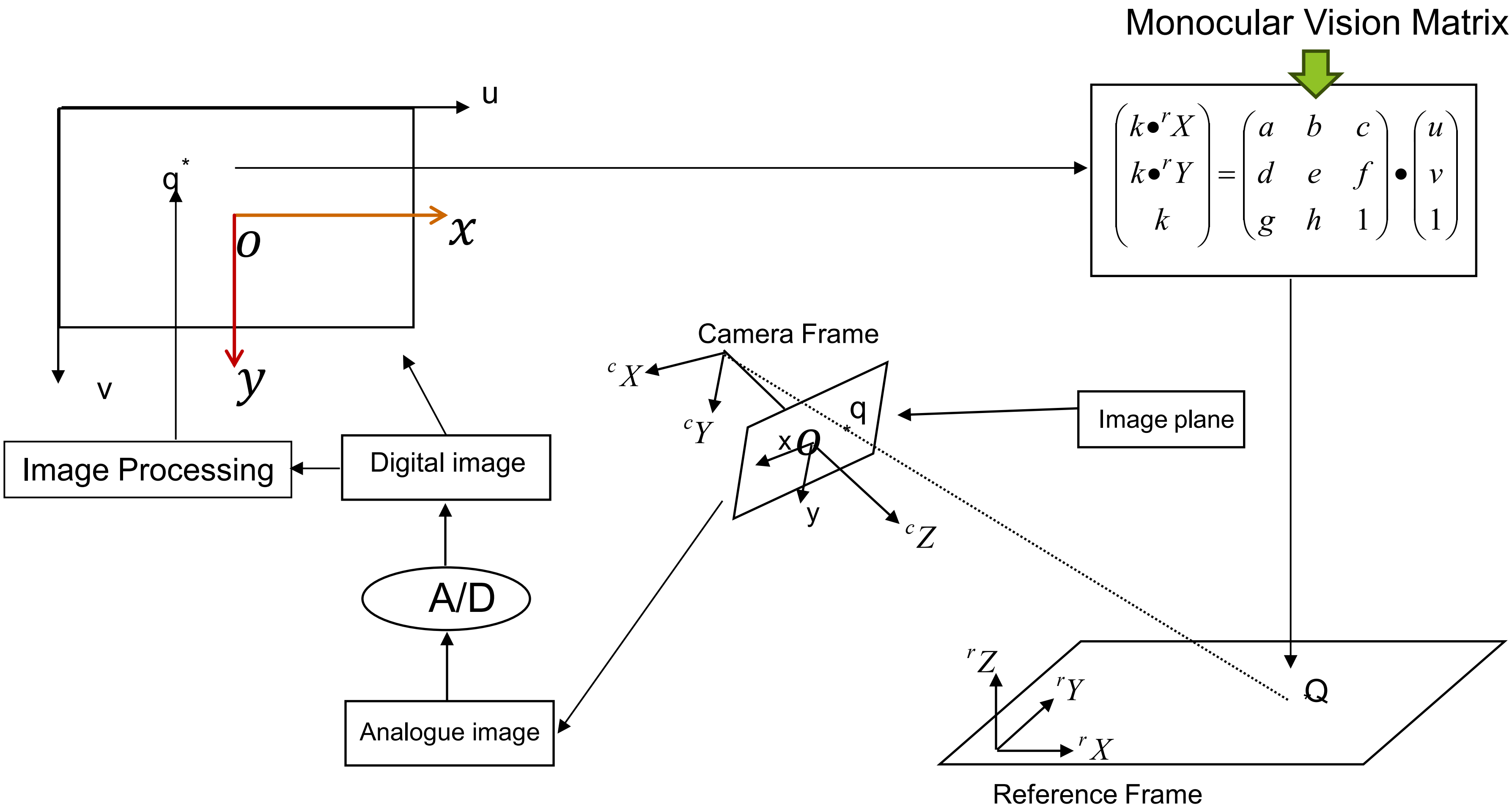


Motion Flow in Monocular Vision …

What is a motion flow? It refers to a flow of coordinate systems

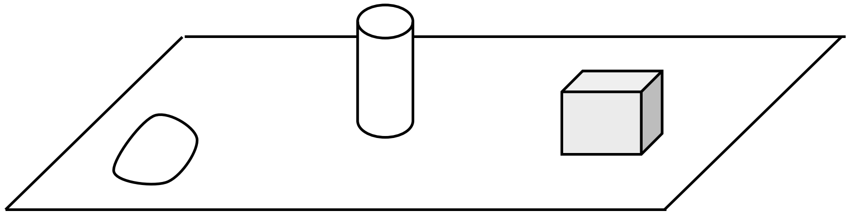


Equations of Monocular Vision



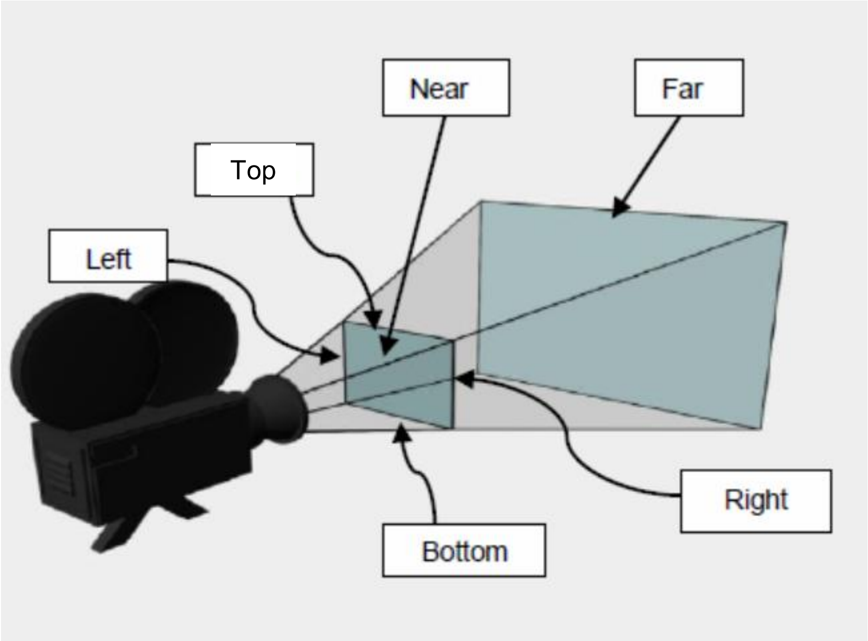
Proof Step 1:

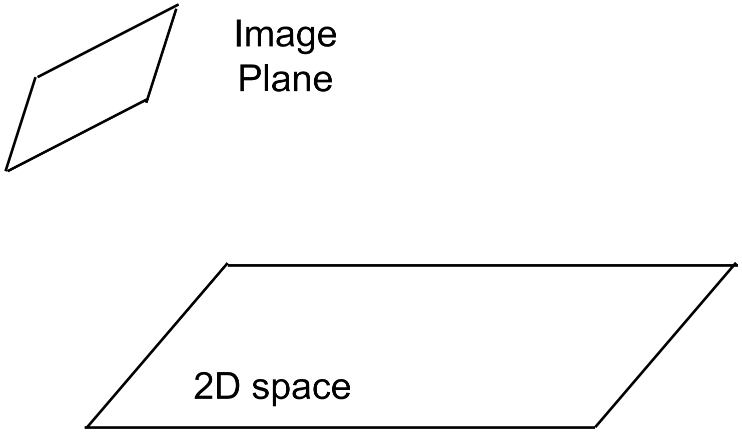
- We consider the case of the measurement of points on a 2D plane:



Proof Step 2:

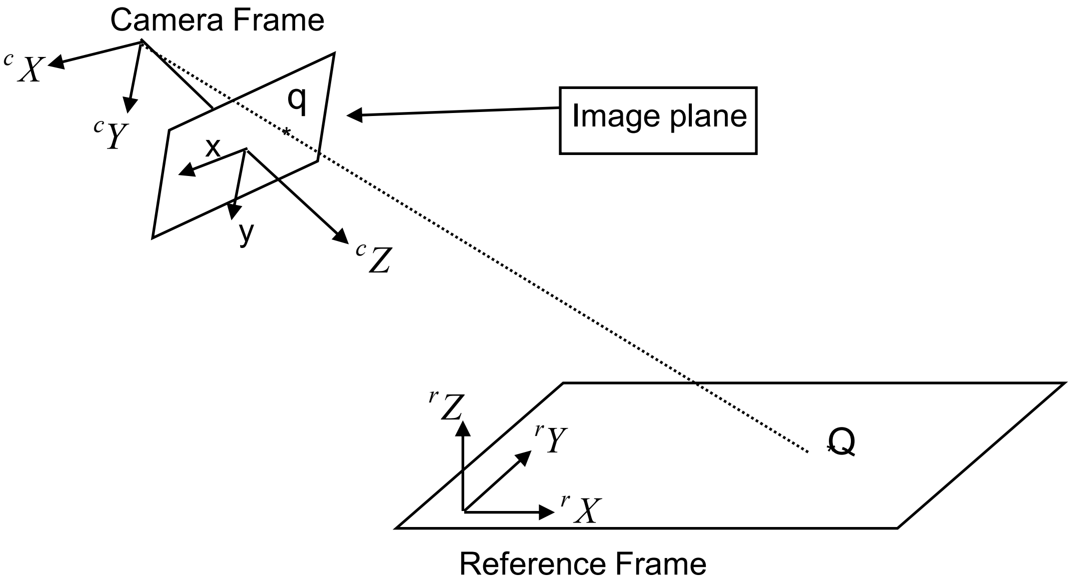
- We use a single camera as the sensing device:





Proof Step 3:

- The coordinates in the reference frame can be transformed into the coordinates in the camera frame:



$$^cH_r = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

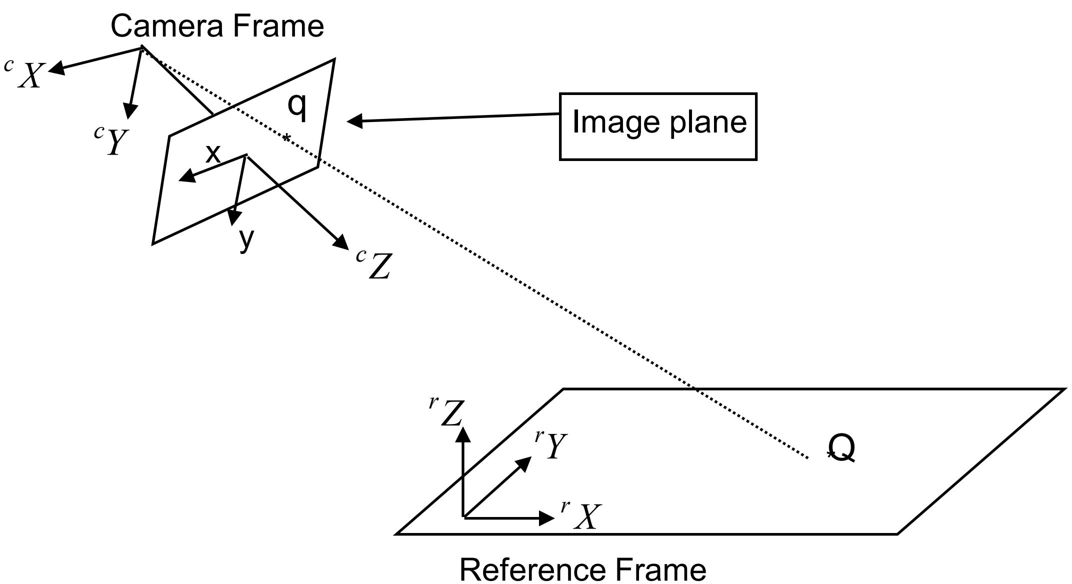
$$\begin{bmatrix} ^cX \\ ^cY \\ ^cZ \\ 1 \end{bmatrix} = {}^cH_r \bullet \begin{bmatrix} ^rX \\ ^rY \\ ^rZ \\ 1 \end{bmatrix}$$

Proof Step 4:

- The coordinates in camera frame can be projected into the coordinates in image plane:

$$x = f \bullet \frac{{}^{c}X}{{}^{c}Z} \quad \text{and} \quad y = f \bullet \frac{{}^{c}Y}{{}^{c}Z}$$

$$\begin{pmatrix} s \bullet x \\ s \bullet y \\ s \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \bullet \begin{pmatrix} {}^{c}X \\ {}^{c}Y \\ {}^{c}Z \\ 1 \end{pmatrix}$$

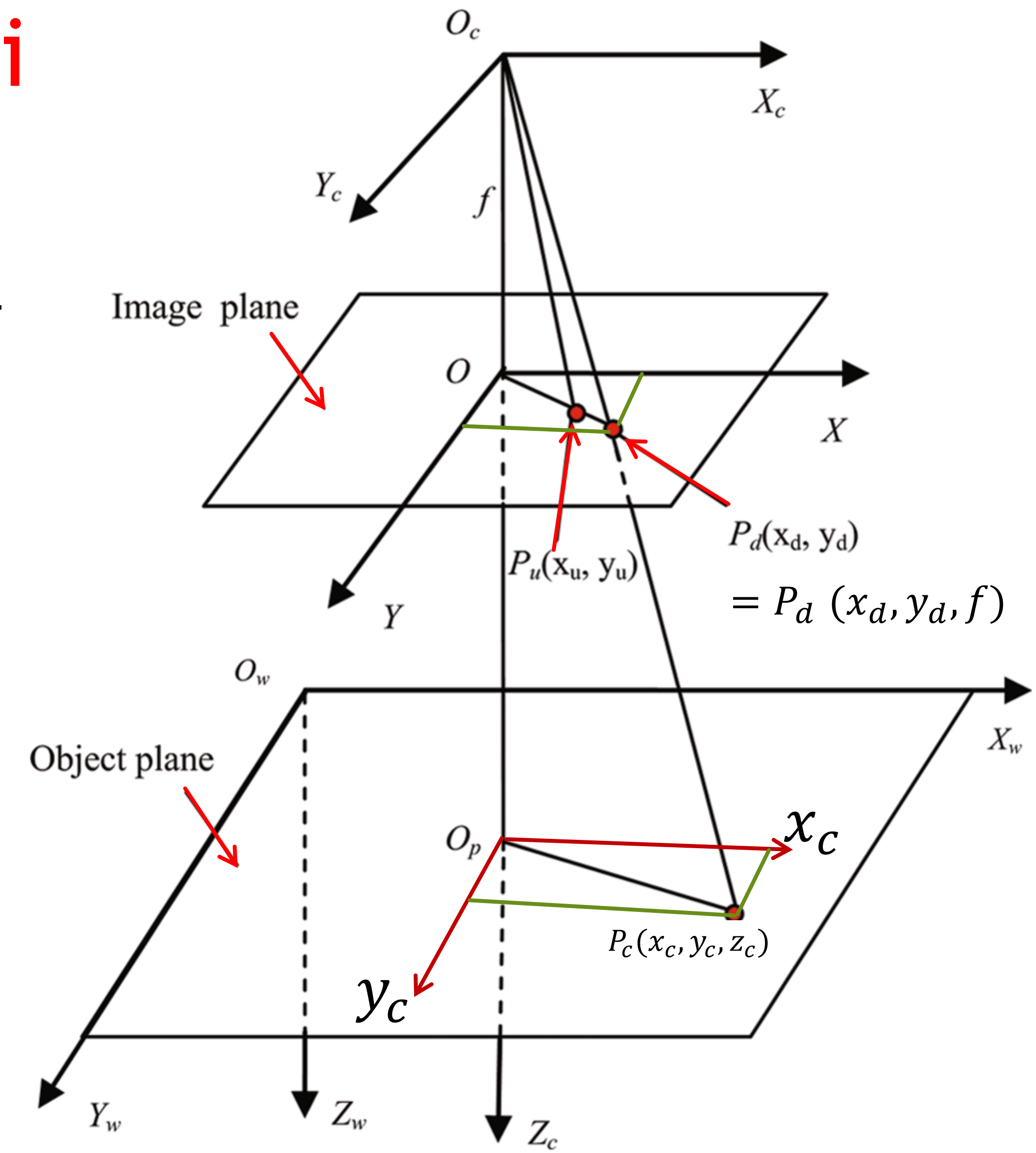


Further Proof Detai

$$\frac{O_pO_c}{OO_c} = \frac{P_cO_p}{P_dO} = \frac{z_c}{f} = \frac{x_c}{x_d} = \frac{y_c}{y_d}$$

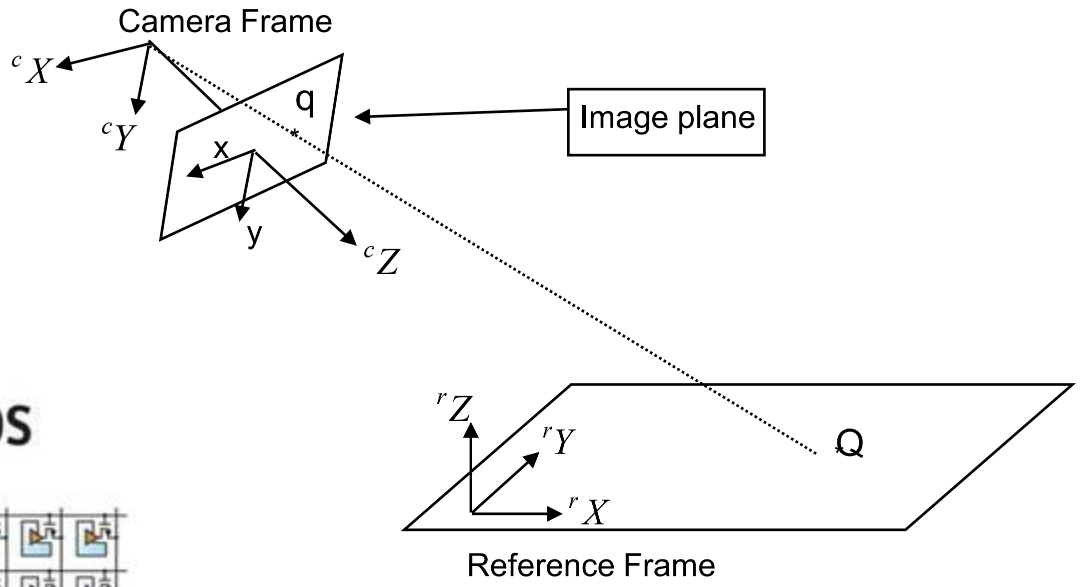
$$\frac{z_c}{f} = \frac{x_c}{x_d} \Rightarrow x_d = f\frac{x_c}{z_c}$$

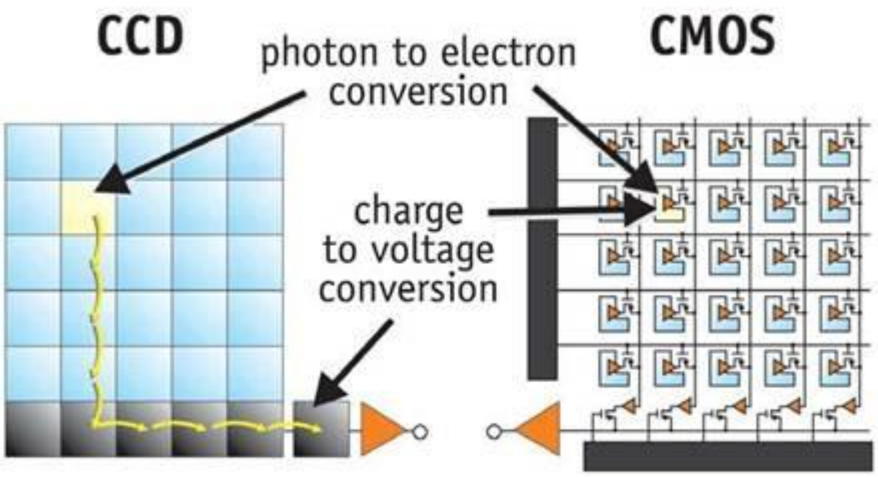
$$\frac{z_c}{f} = \frac{y_c}{y_d} \Rightarrow y_d = f\frac{y_c}{z_c}$$



Proof Step 5:

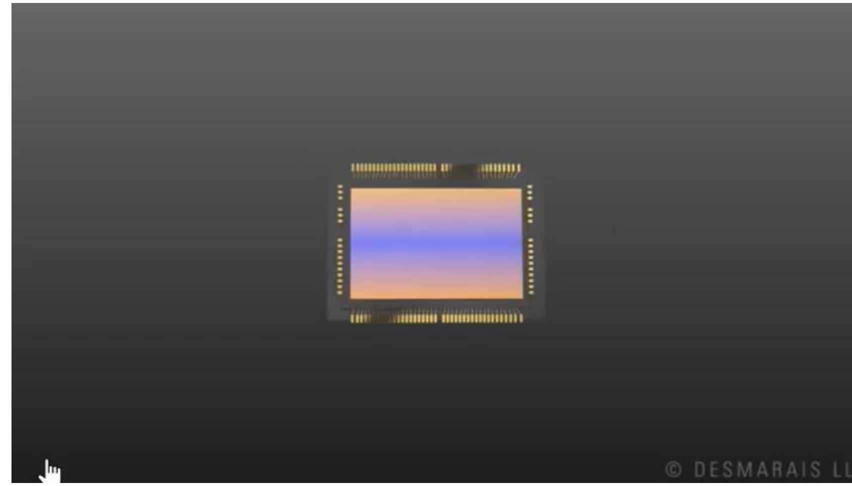
- The image sensor creates an analogue image from the light pattern in the image plane:

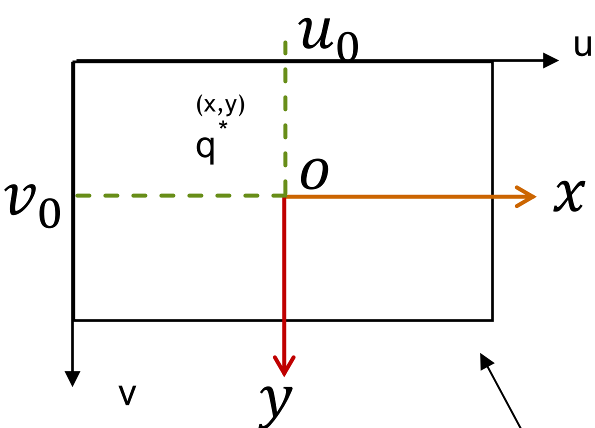


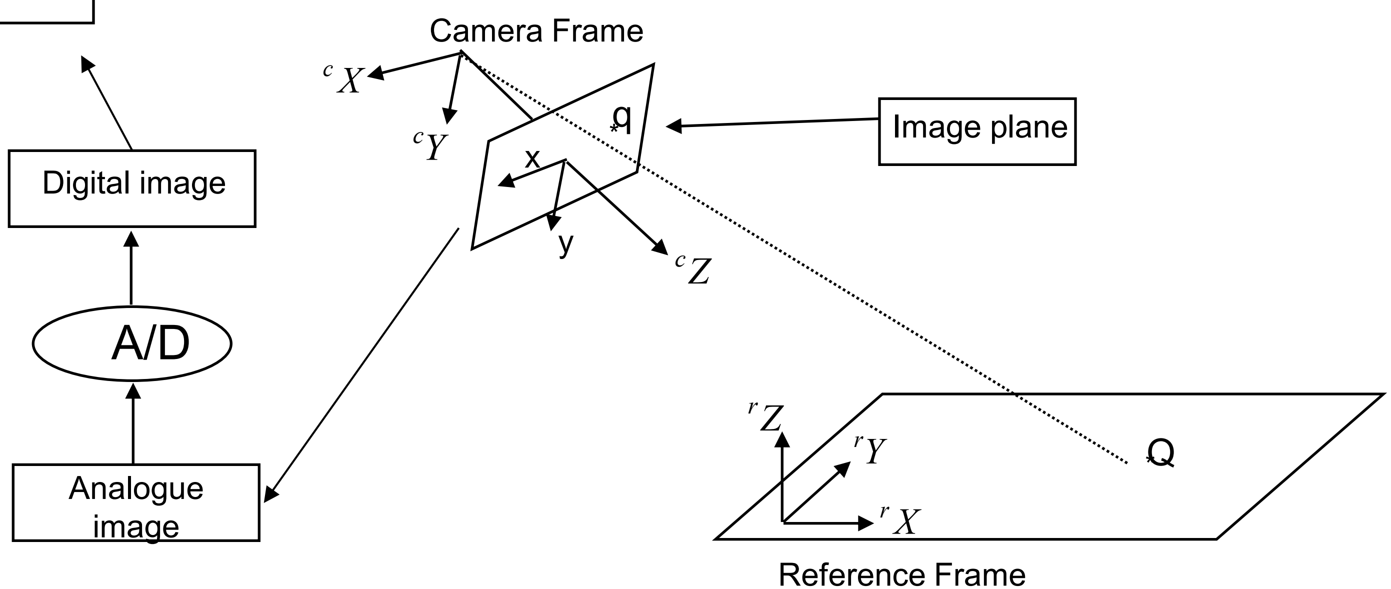


Proof Step 6:

- The analogue image produced at the image plane can be digitized into the corresponding digital image:





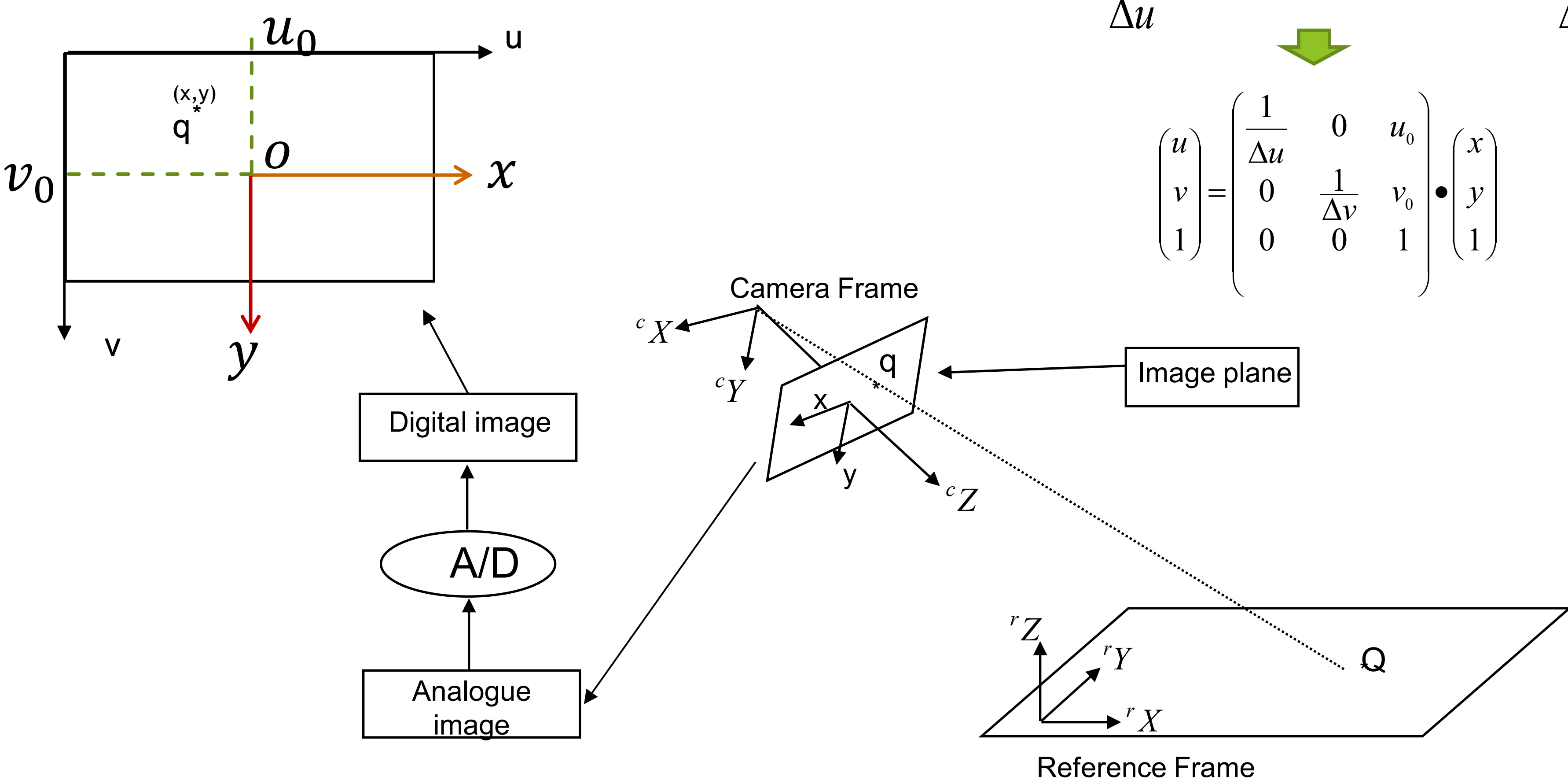


Proof Step 7:

- The coordinates (x, y) in image plane can be converted into the coordinates (u, v) inside image matrix:

$$u = u_0 + \frac{x}{\Delta u} \quad \text{and} \quad v = v_0 + \frac{y}{\Delta v}$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\Delta u} & 0 & u_0 \\ 0 & \frac{1}{\Delta v} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

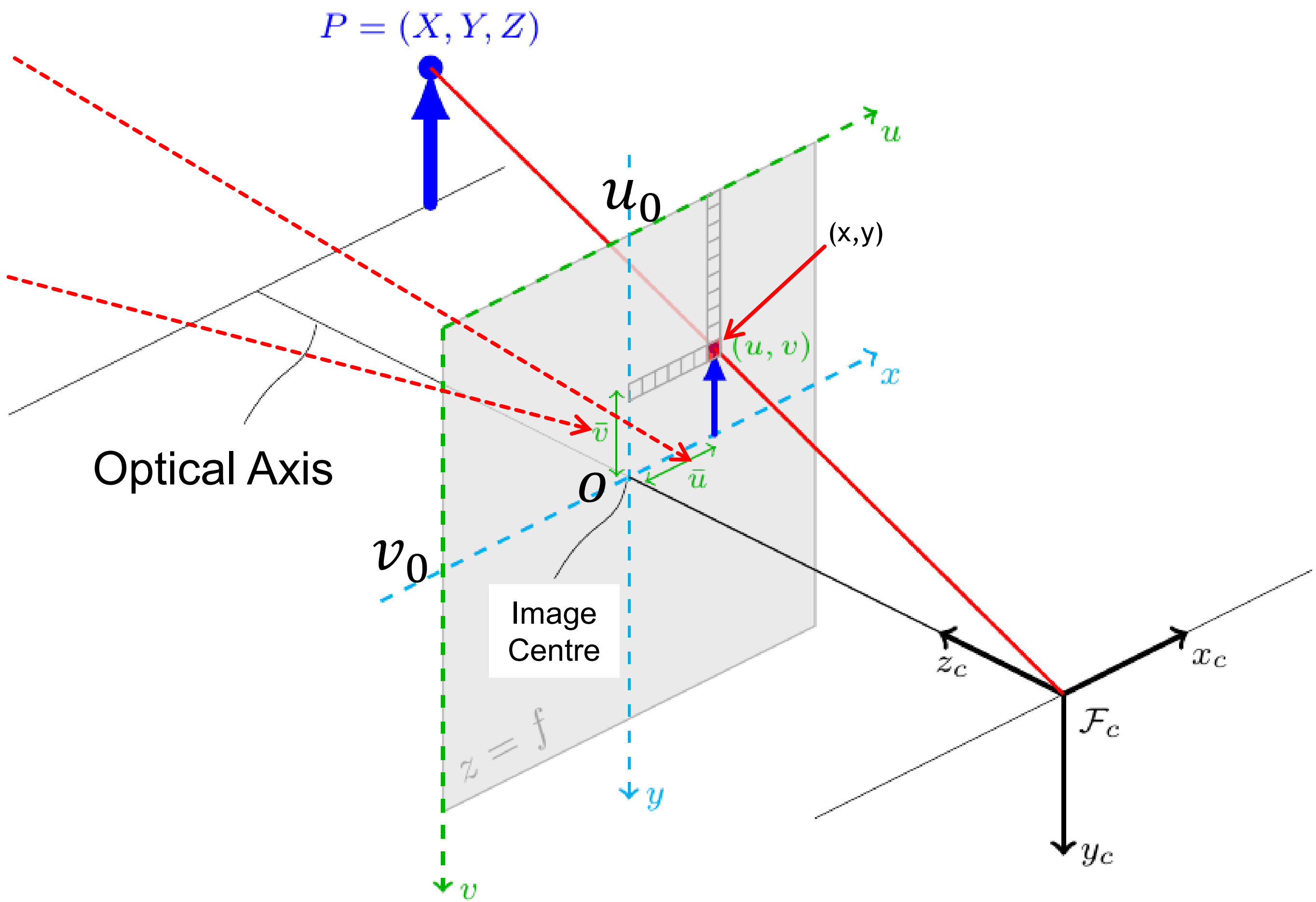


Further Proof Detail …

$$u = u_0 + \bar{u} = u_0 + \frac{x}{\Delta u}$$

$$v = v_0 + \bar{v} = v_0 + \frac{y}{\Delta v}$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\Delta u} & 0 & u_0 \\ 0 & \frac{1}{\Delta v} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$



Proof Step 8:

- The coordinates (X, Y, Z) in reference frame can be related to the coordinates (x, y) in image plane:

$$\begin{bmatrix} {}^{c}X \\ {}^{c}Y \\ {}^{c}Z \\ 1 \end{bmatrix} = {}^{c}H_{r} \bullet \begin{bmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} s \bullet x \\ s \bullet y \\ s \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \bullet \begin{pmatrix} {}^{c}X \\ {}^{c}Y \\ {}^{c}Z \\ 1 \end{pmatrix}$$

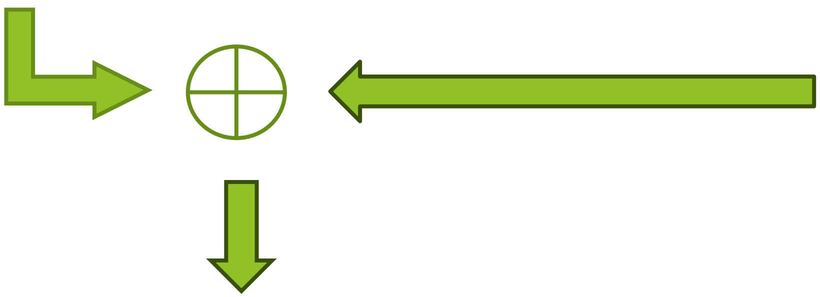
$${}^{c}H_{r} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_{x} \\ r_{21} & r_{22} & r_{23} & t_{y} \\ r_{31} & r_{32} & r_{33} & t_{z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} s \bullet x \\ s \bullet y \\ s \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_{x} \\ r_{21} & r_{22} & r_{23} & t_{y} \\ r_{31} & r_{32} & r_{33} & t_{z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{pmatrix}$$

Proof Step 9:

- The coordinates (X, Y, Z) in reference frame can further be related to the coordinates (u, v) inside image matrix:

$$\begin{pmatrix} s \bullet x \\ s \bullet y \\ s \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} s \bullet u \\ s \bullet v \\ s \end{pmatrix} = \begin{pmatrix} \frac{1}{\Delta u} & 0 & u_0 \\ 0 & \frac{1}{\Delta v} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} s \bullet x \\ s \bullet y \\ s \end{pmatrix}$$

$$\begin{pmatrix} s \bullet u \\ s \bullet v \\ s \end{pmatrix} = \begin{pmatrix} \frac{f}{\Delta u} & 0 & u_0 & 0 \\ 0 & \frac{f}{\Delta v} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix}$$

Proof Step 10:

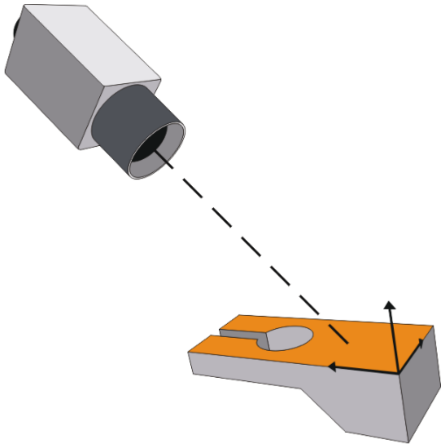
- Then, we obtain the equation of camera's forward projection:

$$\begin{pmatrix} s \bullet u \\ s \bullet v \\ s \end{pmatrix} = \begin{pmatrix} \frac{f}{\Delta u} & 0 & u_0 & 0 \\ 0 & \frac{f}{\Delta v} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{pmatrix}$$

Camera's Forward Projection Matrix

$$\begin{pmatrix} s \bullet u \\ s \bullet v \\ s \end{pmatrix} = C_{3\times 4} \bullet \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{pmatrix}$$

Equation of Camera's Forward Projection



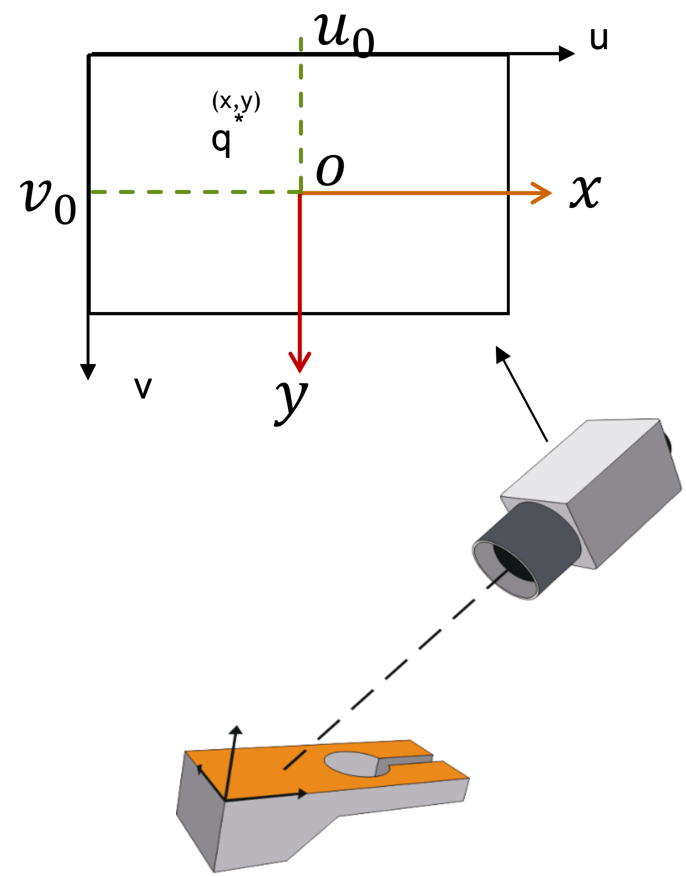
Proof Step 11:

- If we let Z coordinate to be zero, then we obtain the equation of monocular vision's forward projection:

$$\begin{pmatrix} s \bullet u \\ s \bullet v \\ s \end{pmatrix} = C_{3\times4} \bullet \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} s \bullet u \\ s \bullet v \\ s \end{pmatrix} = \mathrm{D}_{3\times3} \bullet \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ 1 \end{pmatrix}$$

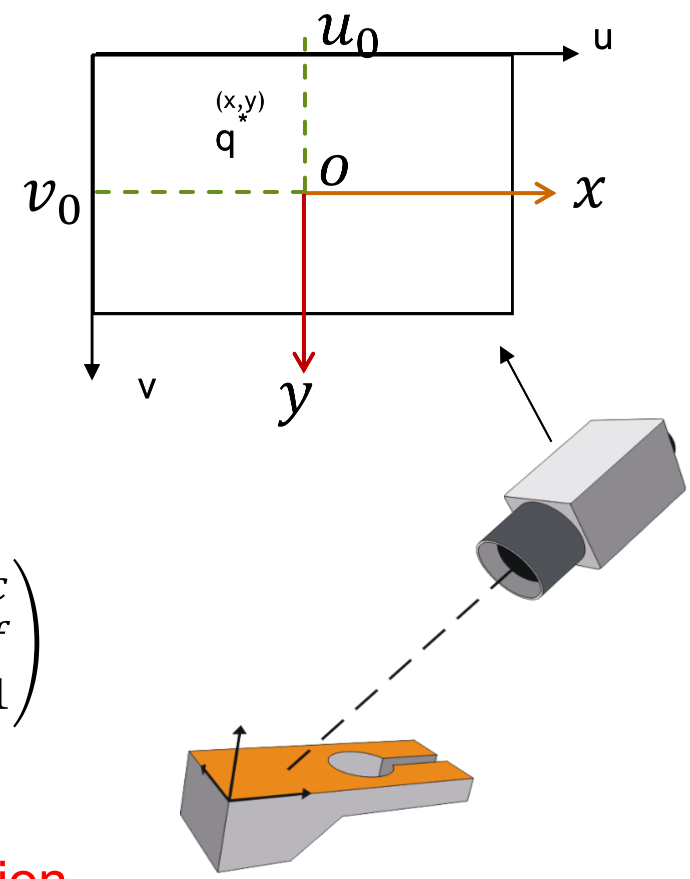
Equation of Monocular Vision's Forward Projection



Proof Step 12:

- By inverting matrix D, then we obtain the equation of monocular vision's inverse projection:

$$\begin{pmatrix} s \bullet u \\ s \bullet v \\ s \end{pmatrix} = \mathrm{D}_{3\times3} \bullet \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ 1 \end{pmatrix}$$

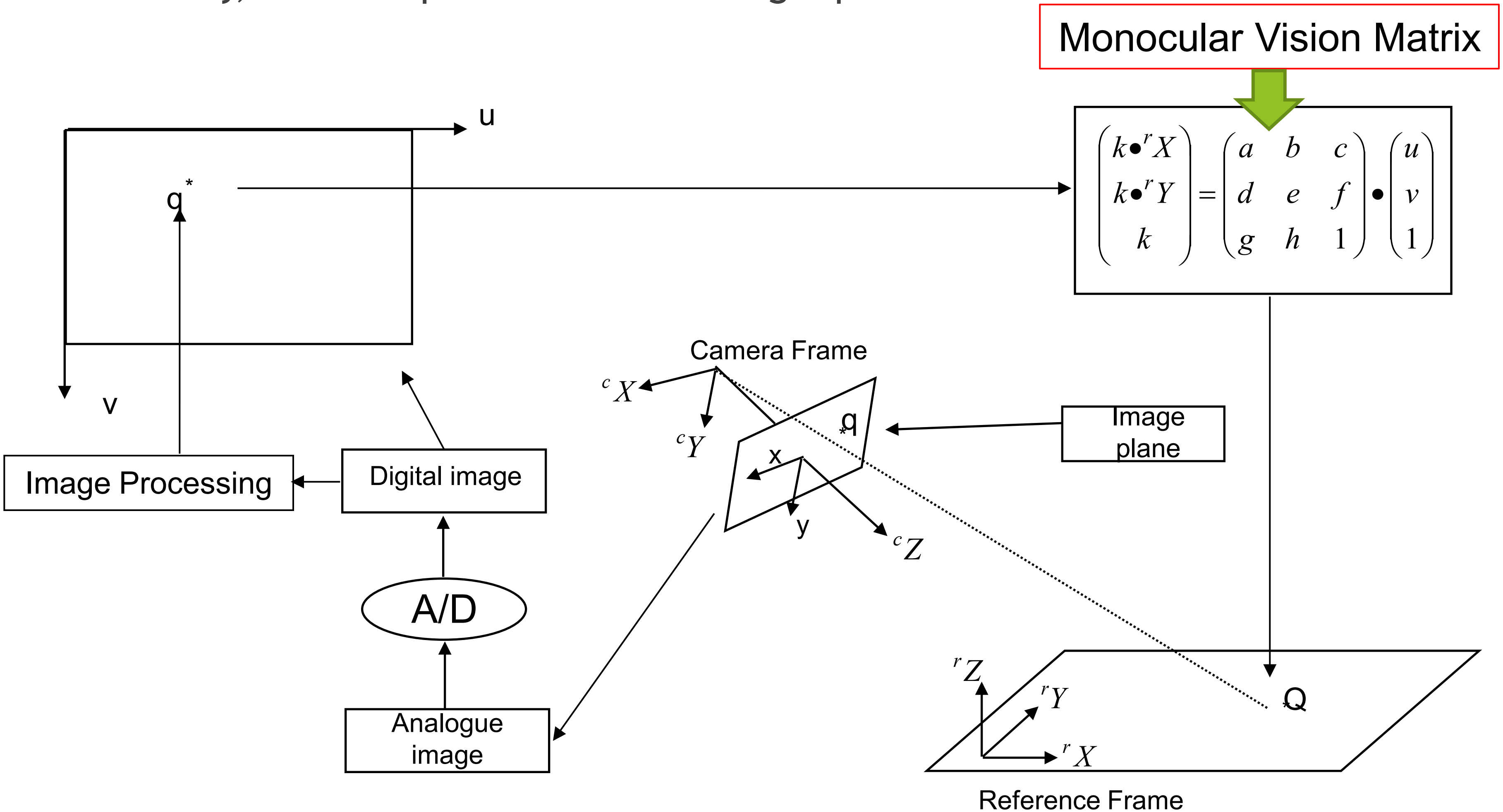


$$\begin{pmatrix} k \bullet {}^{r}X \\ k \bullet {}^{r}Y \\ k \end{pmatrix} = \mathrm{M}_{3\times3} \bullet \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \text{ with } M_{3\times3} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{pmatrix}$$

Equation of Monocular Vision's Inverse Projection

Summary of Proved Result:

- Finally, we have proven the following equation:

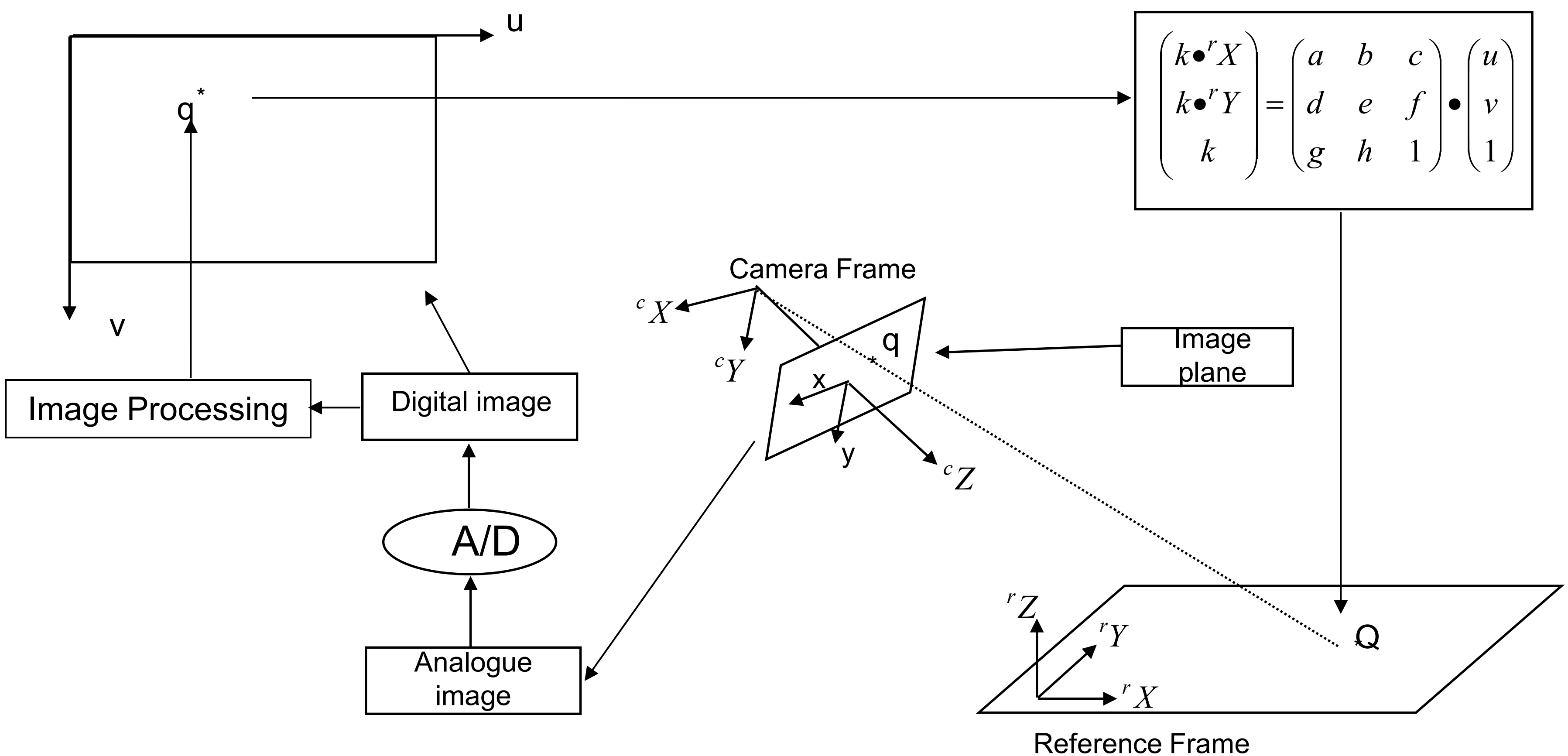


$$\begin{pmatrix} k \bullet {}^{r}X \\ k \bullet {}^{r}Y \\ k \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{pmatrix} \bullet \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

Discussion: How to calibrate monocular vision?

- How to determine the coefficients inside the monocular vision matrix?

Use of At Least Four Pairs of {(u,v), (X,Y)}

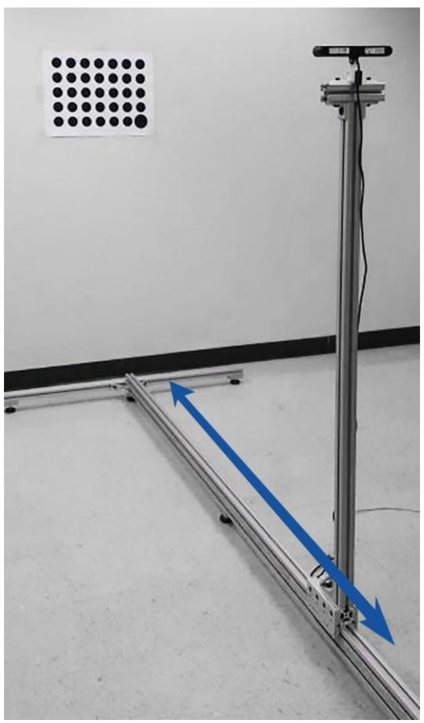


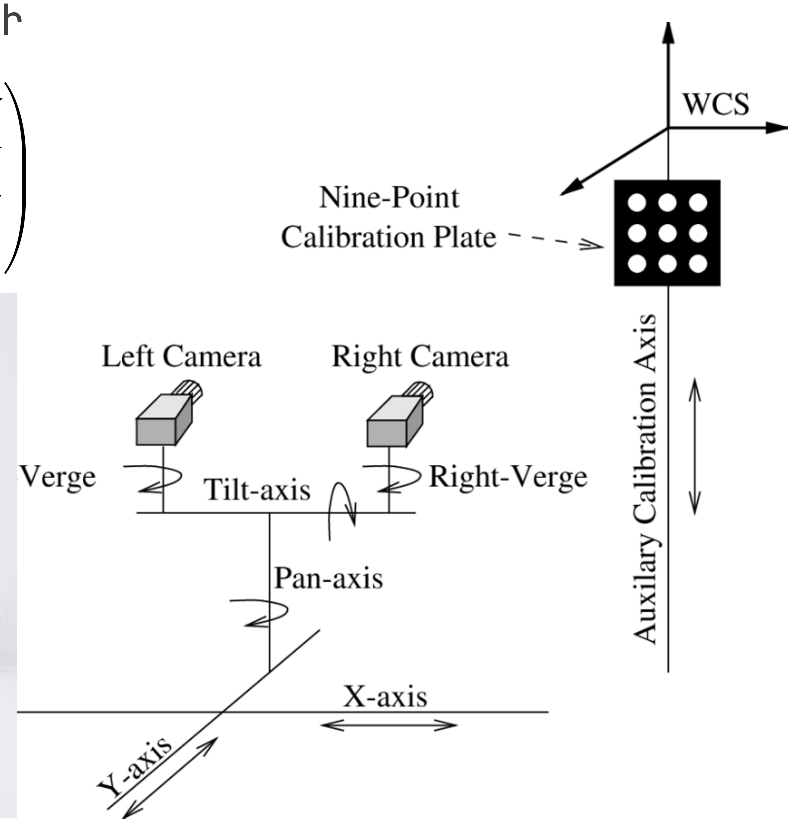
Discussion: How to calibrate a camera?

- The Details of Individual Camera Calibration:
  - There are 11 coefficients inside each M matrix.
  - One pair of {(u,v), (X,Y,Z)} provides 2 equations for each M matrix.
  - Six (6) pairs of {(u,v), (X,Y,Z)} are sufficient enough

$$\begin{pmatrix} s \bullet u_l \\ s \bullet v_l \\ s \end{pmatrix} = \begin{bmatrix} c_{l1} & c_{l2} & c_{l3} & c_{l4} \\ c_{l5} & c_{l6} & c_{l7} & c_{l8} \\ c_{l9} & c_{l10} & c_{l11} & 1 \end{bmatrix} \bullet \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix}$$

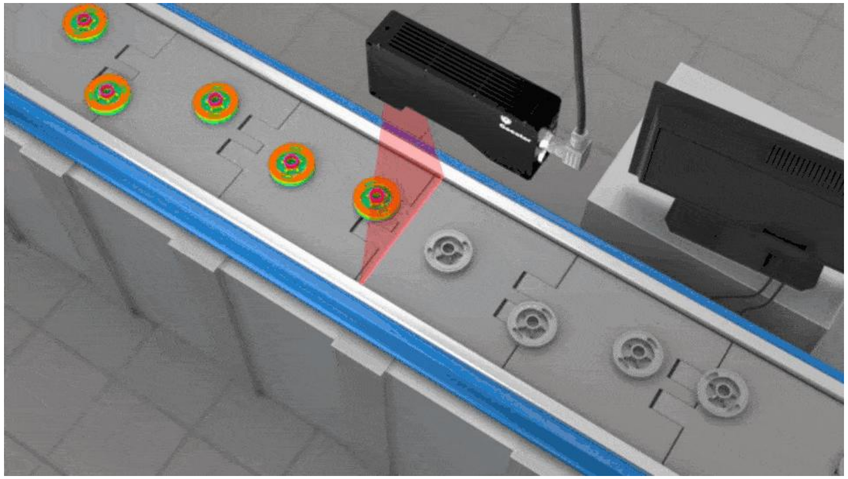
$$\begin{pmatrix} s \bullet u_r \\ s \bullet v_r \\ s \end{pmatrix} = \begin{bmatrix} c_{r1} & c_{r2} & c_{r3} & c_{r4} \\ c_{r5} & c_{r6} & c_{r7} & c_{r8} \\ c_{r9} & c_{r10} & c_{r11} & 1 \end{bmatrix} \bullet \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix}$$



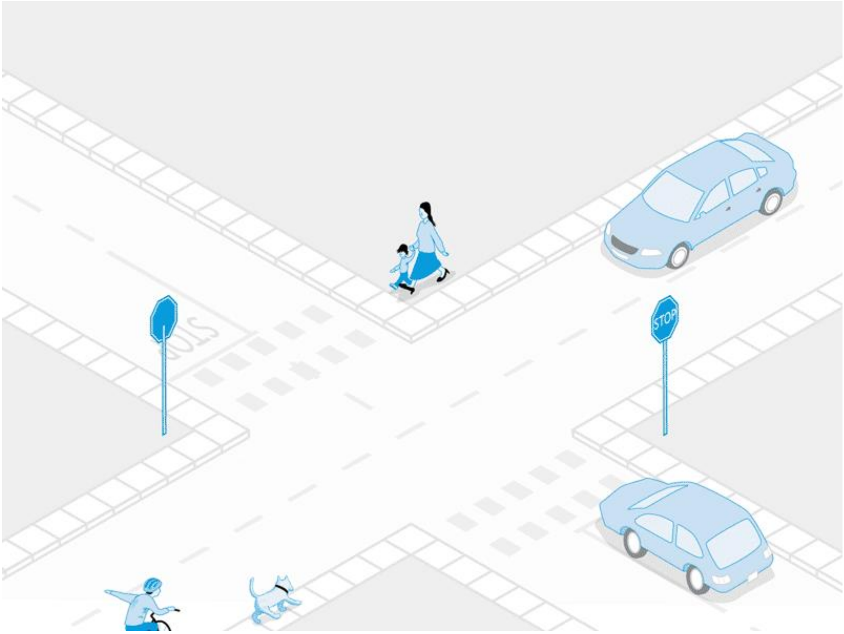


Summary of Lecture 2

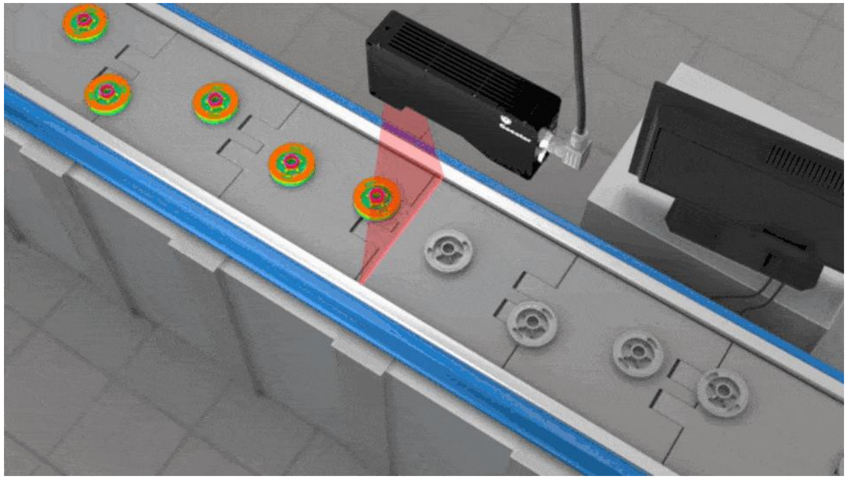
- Basics of 2D Geometry
- Parameters of 2D Geometry
- Measurement of 2D Geometry



Outline of Module 2



- Perception of Photometry
- Perception of 2D Geometry
- Perception of 3D Geometry



Module 2

MA4825 Robotics

Lecture 3

Perception of 3D Geometry

Xie Ming, PhD (France)
http://personal.ntu.edu.sg/mmxie

Outline of Lecture 3

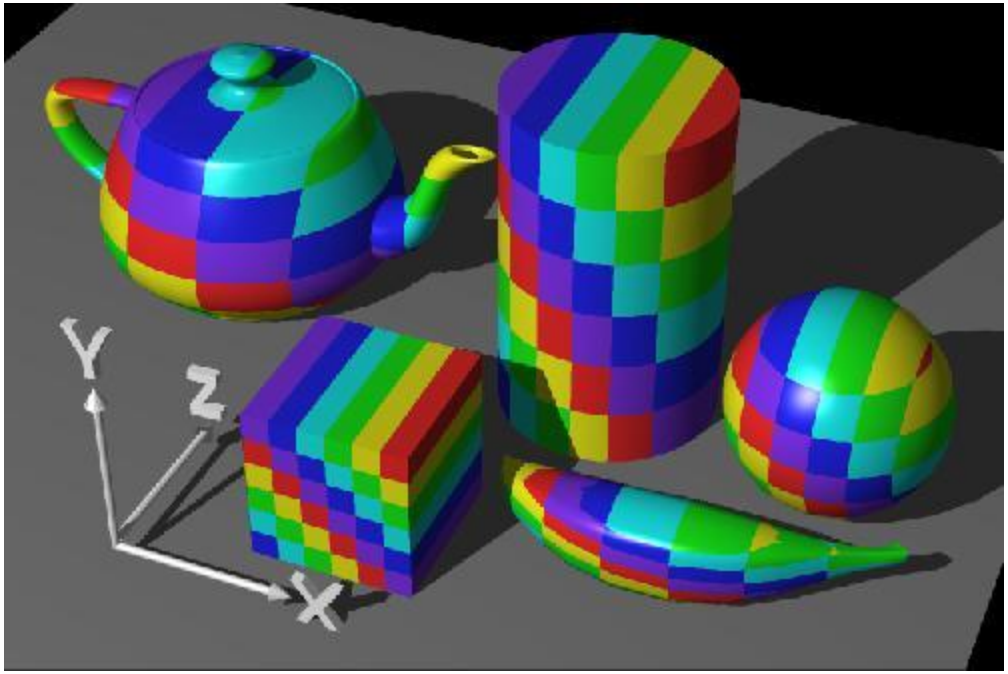
- Basics of 3D Geometry
- Parameters of 3D Geometry
- Measurement of 3D Geometry

Outline of Lecture 3

- Basics of 3D Geometry
- Parameters of 3D Geometry
- Measurement of 3D Geometry

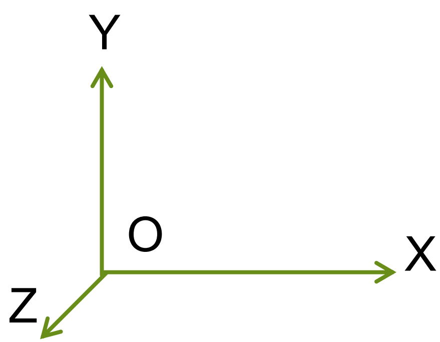
Understanding 3D Geometry (1)

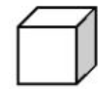
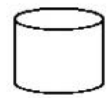
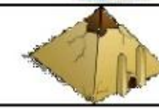
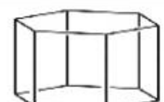
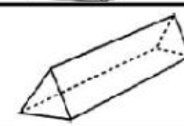
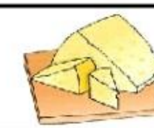
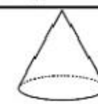
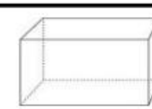
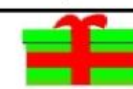
- 3D geometry refers to the appearance of physical entities in a three-dimensional space.
- 3D space consists of a set of positions which are fully determined with three coordinates.



Understanding 3D Geometry (2)

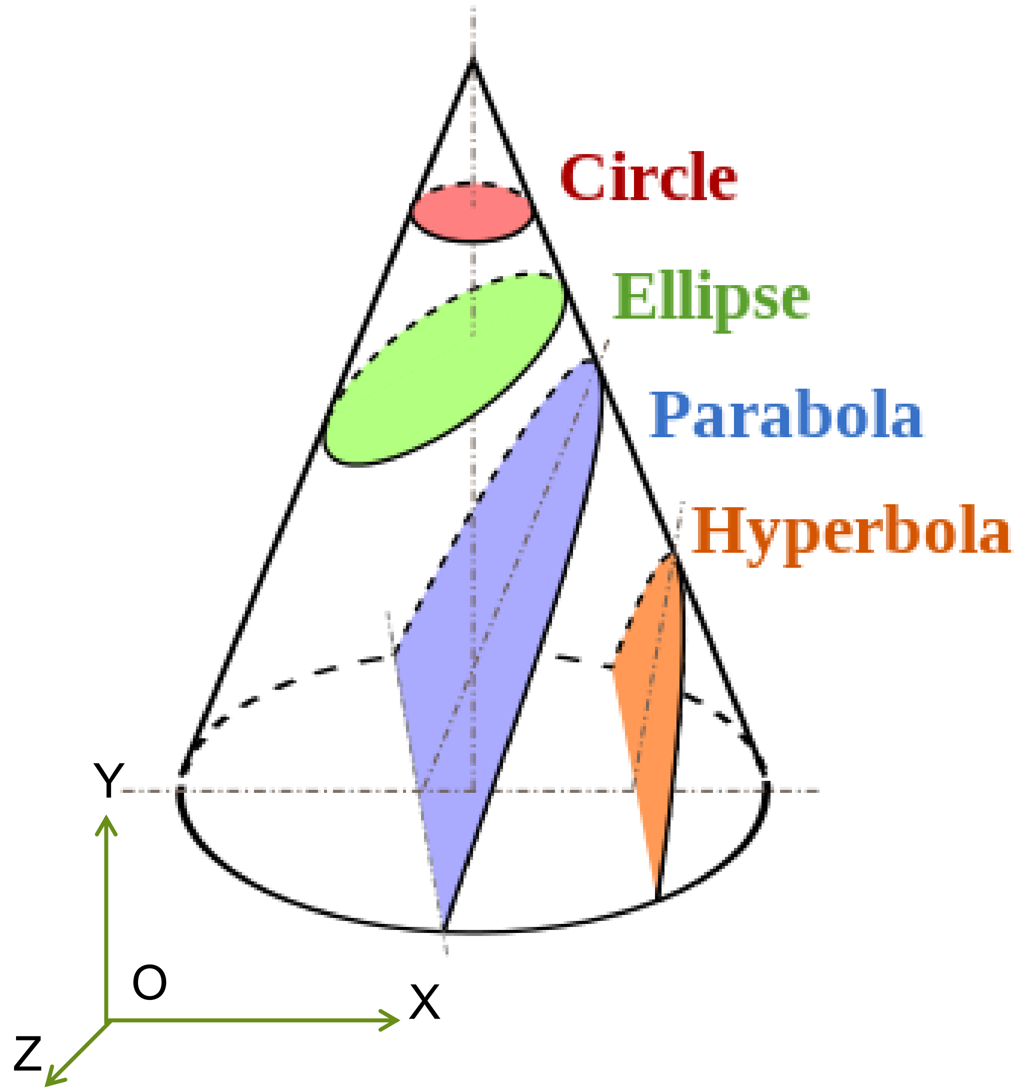
- The appearance of physical entities in a 3D space is manifested in the form of 3D shapes which are also called objects.



3-D Shape	Picture	Example
cube		
cylinder		
sphere		
pyramid		
hexagonal prism		
triangular prism		
cone		
rectangular prism		

Example

- Intersections between 3D shape and planes will result in 2D shapes.
- 3D shape has outer surface, inner surface and body between them.
- 2D shape has outer edge, inner edge and body between them.



Understanding 3D Geometry (3)

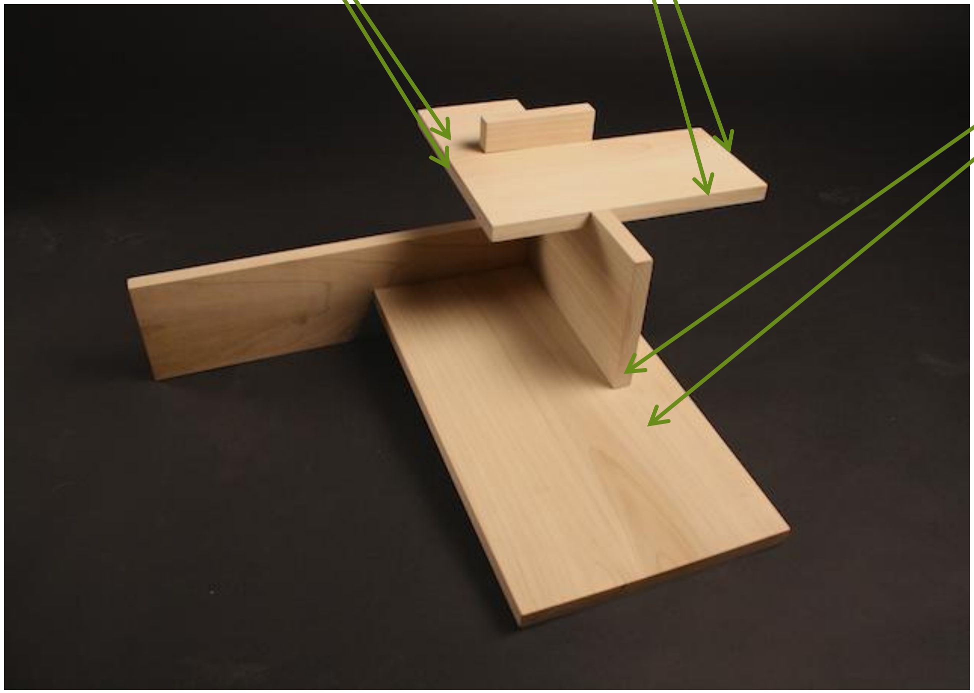
- Complex shapes in a 3D space are the results of compositional rules such as:
  - Connect Between Points(point of shape 1, point of shape 2) at Angle(angle between shape 1 and shape 2)
  - Connect Between Curves(curve of shape 1, curve of shape 2) with Offset(distance between the endpoints of two curves)
  - Connect Between Surfaces(surface of shape 1, surface of shape 2) with Orientation(angle about normal to surface) and Offset(distance between the origins of two surfaces)

Example

Edge Lines
Connected Between Points

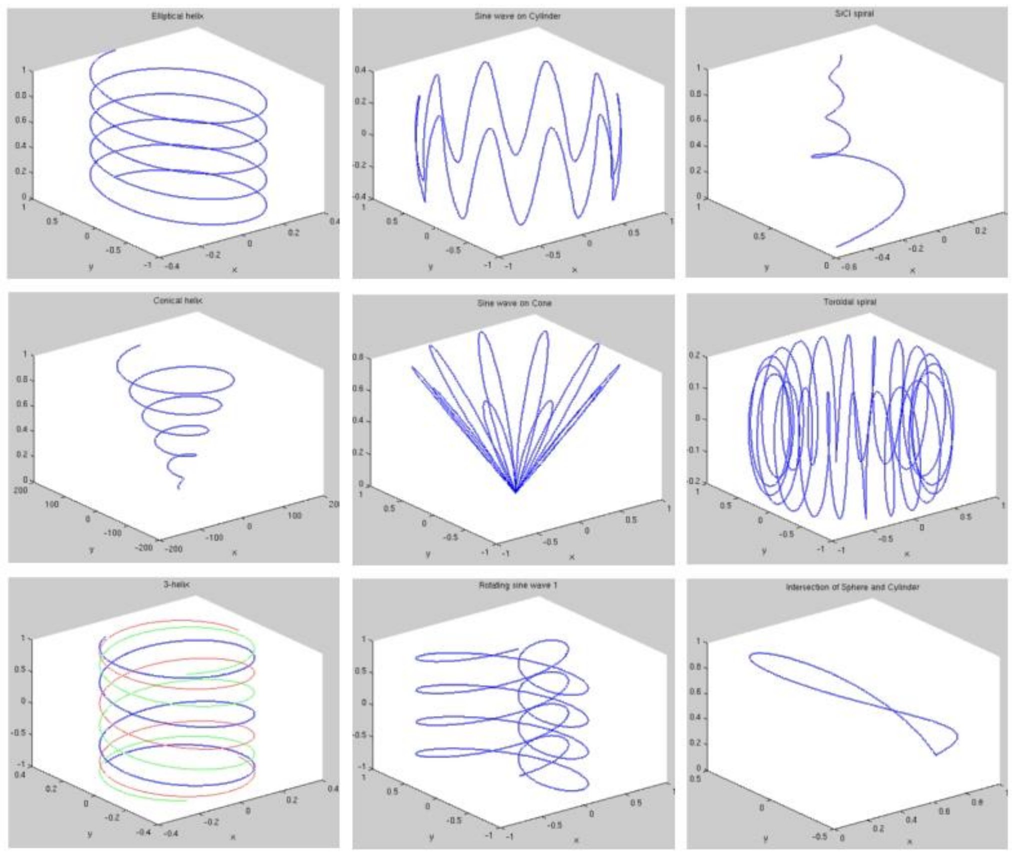
Surfaces
Connected Between Lines

Blocks
Connected Between Surfaces



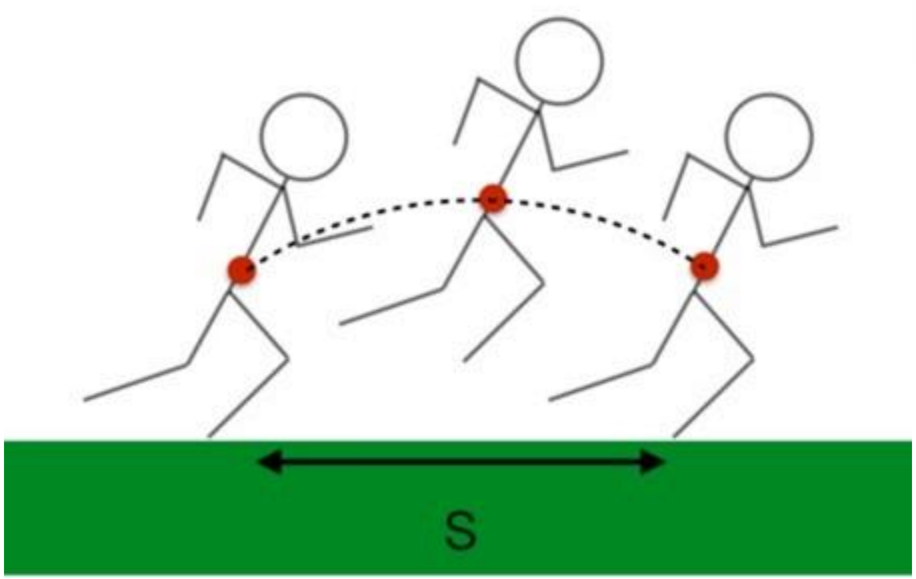
Understanding 3D Geometry (4)

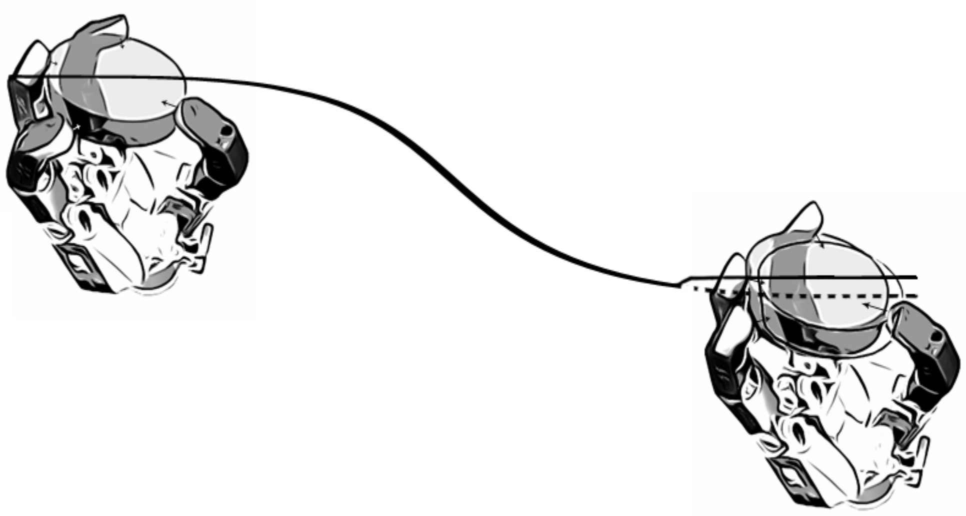
- The appearance of physical entities in a 3D space also includes the travelled locations.



Understanding 3D Geometry (5)

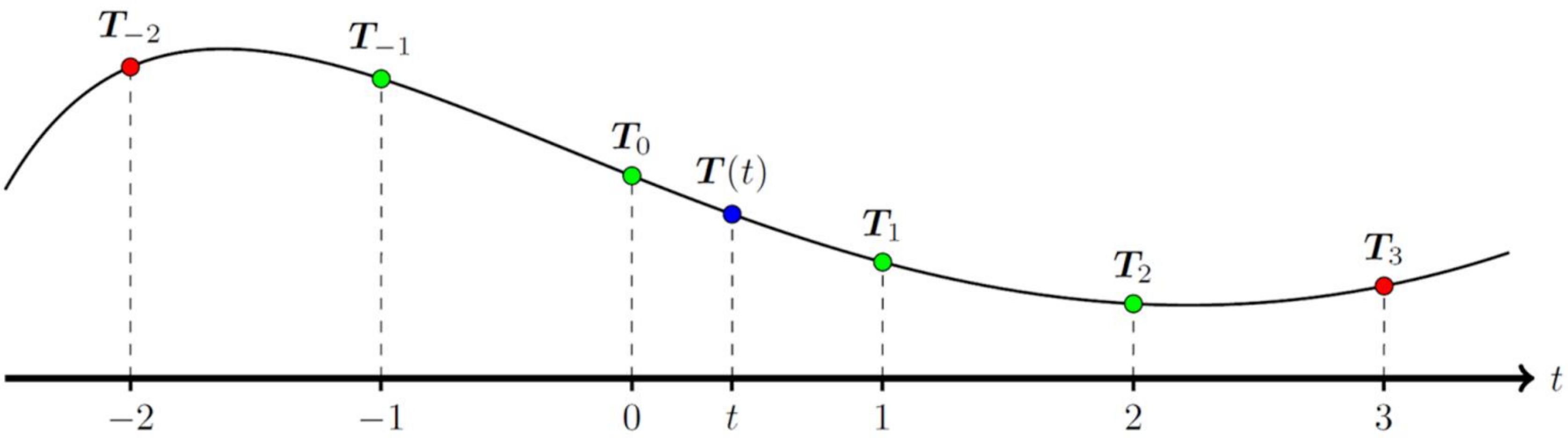
- The spatial locations travelled or to be travelled are called paths.



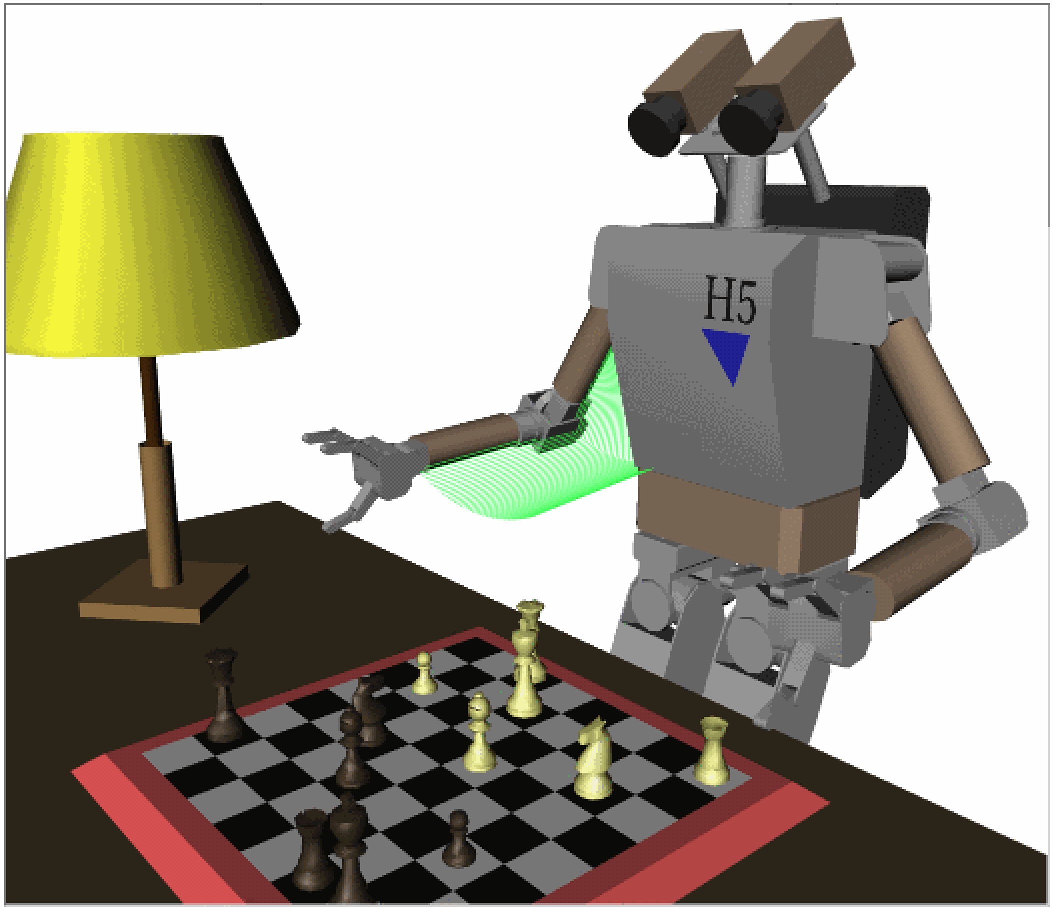


Understanding 3D Geometry (6)

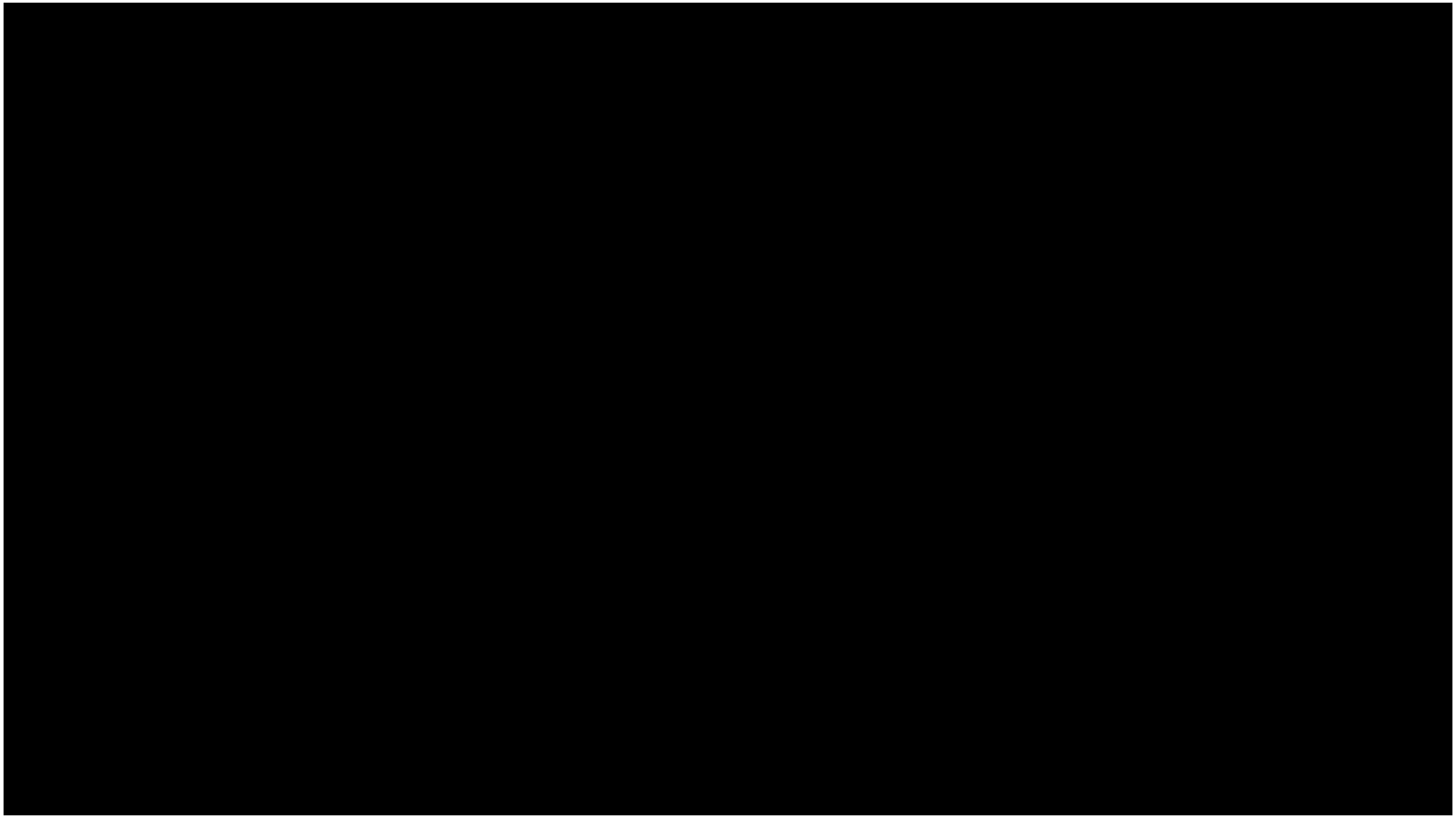
- The spatial locations with time constraint are called trajectories.



Example of Motion Planning and Control



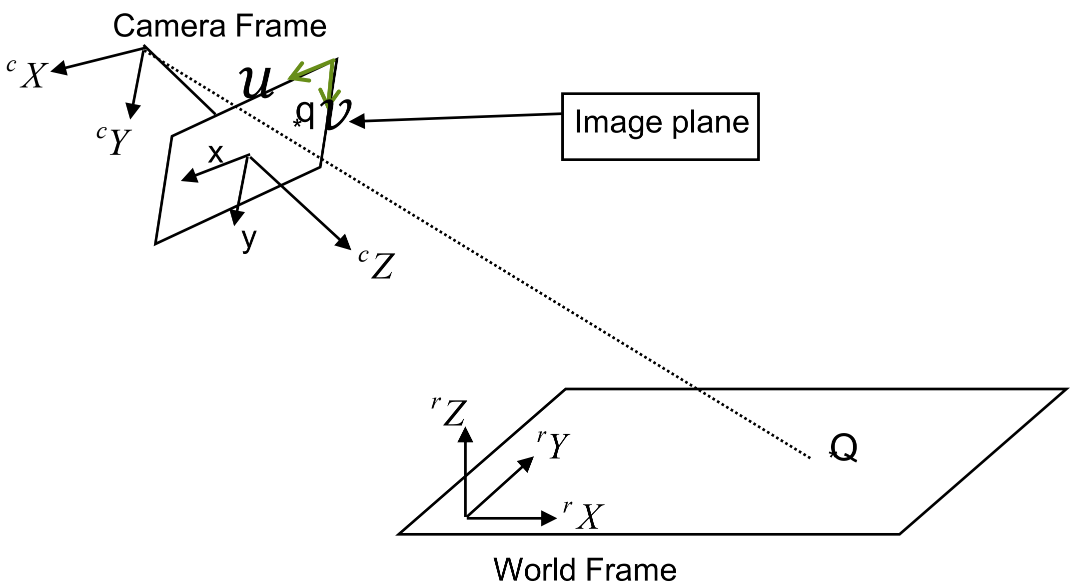
What is the scenario of coordinate transformations in general?



Basics of Homogeneous Transformation

$$H_{camera} = \begin{bmatrix} R_{camera} & T_{camera} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

$$H_{world} = \begin{bmatrix} R_{camera}^{-1} & -R_{camera}^{-1} \times T_{camera} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$



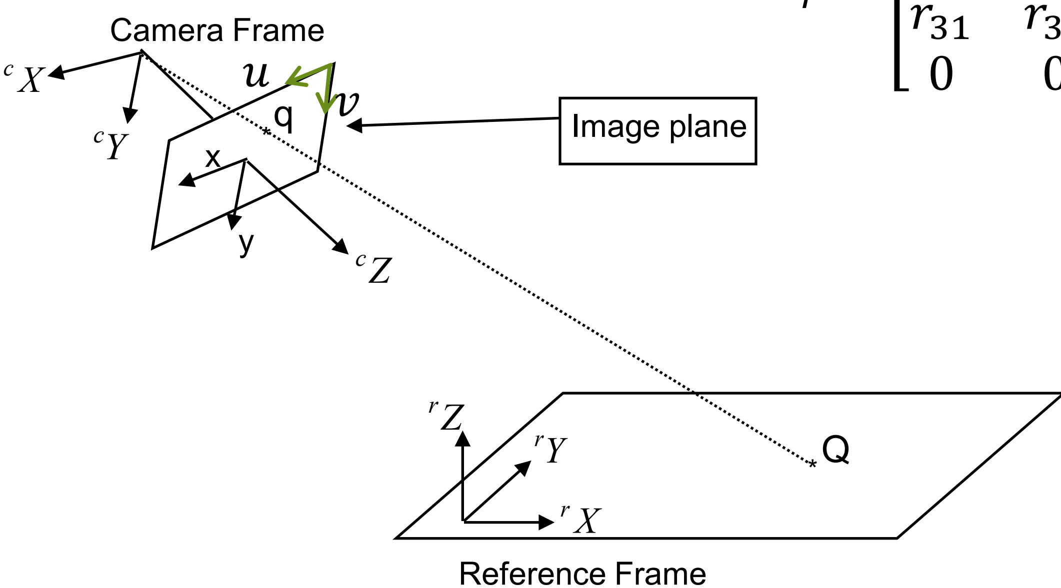
$$H_{camera} \times H_{world} = I_{4\times4}$$

$$H_{camera} = H_{world}^{-1}$$

Transformation of Coordinates in 3D Space

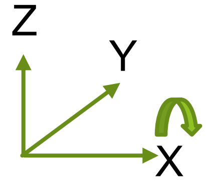
- The coordinates in the reference frame can be transformed into the coordinates in the camera frame.

$$ {}^{c}H_{r} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



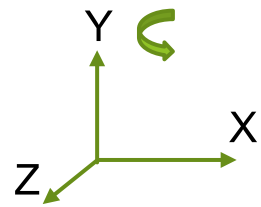
$$ \begin{bmatrix} {}^{c}X \\ {}^{c}Y \\ {}^{c}Z \\ 1 \end{bmatrix} = {}^{c}H_{r} \bullet \begin{bmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{bmatrix} $$

Practices with Rotational Transformation

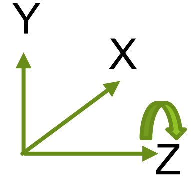


Before Rotation

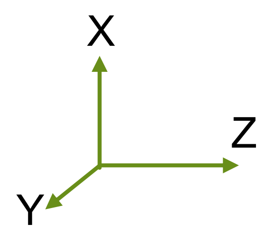
After Rotation About X Axis

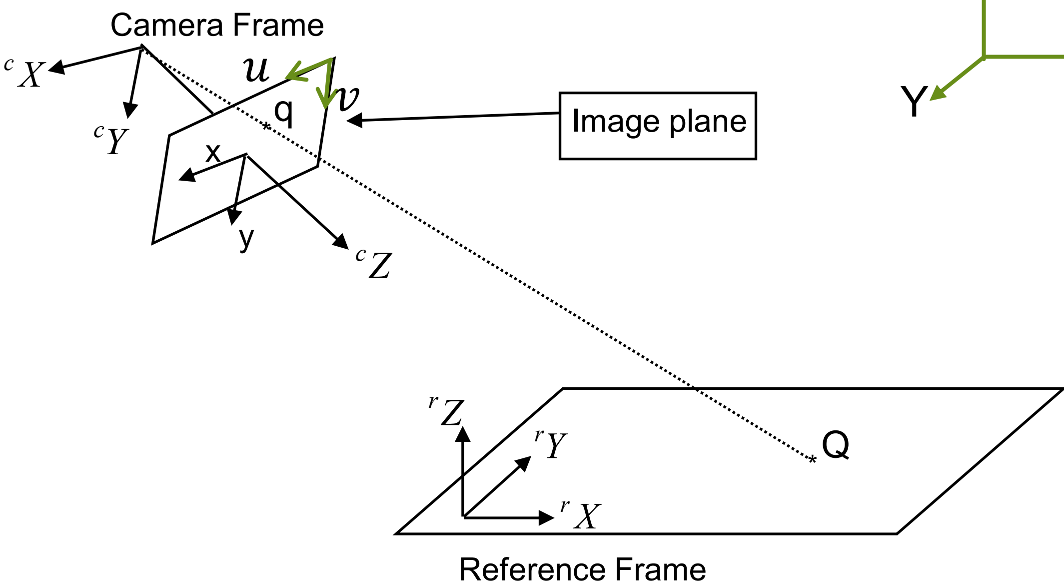


After Rotation About Y Axis



After Rotation About Z Axis






```
ax = 90*pi/180;
rx = [1    0          0 ;
      0  cos(ax)  -sin(ax);
      0  sin(ax)   cos(ax)];

ay = 90*pi/180;
ry = [cos(ay)    0     sin(ay);
        0        1     0 ;
       -sin(ay)  0    cos(ay)];

az = 90*pi/180;
rz = [cos(az)   -sin(az)   0;
      sin(az)    cos(az)   0;
      0          0         1];

r = rx*ry*rz
```

Command Window

New to MATLAB? See resources for Getting Started.

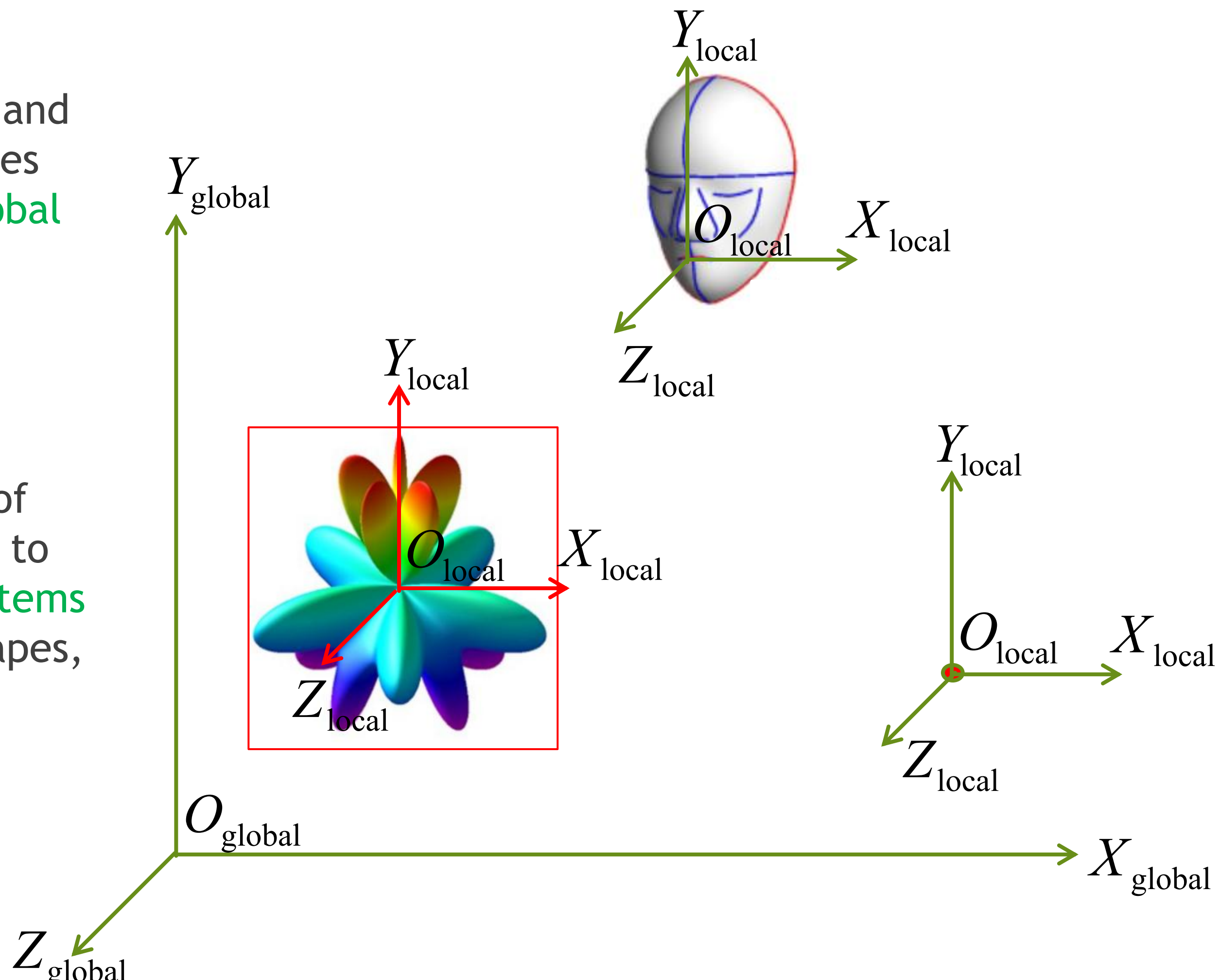
```
r =

    0.0000   -0.0000    1.0000
    0.0000   -1.0000   -0.0000
    1.0000    0.0000    0.0000
```

Outline of Lecture 3

- Basics of 3D Geometry
- Parameters of 3D Geometry
- Measurement of 3D Geometry

There Are Two Types of Parameters

- Pose:
  - It refers to positions and orientations of entities with respect to a Global Coordinate System.
- Shape:
  - It refers to surfaces of entities with respect to Local Coordinate Systems assigned to these shapes, respectively.

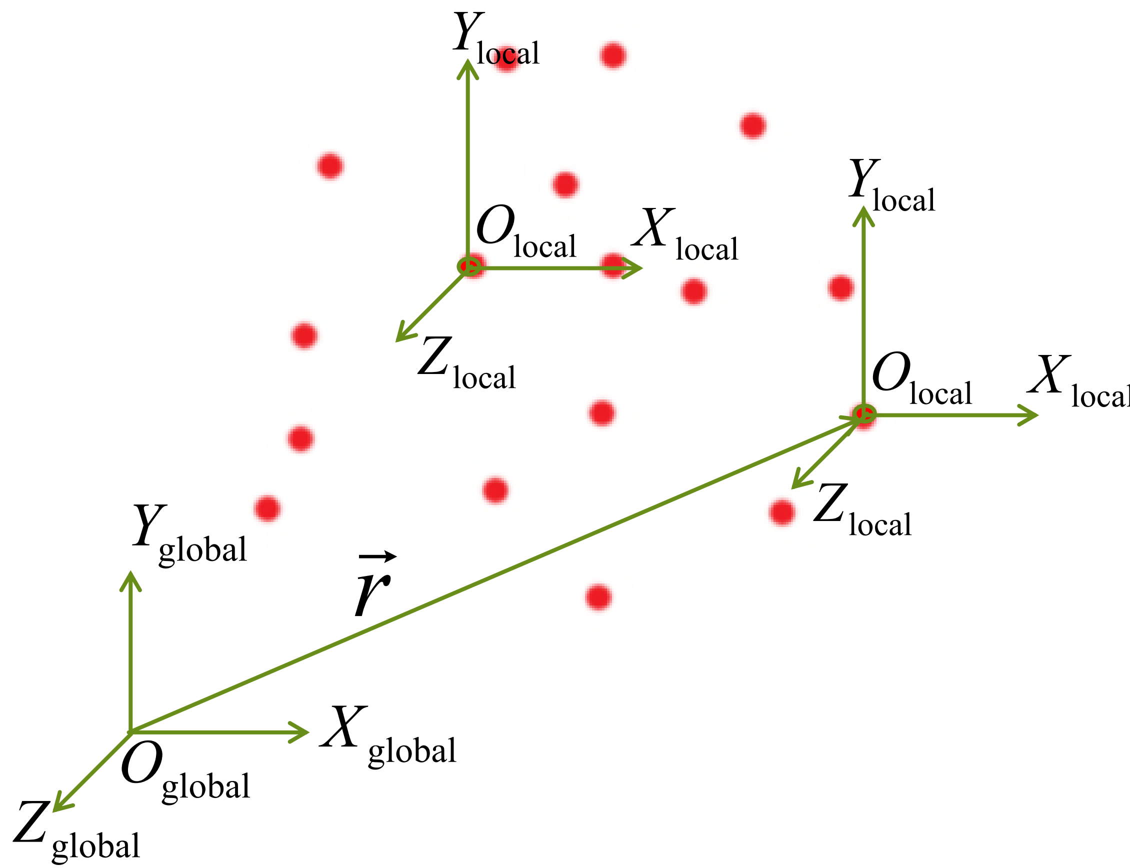


Representation of Positions in 3D Space without Time Constraints

$$\vec{r} = (x, y, z)$$

$$f(x, y) = 0$$

$$f(x, z) = 0$$

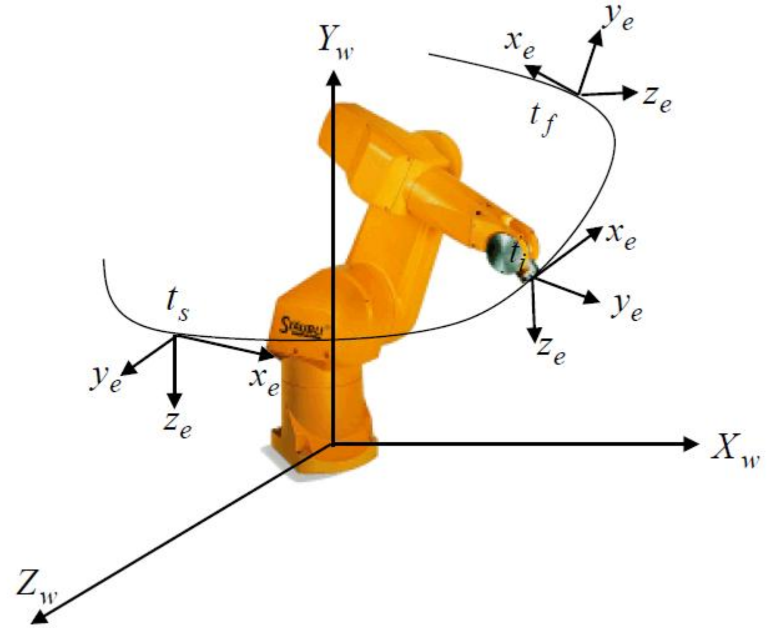


Representation of Positions in 3D Space with Time Constraints

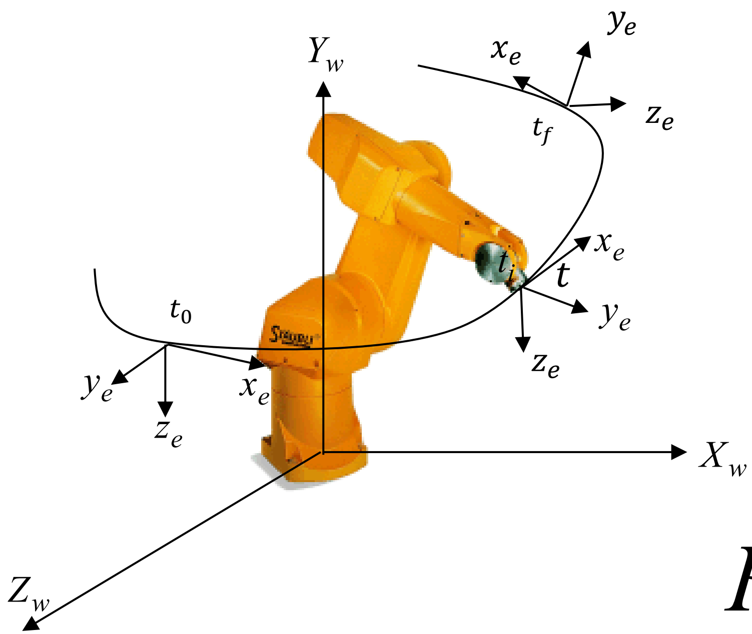
$$\vec{r}(t)=\begin{pmatrix}x(t)\\ y(t)\\ z(t)\end{pmatrix}=\begin{pmatrix}a_x t^3+b_x t^2+c_x t+d_x\\ a_y t^3+b_y t^2+c_y t+d_y\\ a_z t^3+b_z t^2+c_z t+d_z\end{pmatrix}$$

$$\vec{r}(t)=\begin{pmatrix}x(t)\\ y(t)\\ z(t)\end{pmatrix}=\begin{pmatrix}a_x t^2+b_x t+c_x\\ a_y t^2+b_y t+c_y\\ a_z t^2+b_z t+c_z\end{pmatrix}$$

$$\vec{r}(t)=\begin{pmatrix}x(t)\\ y(t)\\ z(t)\end{pmatrix}=\begin{pmatrix}a_x t+b_x\\ a_y t+b_y\\ a_z t+b_z\end{pmatrix}$$



Representation of Orientations in 3D Space



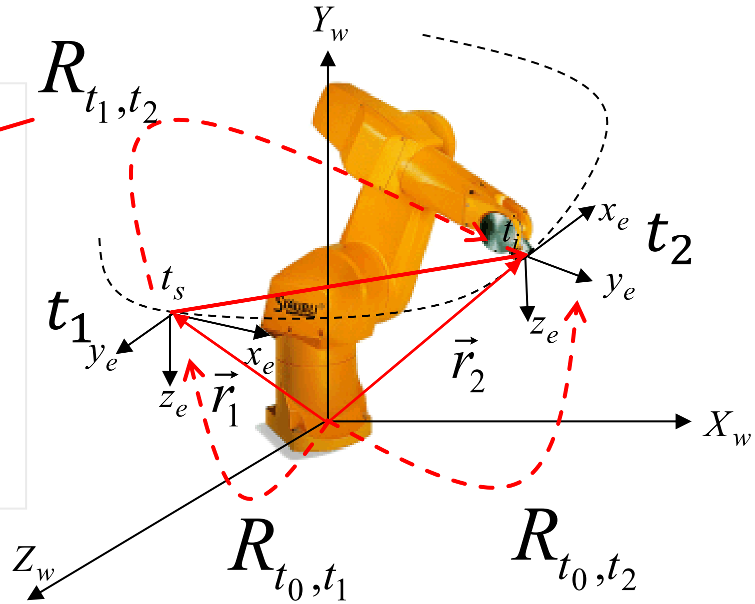
X Axis's Unit Vector | Y Axis's Unit Vector | Z Axis's Unit Vector

$$R_{t_0,t} = \begin{bmatrix} x_i(t) & x_j(t) & x_k(t) \\ y_i(t) & y_j(t) & y_k(t) \\ z_i(t) & z_j(t) & z_k(t) \end{bmatrix}$$

This is the orientation at time t, as the result of the change of orientation from time t0 to time t.

Equivalent Axis and Angle of Rotation in 3D Space

$$R_{t_1,t_2} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$



This is the orientation at time t2, as the result of the change of orientation from time t1 to time t2.

Equivalent Axis of Rotation:

$$\vec{r}\Big|_{t_1,t_2} = \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

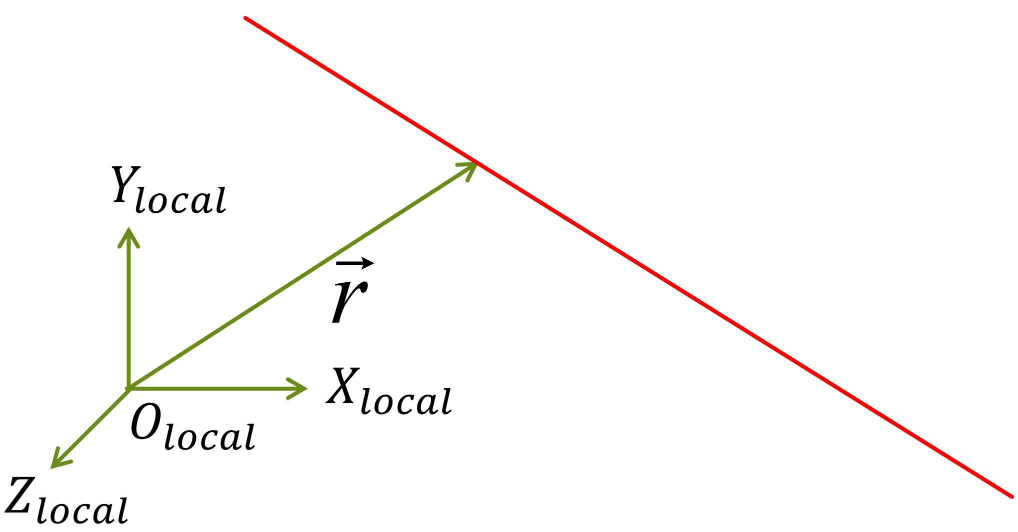
Equivalent Angle of Rotation:

$$\theta_{t_1,t_2} = \arccos\left(\frac{r_{11} + r_{22} + r_{33} - 1}{2}\right)$$

Simple and Useful for Adding Time Constraint

Parameters of Lines in 3D Space

$$ax + by + cz + d = 0$$
$$ex + fy + gz + h = 0$$



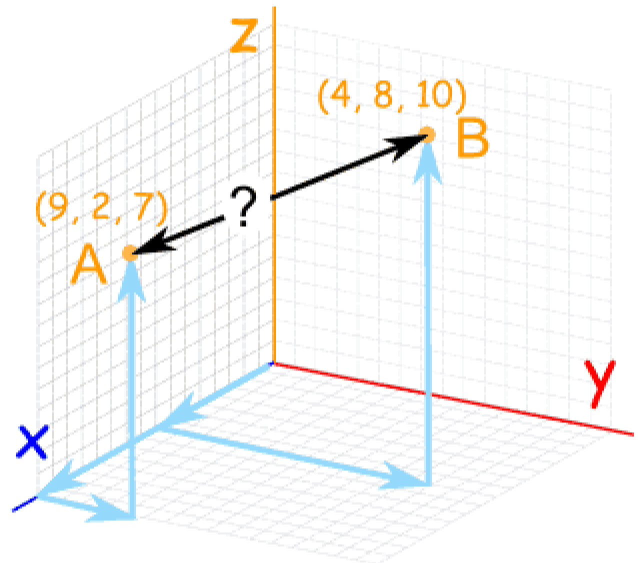
Example

- What are the parameters of the line which passes through points A(9,2,7) and B(4,8,10)?

- Answer:

$$9 = 7a + b \qquad 4 = 10a + b$$

$$2 = 7c + d \qquad 8 = 10c + d$$



$$\begin{pmatrix} 7 & 1 & 0 & 0 \\ 0 & 0 & 7 & 1 \\ 10 & 1 & 0 & 0 \\ 0 & 0 & 10 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 4 \\ 8 \end{pmatrix} \qquad \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 7 & 1 & 0 & 0 \\ 0 & 0 & 7 & 1 \\ 10 & 1 & 0 & 0 \\ 0 & 0 & 10 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 9 \\ 2 \\ 4 \\ 8 \end{pmatrix}$$

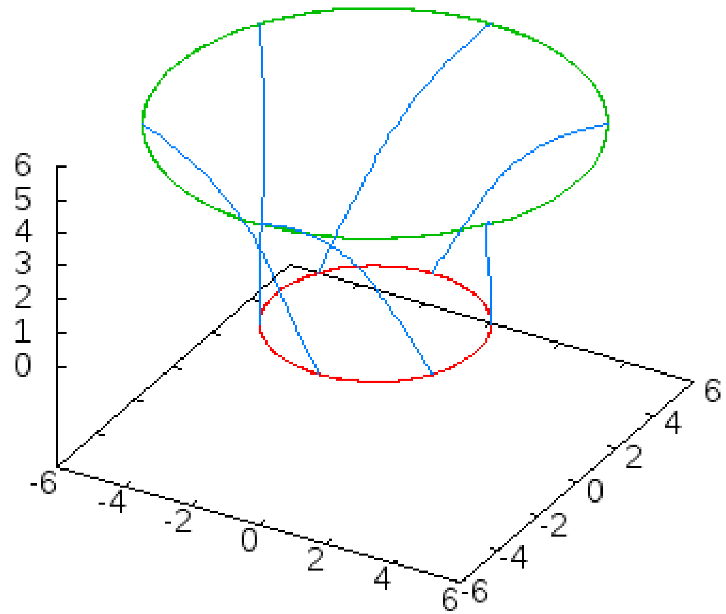
$$x = az + b$$

$$y = cz + d$$

Parameters of Circles in 3D Space

$$(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 = r^2$$

$$ax + by + cz + d = 0$$



A 3D circle is the intersection between a sphere and a plane.

Example

- You are given three points on a circle, List down the equations which are sufficient enough to determine the parameters of the circle.

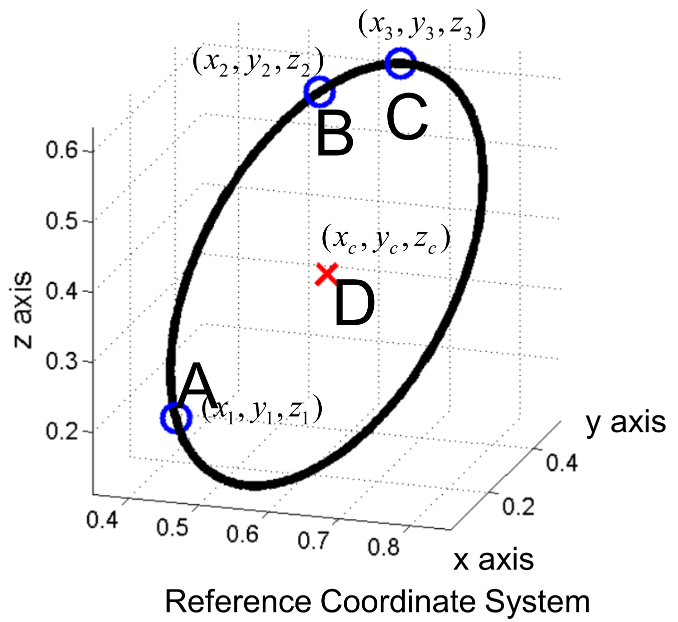
- Answer:

$$(x_1 - x_c)^2 + (y_1 - y_c)^2 + (z_1 - z_c)^2 = r^2$$

$$(x_2 - x_c)^2 + (y_2 - y_c)^2 + (z_2 - z_c)^2 = r^2$$

$$(x_3 - x_c)^2 + (y_3 - y_c)^2 + (z_3 - z_c)^2 = r^2$$

$$ax + by + cz + 1 = 0$$



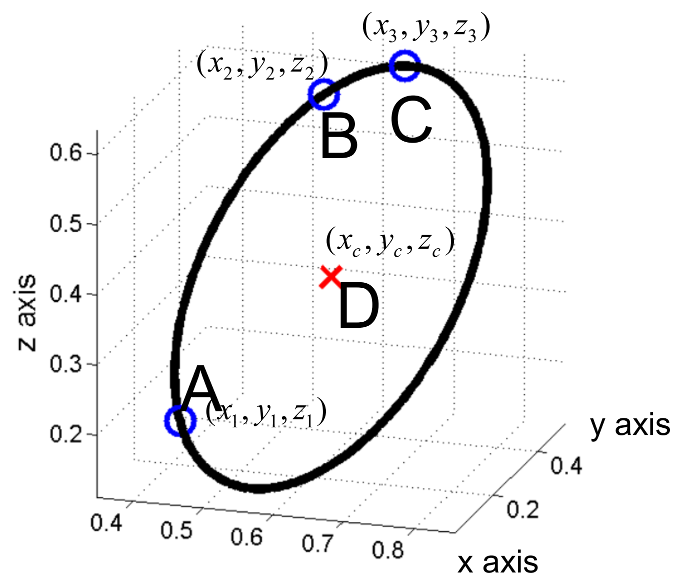
How to estimate the parameters?

- First step to estimate the equation of supporting plane.
- Second step to estimate the center of the circle.
- Final step to estimate the radius.

- Hint to first step:

$$ax + by + cz + 1 = 0$$

$$\begin{cases} ax_1 + by_1 + cz_1 + 1 = 0 \\ ax_2 + by_2 + cz_2 + 1 = 0 \\ ax_3 + by_3 + cz_3 + 1 = 0 \end{cases}$$

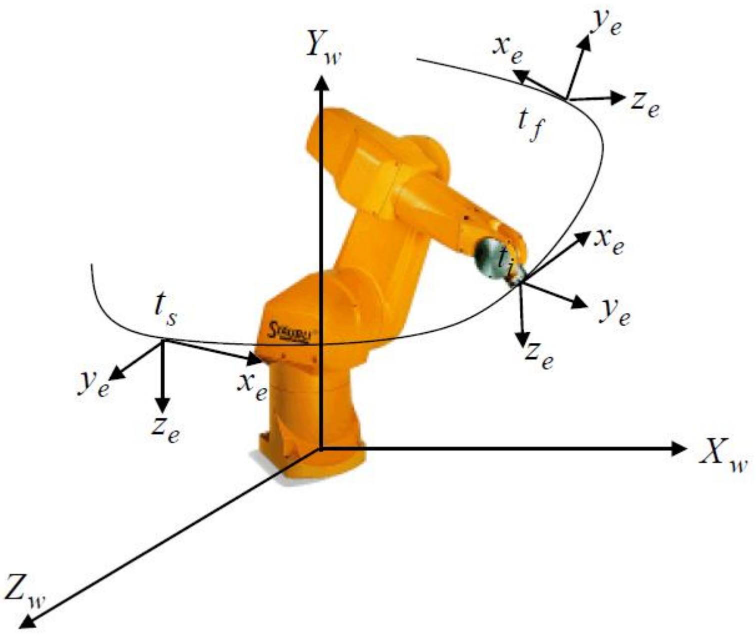


Reference Coordinate System

Parameters of Curves in 3D Space

$$\begin{bmatrix} x^n & x^{n-1} & \cdots & 1 \end{bmatrix} A_{n\times n} \begin{bmatrix} z^n \\ z^{n-1} \\ \cdots \\ 1 \end{bmatrix} = 0$$

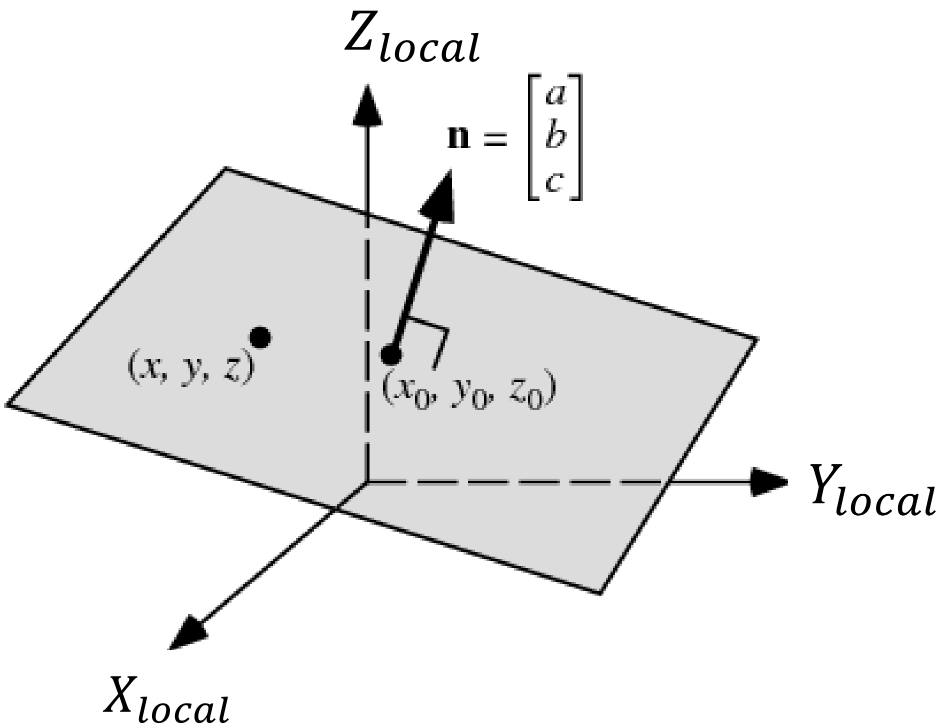
$$\begin{bmatrix} y^n & y^{n-1} & \cdots & 1 \end{bmatrix} B_{n\times n} \begin{bmatrix} z^n \\ z^{n-1} \\ \cdots \\ 1 \end{bmatrix} = 0$$

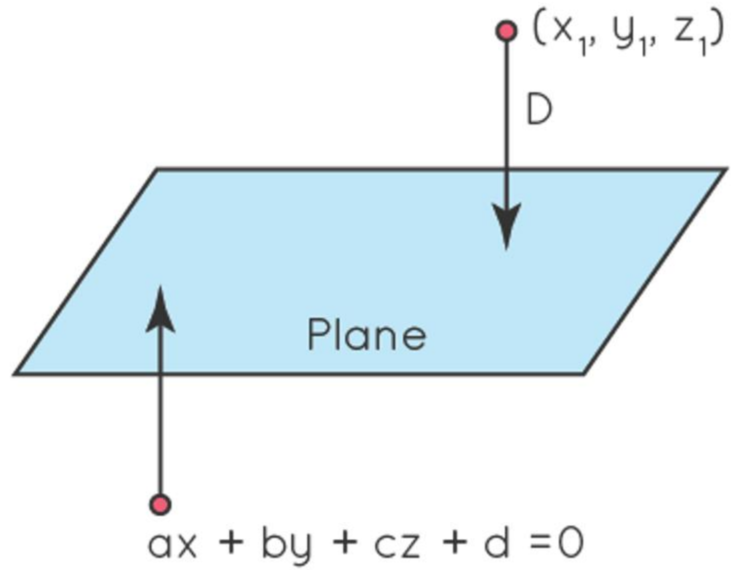


n=1,2,…

Representation of Plane-type Shape (e.g. 3D Rectangle or Triangle) in 3D Space

$$ax + by + cz + d = 0$$

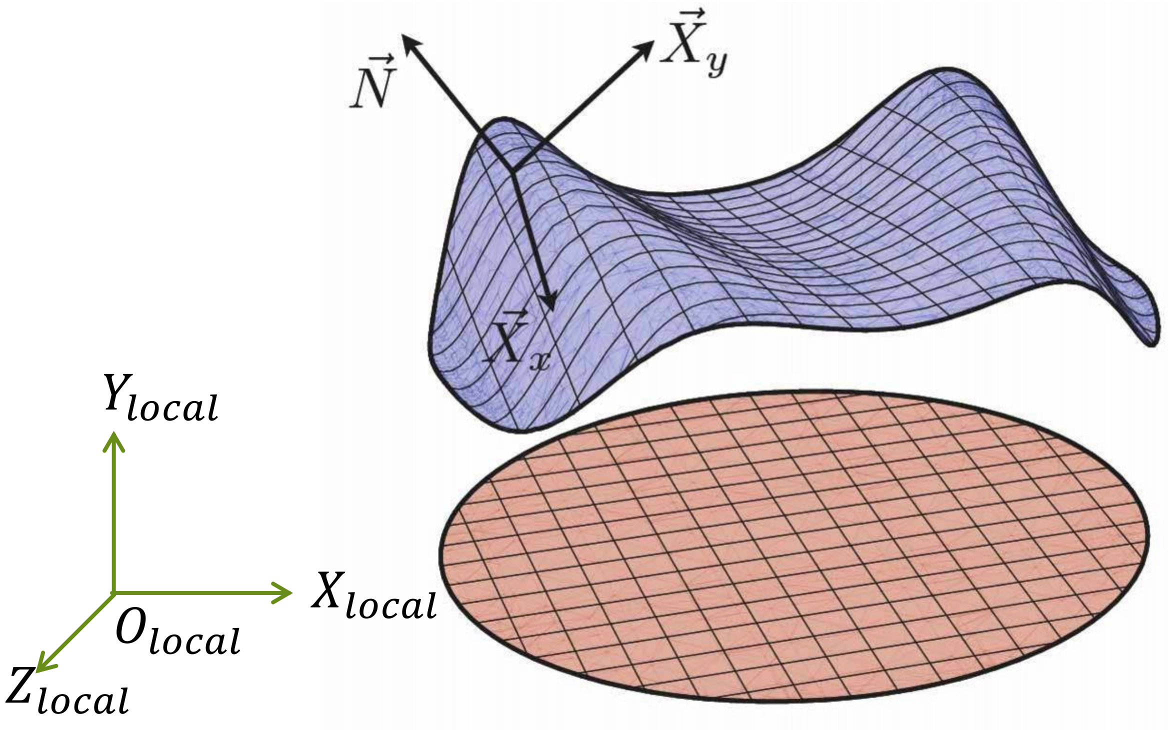




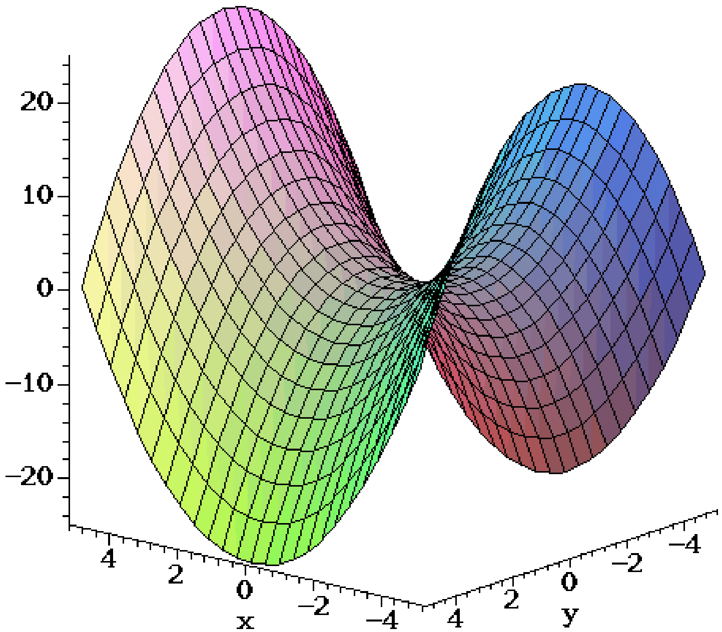
$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

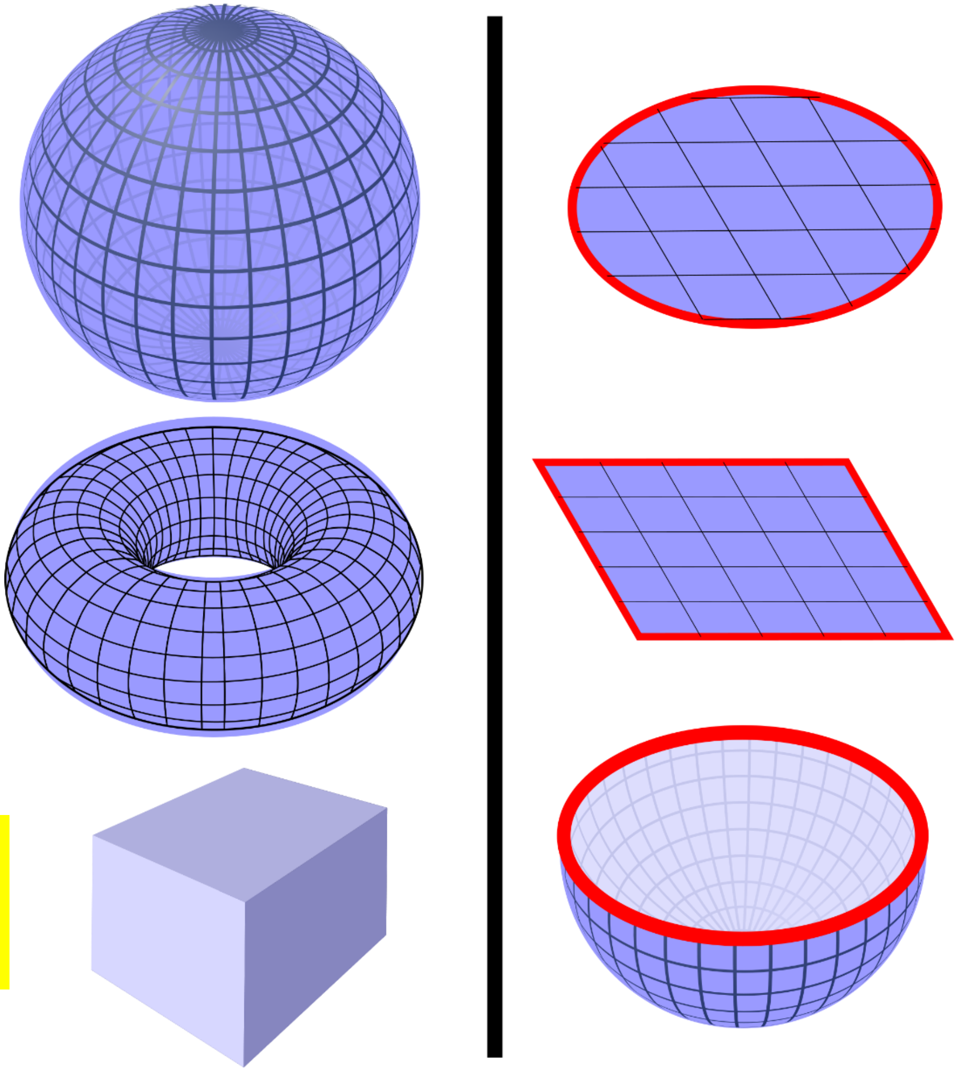
Parameters of Surfaces in 3D Space

$$a_i x + b_i y + c_i z + d_i = 0 \quad i = 0,1,2,3,\ldots$$



Example



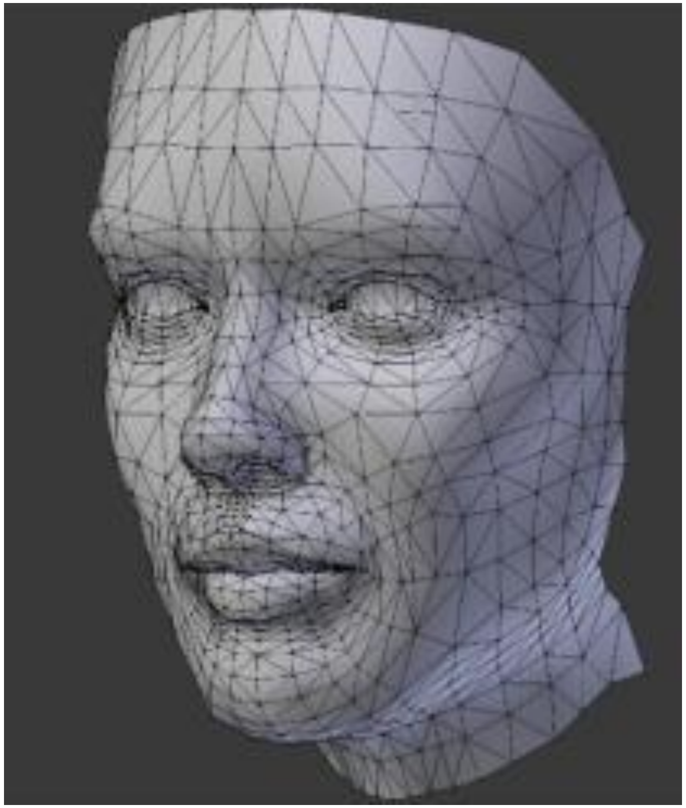


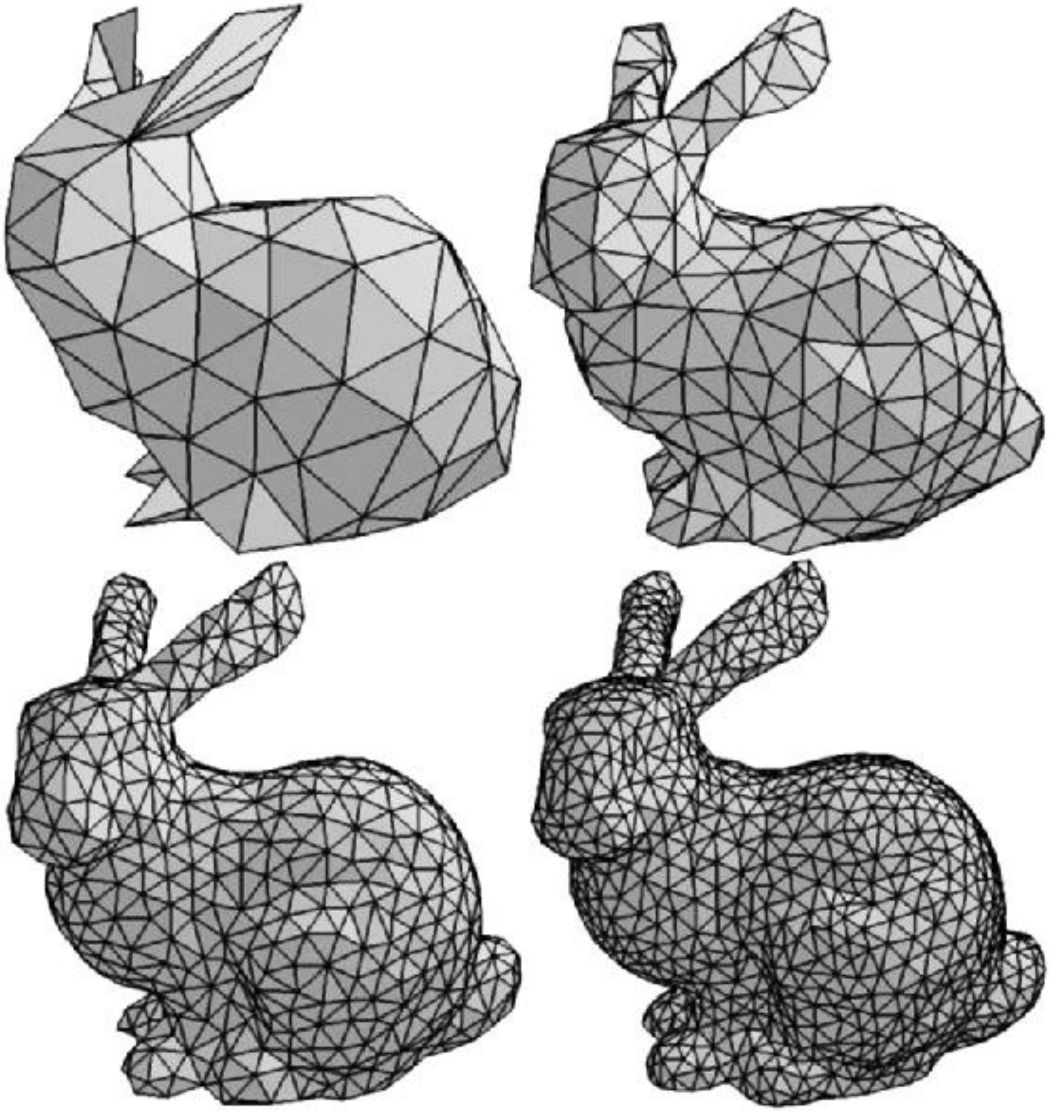
Any surface could be a set of Triangles or Rectangles

Next Slide

Example

More Examples …



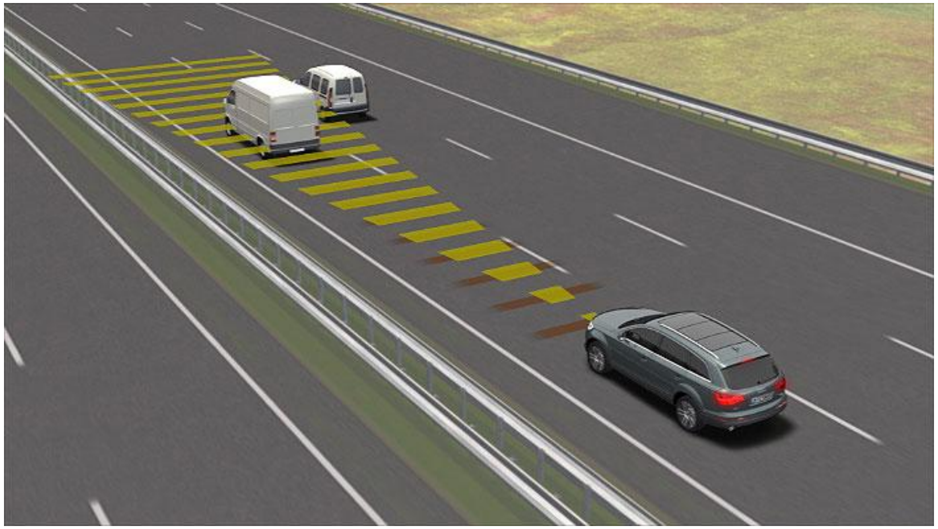


Outline of Lecture 3

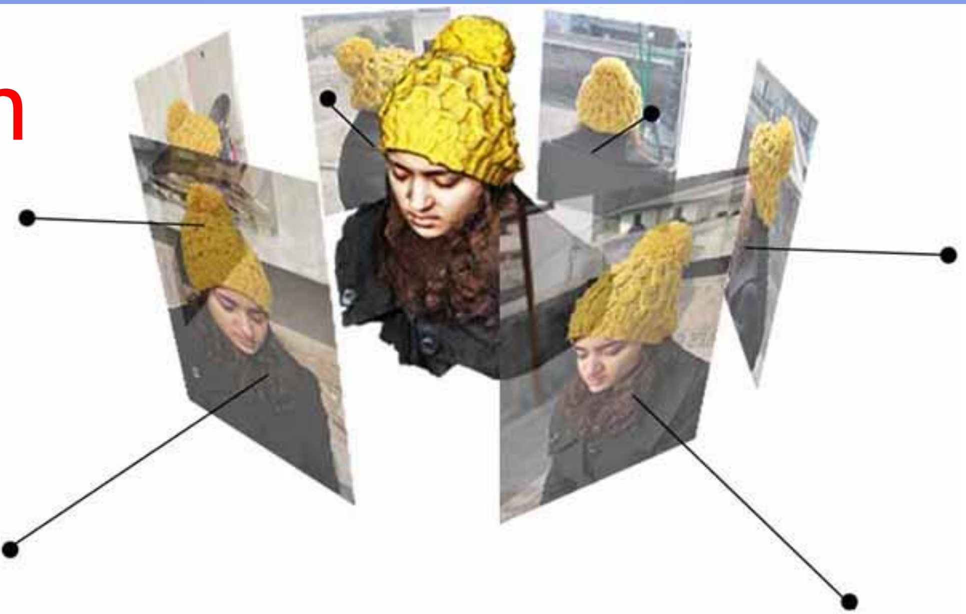
- Basics of 3D Geometry
- Parameters of 3D Geometry
- Measurement of 3D Geometry

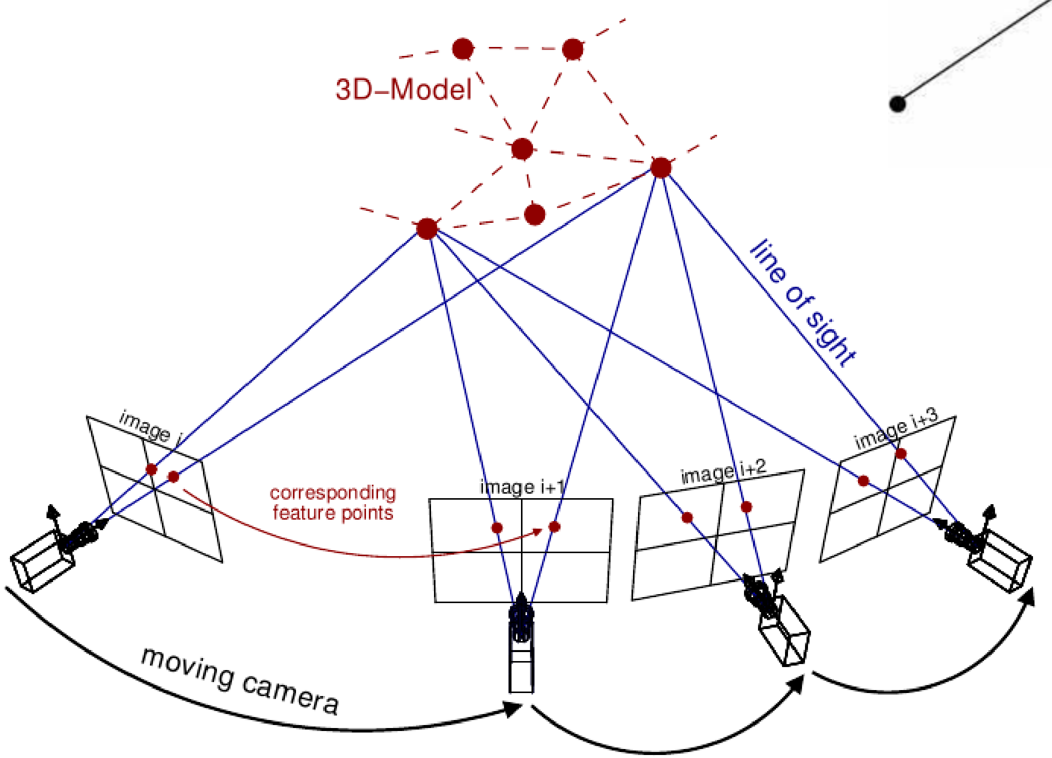
Applications

- Visual Inspection
- Guidance
- Grasping / Manipulation
- Reverse Engineering
- Object Cognition / Recognition
- Scene Cognition / Recognition



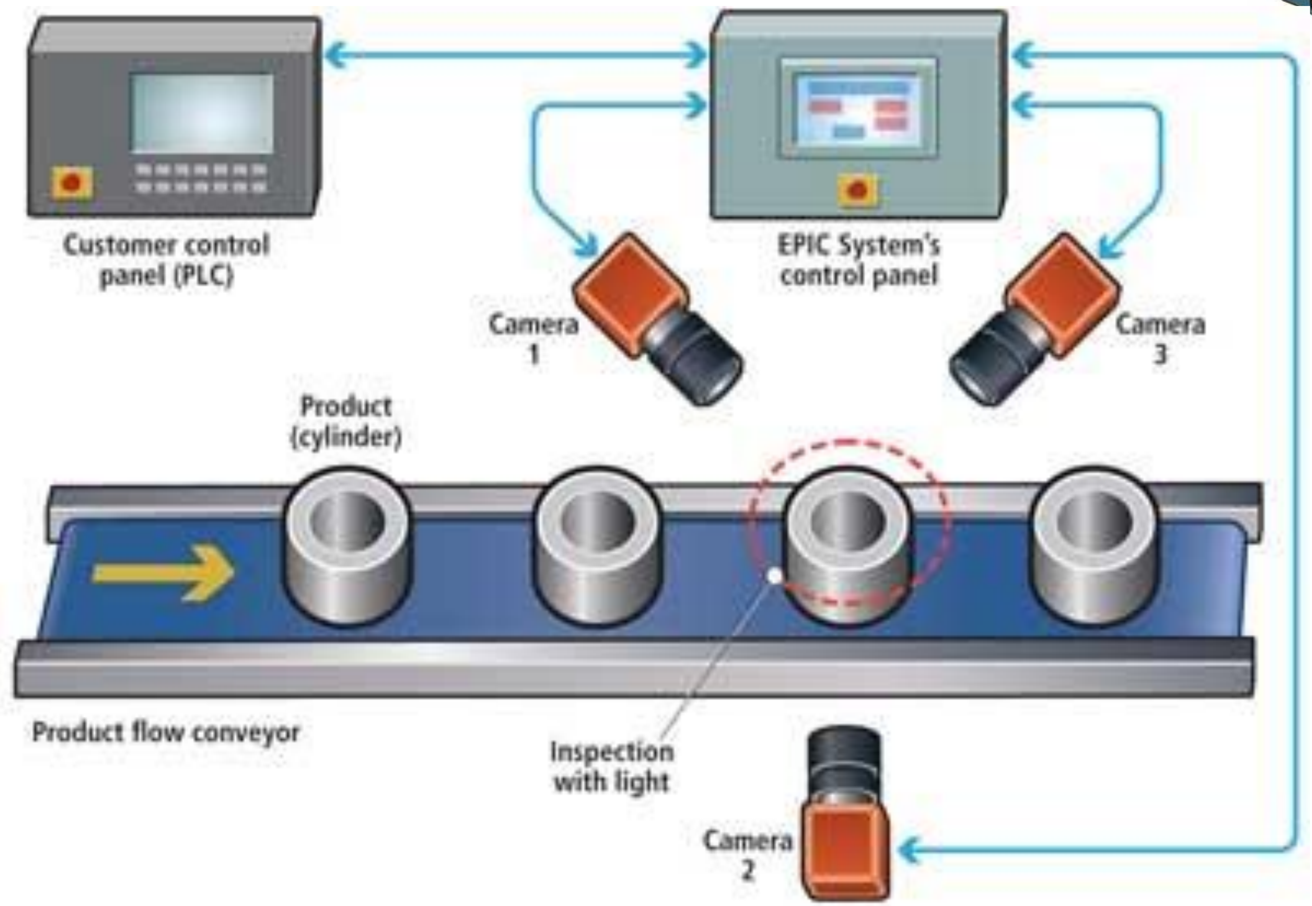
3D Model Acquisition



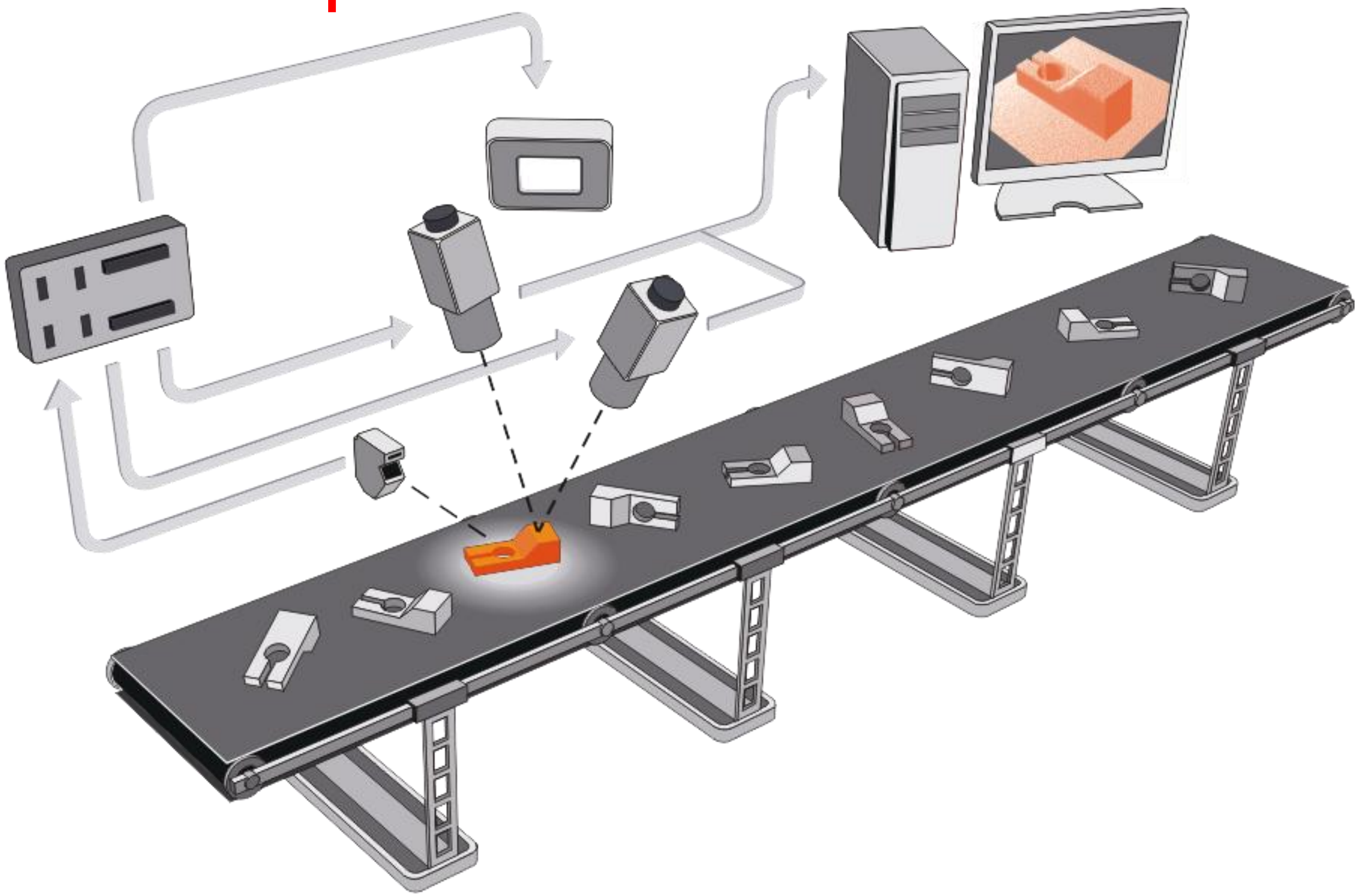


You could set up binocular vision system at home with two laptops

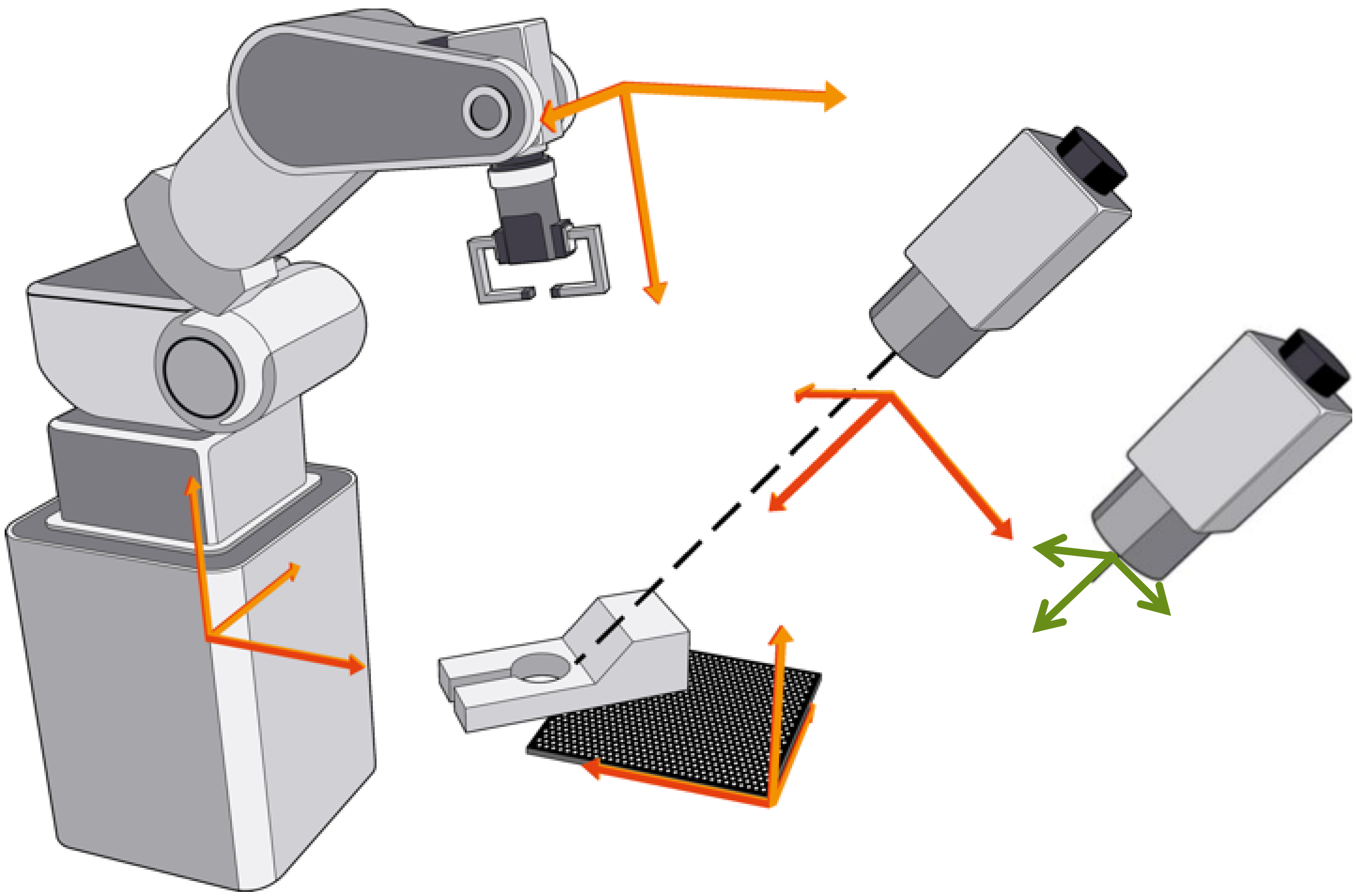
Visual Inspection



Visual Inspection

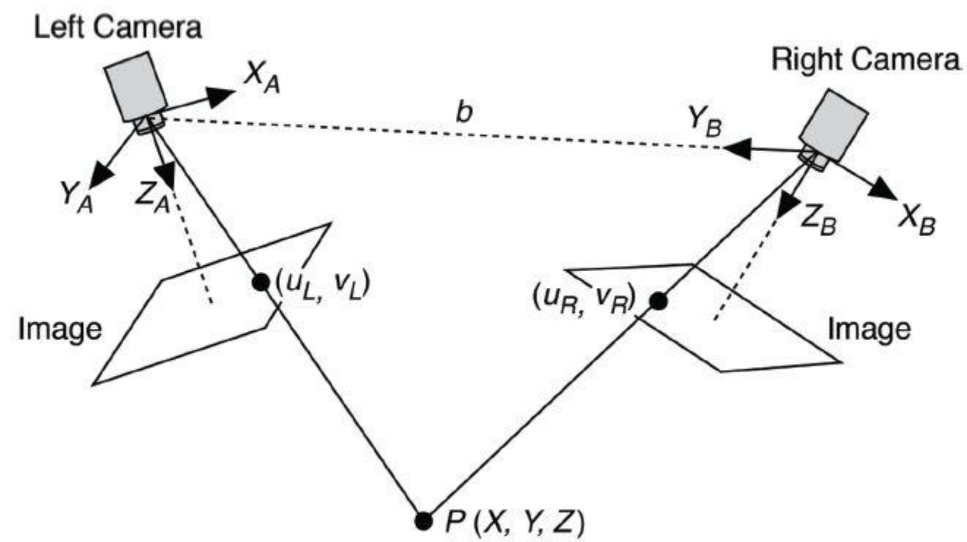


Visual Guidance



Principle of Using Two Cameras

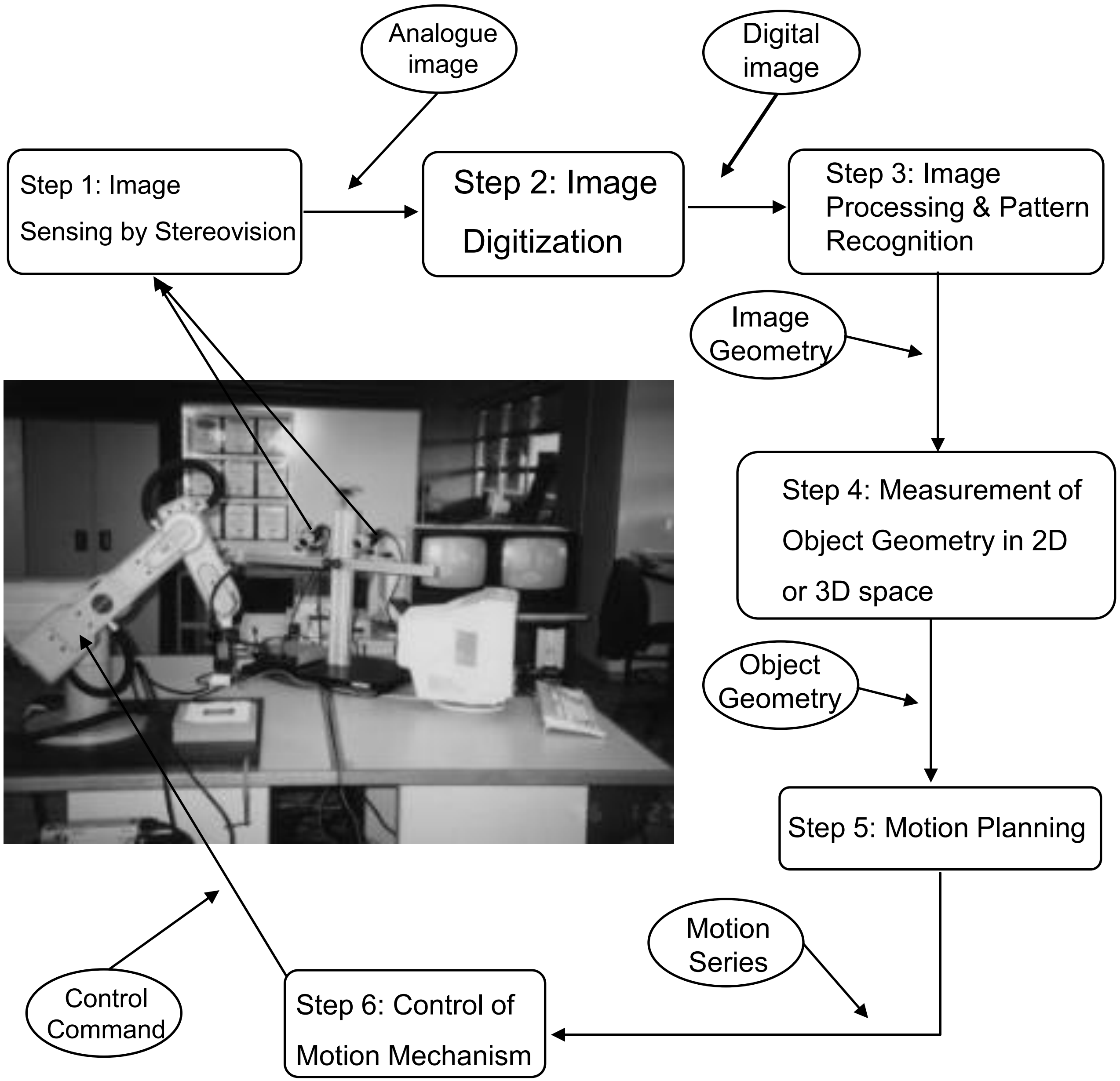
- The geometry of physical entities in a 3D space depends on the coordinates of points.
- The measurement of coordinates of points in a 3D space can be achieved with stereovision or binocular vision system.
- A binocular vision can see a point P and will output two image points.
- The coordinates of the two images allow a machine to uniquely determine the coordinates of point P.



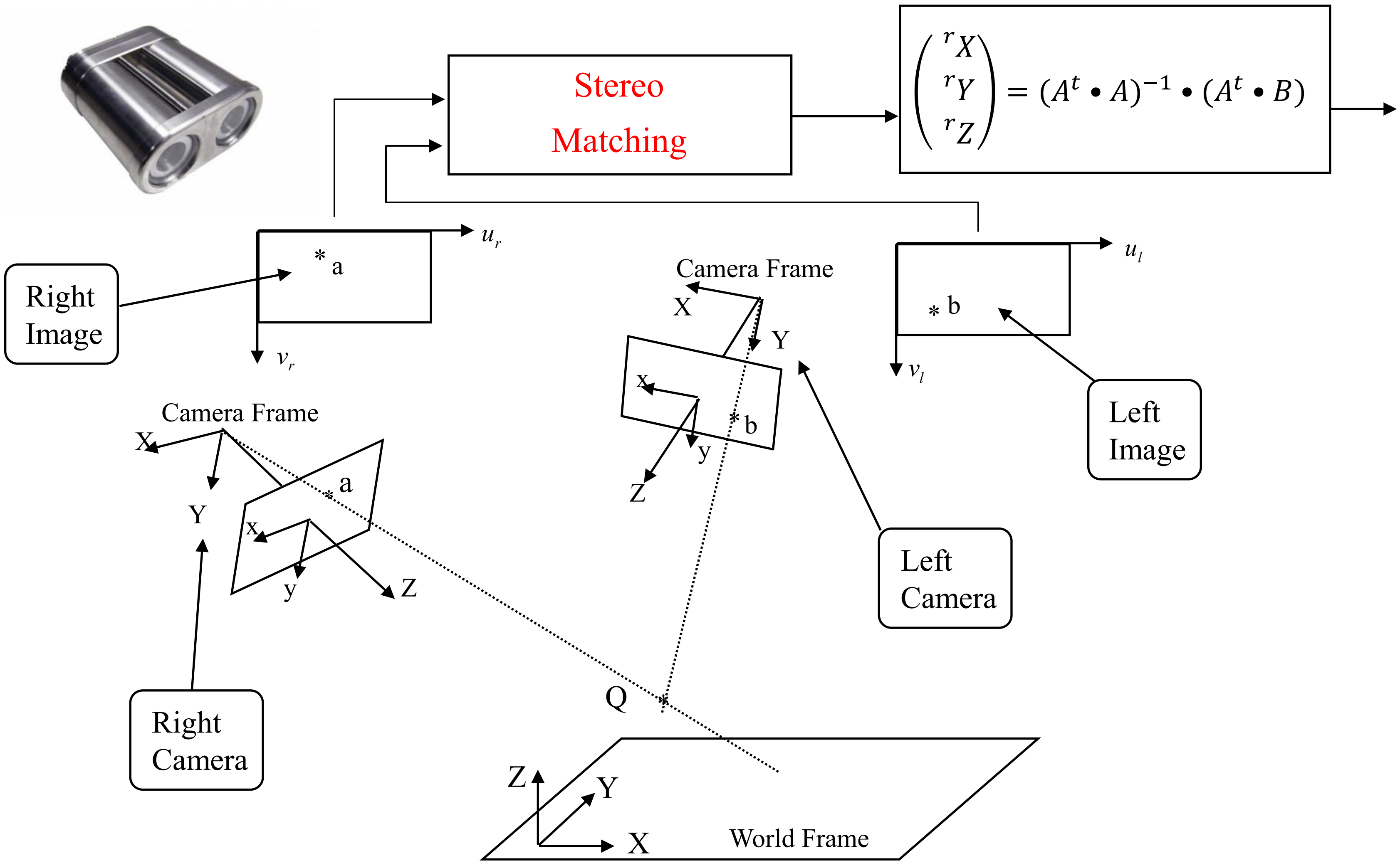
$$[u_l \quad v_l \quad 1]\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0$$

Motion Flow in Binocular Vision …

What is a motion flow? It refers to a flow of coordinate systems



Equation of Binocular Vision

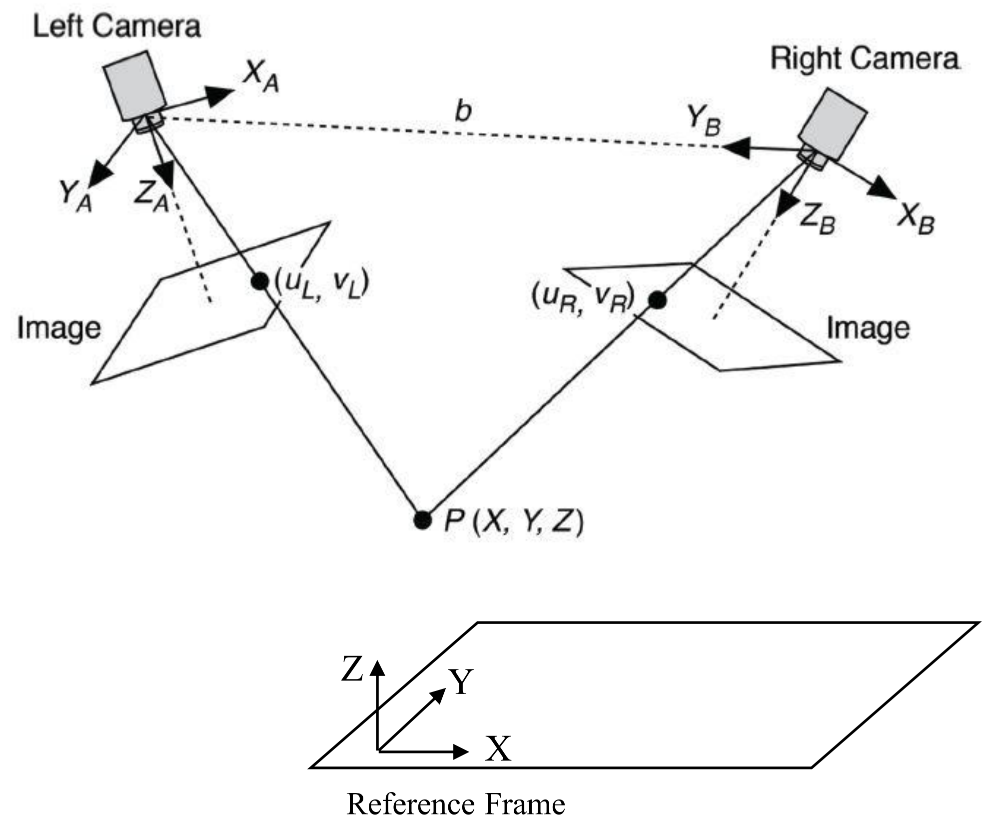


Proof Step 1:

- The equation of left camera's forward projection could be written into a system of equations:

$$\begin{pmatrix} s_l \bullet u_l \\ s_l \bullet v_l \\ s_l \end{pmatrix} = \begin{bmatrix} c_{l1} & c_{l2} & c_{l3} & c_{l4} \\ c_{l5} & c_{l6} & c_{l7} & c_{l8} \\ c_{l9} & c_{l10} & c_{l11} & 1 \end{bmatrix} \bullet \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix}$$

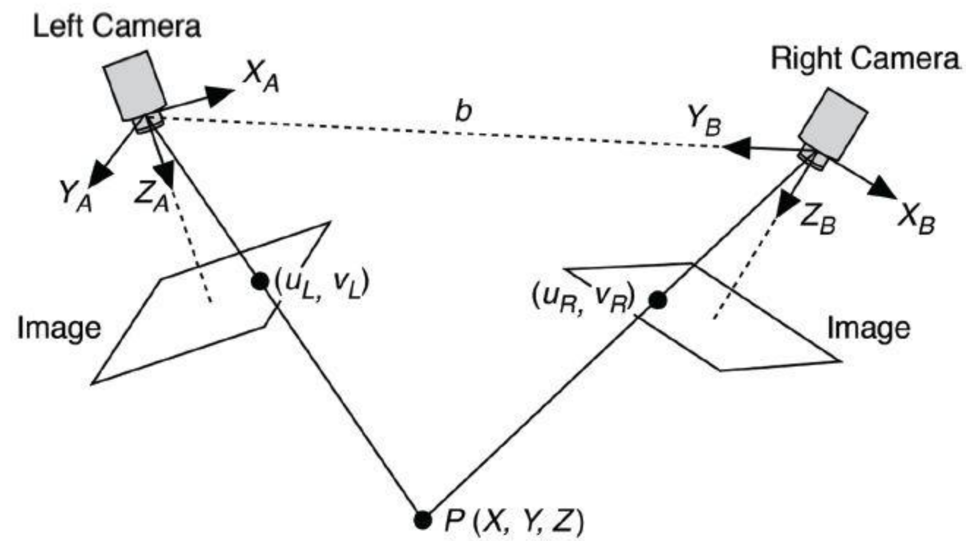
$$\begin{cases} s_l \bullet u_l = c_{l1} \bullet {}^rX + c_{l2} \bullet {}^rY + c_{l3} \bullet {}^rZ + c_{l4} \\ s_l \bullet v_l = c_{l5} \bullet {}^rX + c_{l6} \bullet {}^rY + c_{l7} \bullet {}^rZ + c_{l8} \\ s_l = c_{l9} \bullet {}^rX + c_{l10} \bullet {}^rY + c_{l11} \bullet {}^rZ + 1 \end{cases}$$



$$\begin{cases} (c_{l1} - c_{l9} \bullet u_l) \bullet {}^rX + (c_{l2} - c_{l10} \bullet u_l) \bullet {}^rY + (c_{l3} - c_{l11} \bullet u_l) \bullet {}^rZ + c_{l4} = u_l \\ (c_{l5} - c_{l9} \bullet v_l) \bullet {}^rX + (c_{l6} - c_{l10} \bullet v_l) \bullet {}^rY + (c_{l7} - c_{l11} \bullet v_l) \bullet {}^rZ + c_{l8} = v_l \end{cases}$$

Proof Step 2:

- Also, the equation of right camera's forward projection could be written into a system of equations:



$$\begin{pmatrix} s_r \bullet u_r \\ s_r \bullet v_r \\ s_r \end{pmatrix} = \begin{bmatrix} c_{r1} & c_{r2} & c_{r3} & c_{r4} \\ c_{r5} & c_{r6} & c_{r7} & c_{r8} \\ c_{r9} & c_{r10} & c_{r11} & 1 \end{bmatrix} \bullet \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{pmatrix}$$

$$\begin{cases} s_r \bullet u_r = c_{r1} \bullet {}^{r}X + c_{r2} \bullet {}^{r}Y + c_{r3} \bullet {}^{r}Z + c_{r4} \\ s_r \bullet v_r = c_{r5} \bullet {}^{r}X + c_{r6} \bullet {}^{r}Y + c_{r7} \bullet {}^{r}Z + c_{r8} \\ s_r = c_{r9} \bullet {}^{r}X + c_{r10} \bullet {}^{r}Y + c_{r11} \bullet {}^{r}Z + 1 \end{cases}$$

Reference Frame

$$\begin{cases} (c_{r1} - c_{r9} \bullet u_r) \bullet {}^{r}X + (c_{r2} - c_{r10} \bullet u_r) \bullet {}^{r}Y + (c_{r3} - c_{r11} \bullet u_r) \bullet {}^{r}Z + c_{r4} = u_r \\ (c_{r5} - c_{r9} \bullet v_r) \bullet {}^{r}X + (c_{r6} - c_{r10} \bullet v_r) \bullet {}^{r}Y + (c_{r7} - c_{r11} \bullet v_r) \bullet {}^{r}Z + c_{r8} = v_r \end{cases}$$

Proof Step 3:

- The equations from both cameras could be combined into the following system of equations:

$$\begin{cases} (c_{l1} - c_{l9} \bullet u_l) \bullet {}^rX + (c_{l2} - c_{l10} \bullet u_l) \bullet {}^rY + (c_{l3} - c_{l11} \bullet u_l) \bullet {}^rZ + c_{l4} = u_l \\ (c_{l5} - c_{l9} \bullet v_l) \bullet {}^rX + (c_{l6} - c_{l10} \bullet v_l) \bullet {}^rY + (c_{l7} - c_{l11} \bullet v_l) \bullet {}^rZ + c_{l8} = v_l \\ (c_{r1} - c_{r9} \bullet u_r) \bullet {}^rX + (c_{r2} - c_{r10} \bullet u_r) \bullet {}^rY + (c_{r3} - c_{r11} \bullet u_r) \bullet {}^rZ + c_{r4} = u_r \\ (c_{r5} - c_{r9} \bullet v_r) \bullet {}^rX + (c_{r6} - c_{r10} \bullet v_r) \bullet {}^rY + (c_{r7} - c_{r11} \bullet v_r) \bullet {}^rZ + c_{r8} = v_r \end{cases}$$

We define:

$$A = \begin{pmatrix} (c_{l1} - c_{l9} \bullet u_l) & (c_{l2} - c_{l10} \bullet u_l) & (c_{l3} - c_{l11} \bullet u_l) \\ (c_{l5} - c_{l9} \bullet v_l) & (c_{l6} - c_{l10} \bullet v_l) & (c_{l7} - c_{l11} \bullet v_l) \\ (c_{r1} - c_{r9} \bullet u_r) & (c_{r2} - c_{r10} \bullet u_r) & (c_{r3} - c_{r11} \bullet u_r) \\ (c_{r5} - c_{r9} \bullet v_r) & (c_{r6} - c_{r10} \bullet v_r) & (c_{r7} - c_{r11} \bullet v_r) \end{pmatrix} \qquad B = \begin{pmatrix} u_l - c_{l4} \\ v_l - c_{l8} \\ u_r - c_{r4} \\ v_r - c_{r8} \end{pmatrix}$$

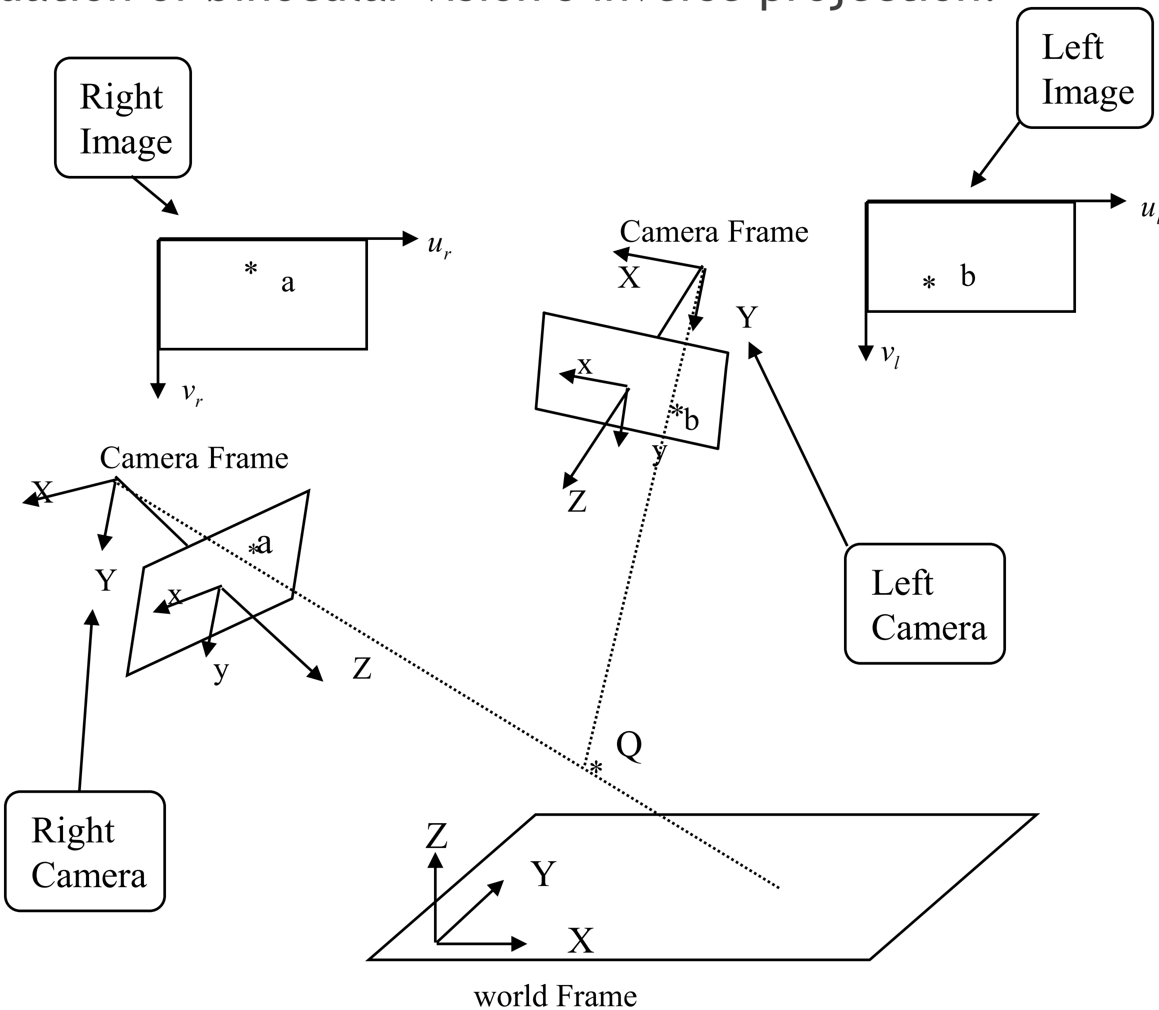
Proof Step 4:

- The previous system of equations could be written into a matrix form. The pseudo-inverse yields the equation of binocular vision's inverse projection:

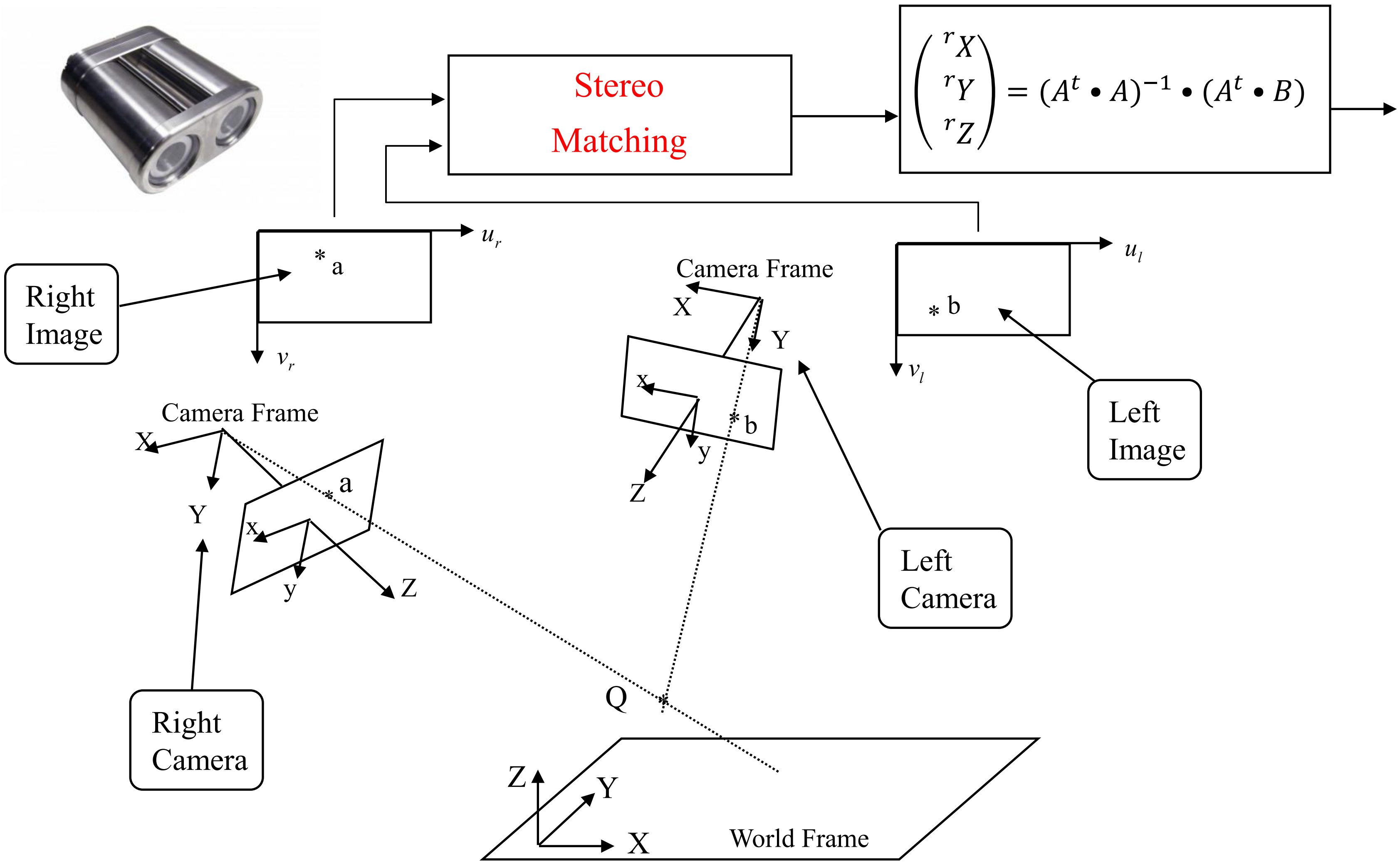
$$A \bullet \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \end{pmatrix} = B$$

$$(A^t \bullet A) \bullet \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \end{pmatrix} = A^t \bullet B$$

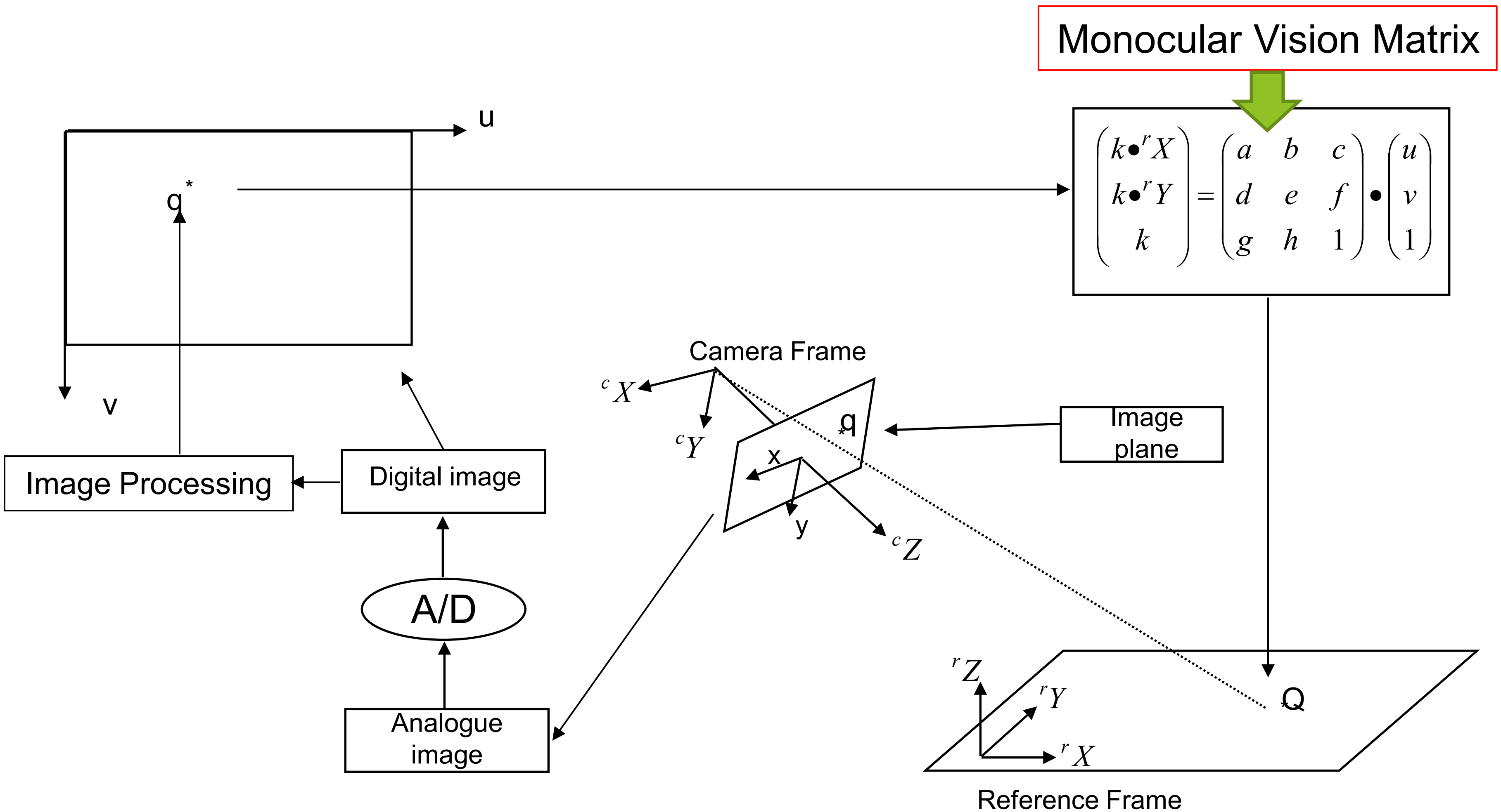
$$\begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \end{pmatrix} = (A^t \bullet A)^{-1} \bullet (A^t \bullet B)$$



Summary of Binocular Vision Solution

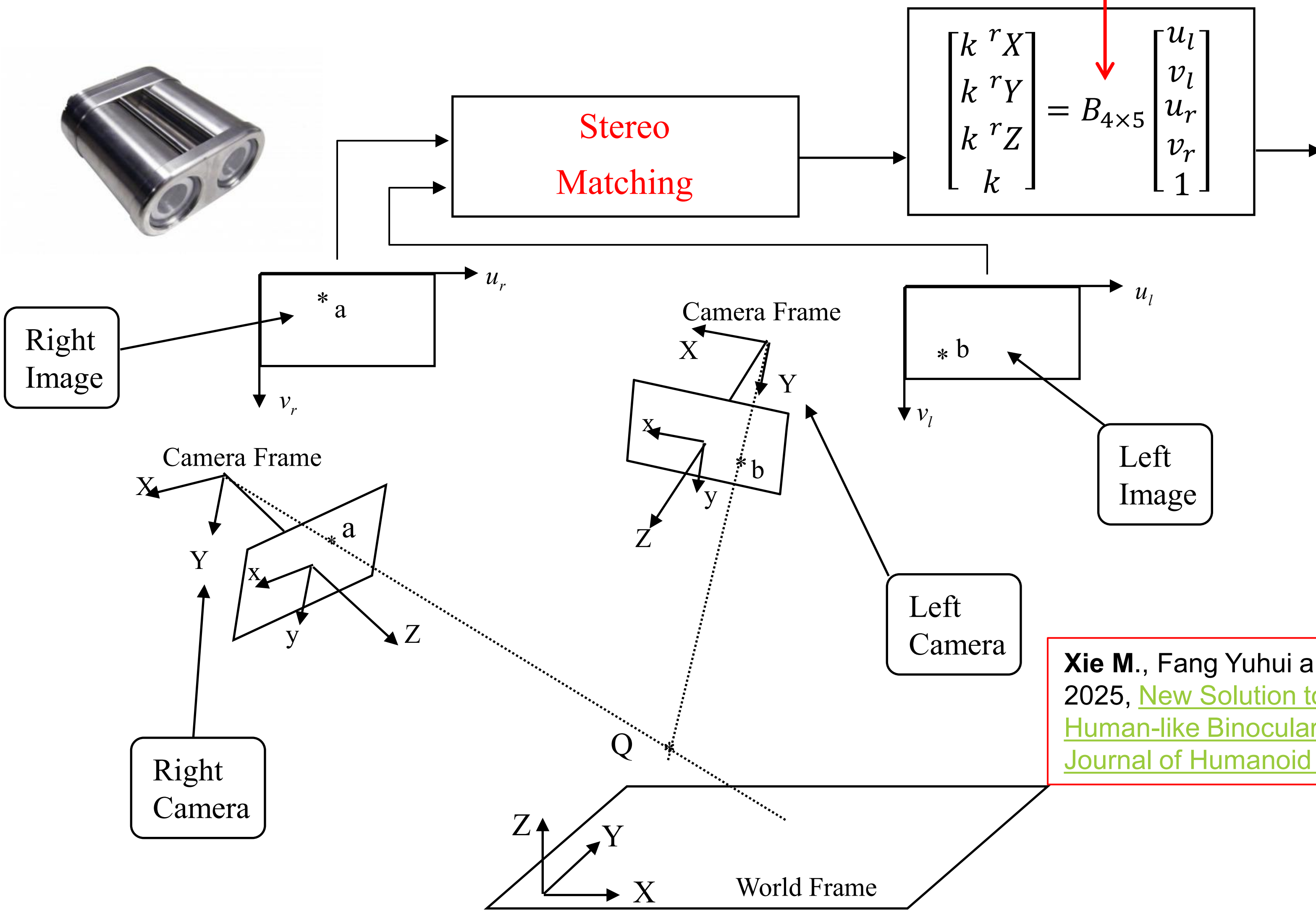


Discussion: Could we obtain a binocular vision matrix which is similar to a monocular vision matrix?



$$\begin{pmatrix} k\bullet {}^rX \\ k\bullet {}^rY \\ k \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{pmatrix} \bullet \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

Yes, binocular vision matrix is:



$$\begin{bmatrix} k\ {}^rX \\ k\ {}^rY \\ k\ {}^rZ \\ k \end{bmatrix} = B_{4\times5} \begin{bmatrix} u_l \\ v_l \\ u_r \\ v_r \\ 1 \end{bmatrix}$$

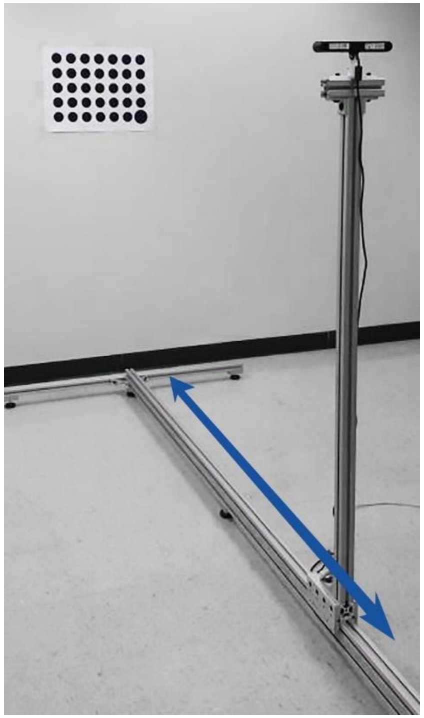
Xie M., Fang Yuhui and Lai Tingfeng, 2025, New Solution to 3D Projection in Human-like Binocular Vision, International Journal of Humanoid Robotics.

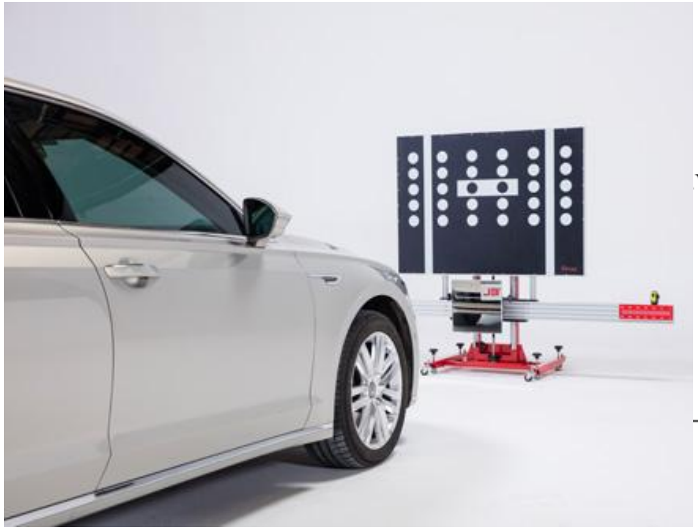
How to calibrate a human-like binocular vision?

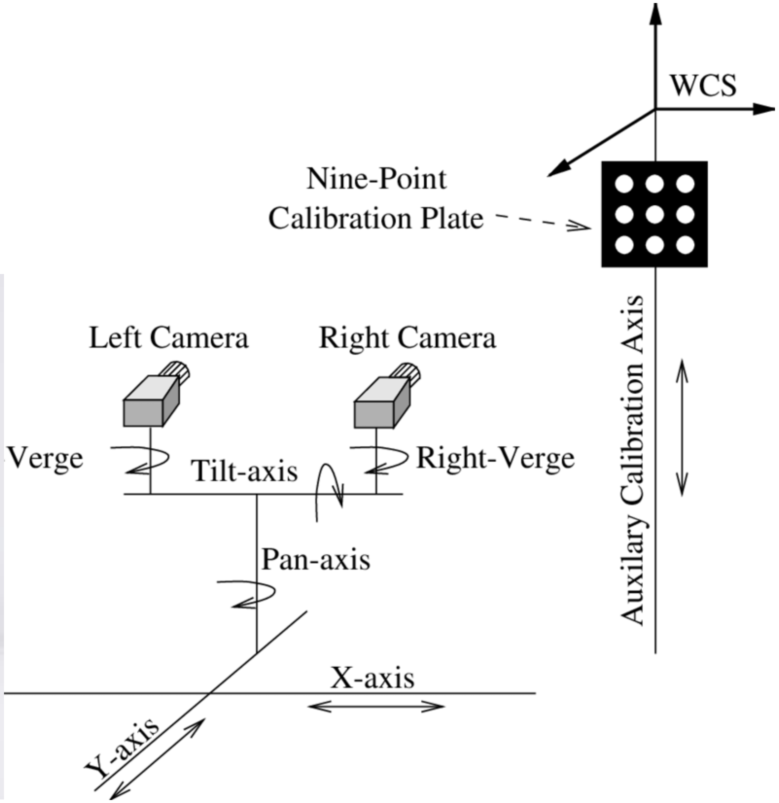
- The Details of Binocular Vision Calibration:
  - There are 19 coefficients inside B matrix.
  - One pair of data provides 3 equations.
  - Seven (7) pairs of data are sufficient enough.

One pair of {(u,v), (X,Y,Z)} provides three equations for calibrating binocular vision.

$$\begin{bmatrix} k\ {}^{r}X \\ k\ {}^{r}Y \\ k\ {}^{r}Z \\ k \end{bmatrix} = B_{4\times5} \begin{bmatrix} u_l \\ v_l \\ u_r \\ v_r \\ 1 \end{bmatrix}$$

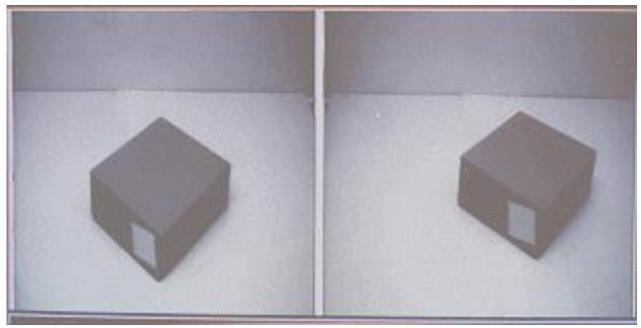




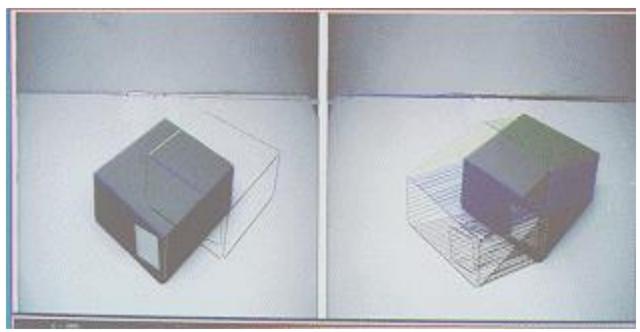


Example of Results

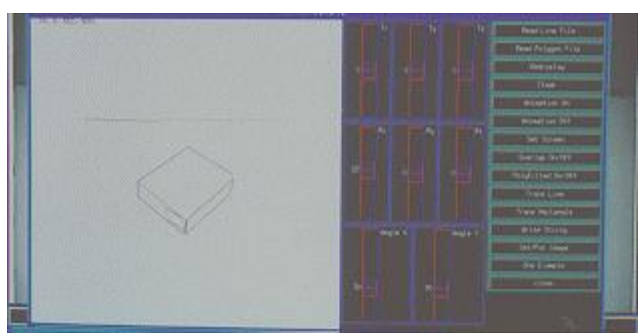
One pair of stereo images



Result of stereo matching

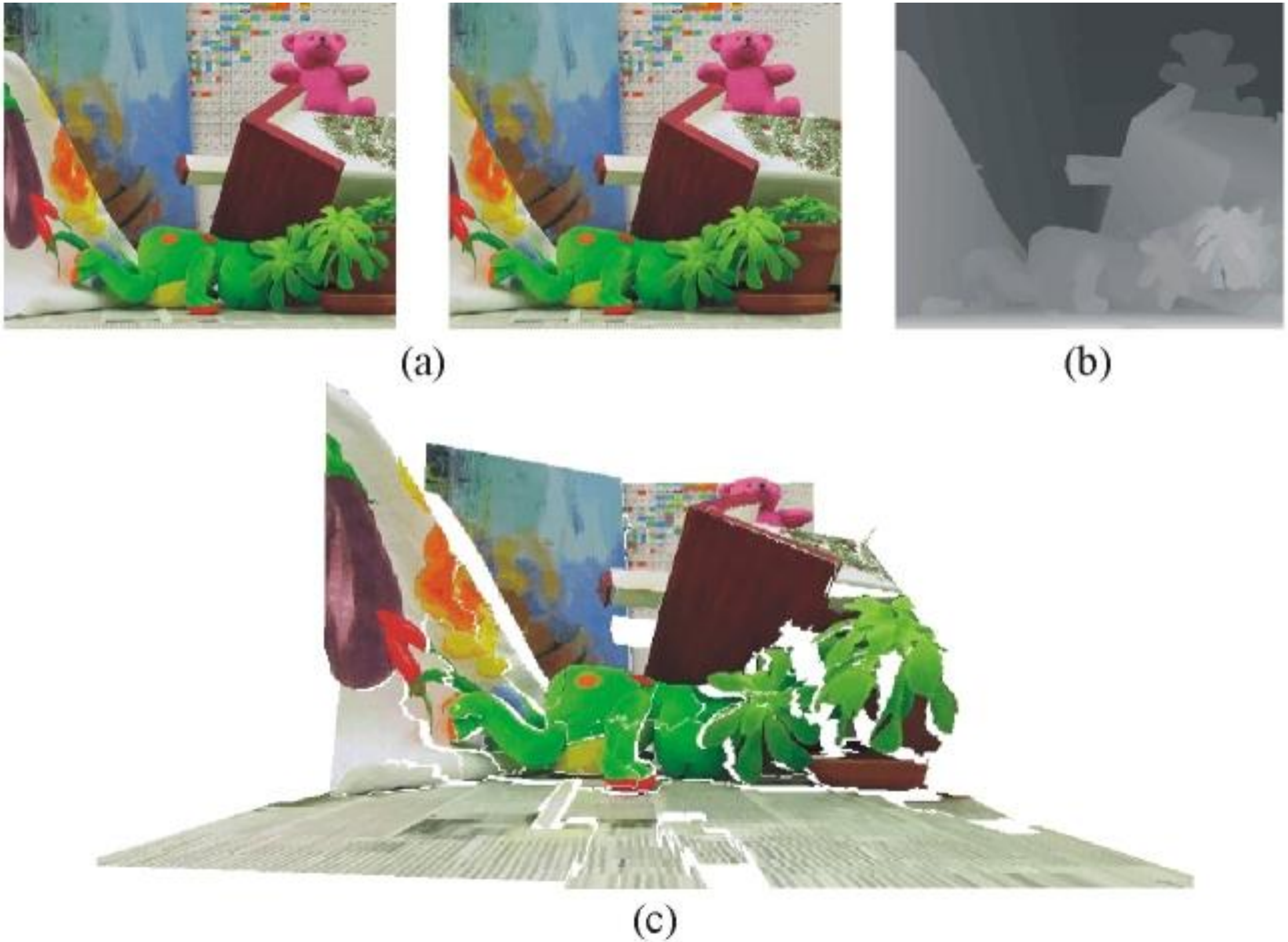


3D view of reconstructed line segments

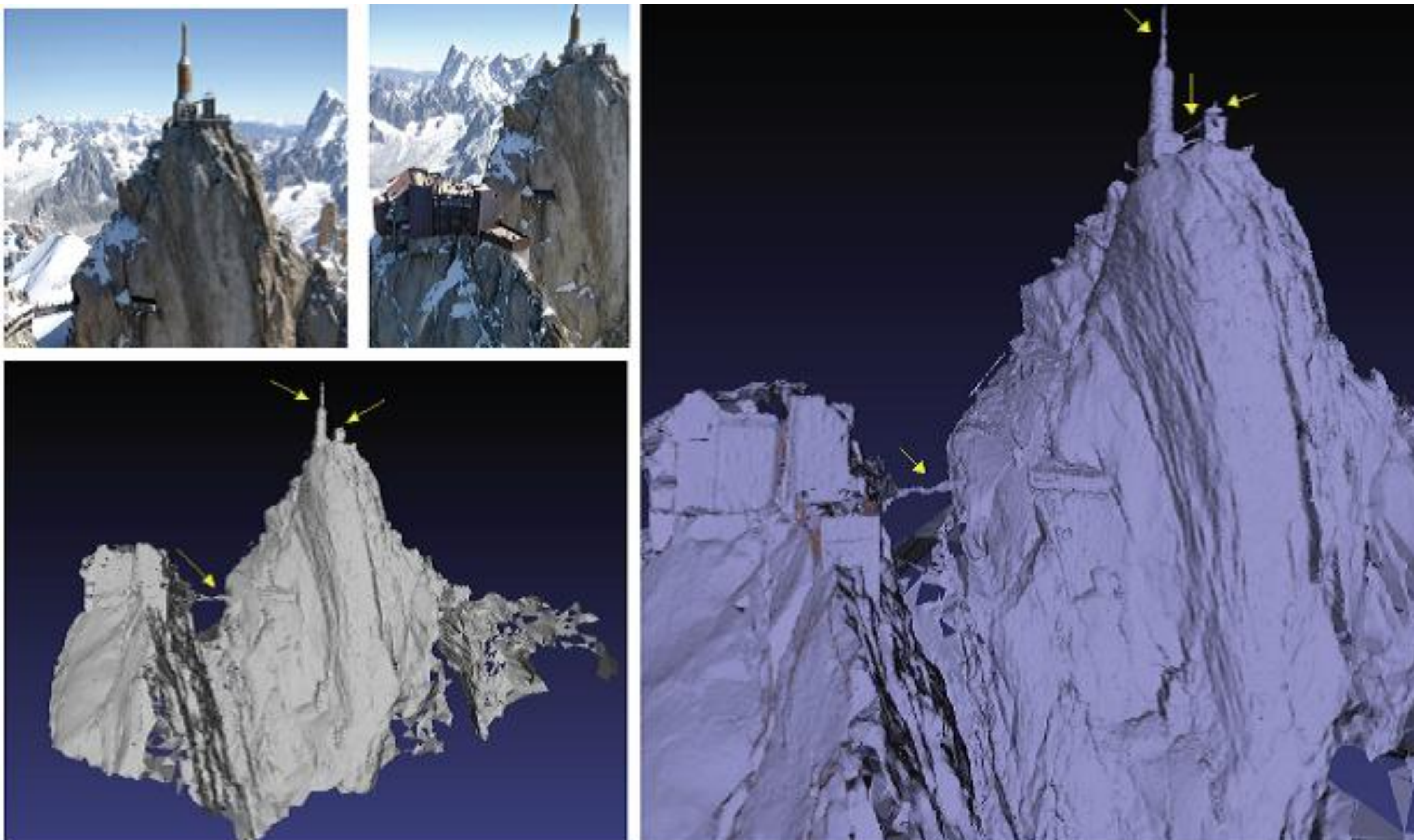


Example of Results

Example of Results

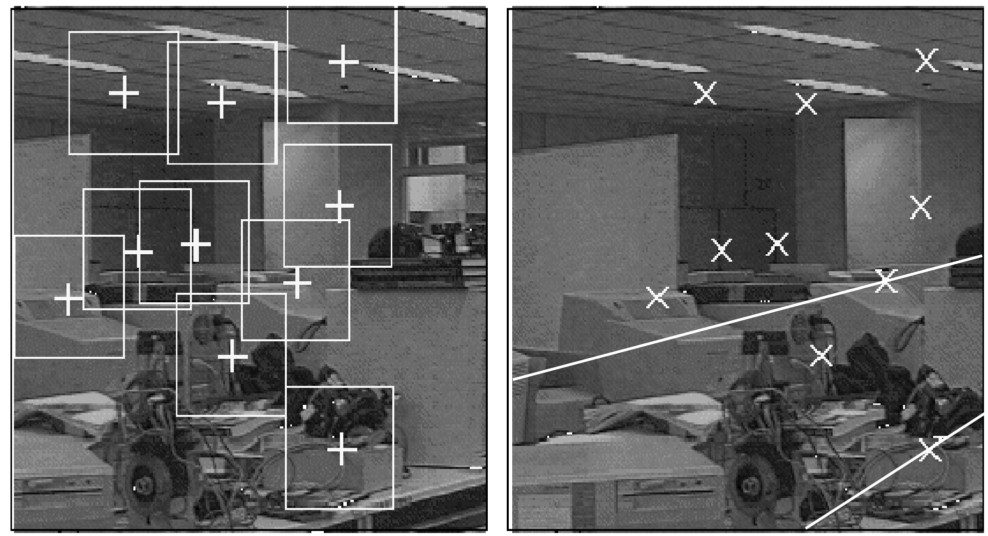


Example of Results

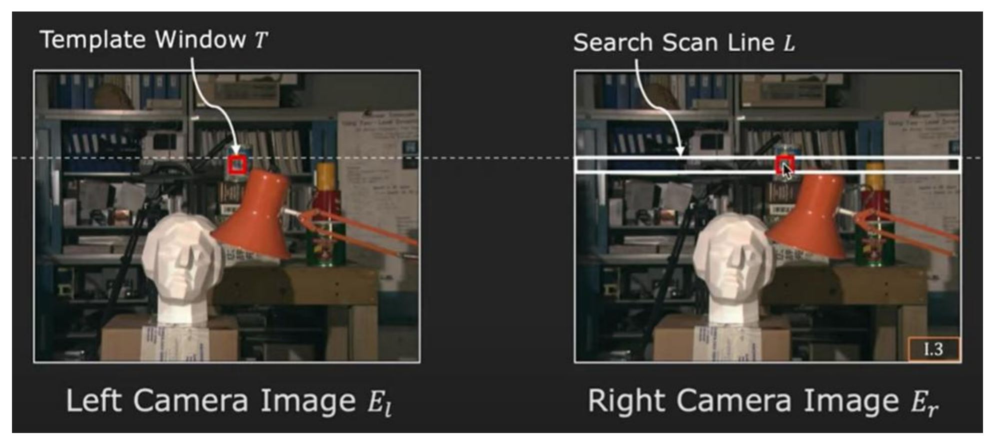


Open Issue of Stereo Matching

- How to find the match inside the right camera, given a pixel inside the left camera?

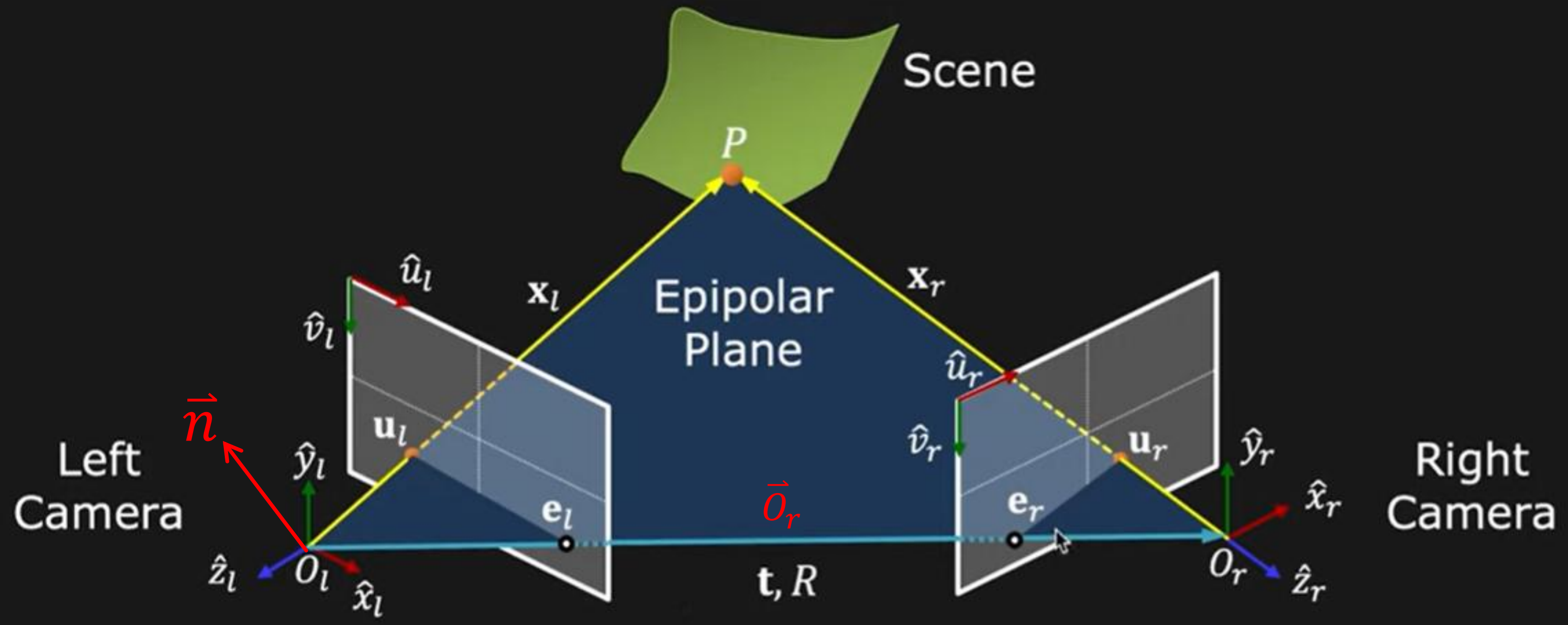


- Definition of Epipolar Line: Given a point in the left image, there is a straight line in the right image, which contains all the possible points to be matched with the given point in the left image. Such a line is called as an Epipolar line.



About Epipolar Plane and Epipoles …

Epipolar Geometry: Epipolar Plane

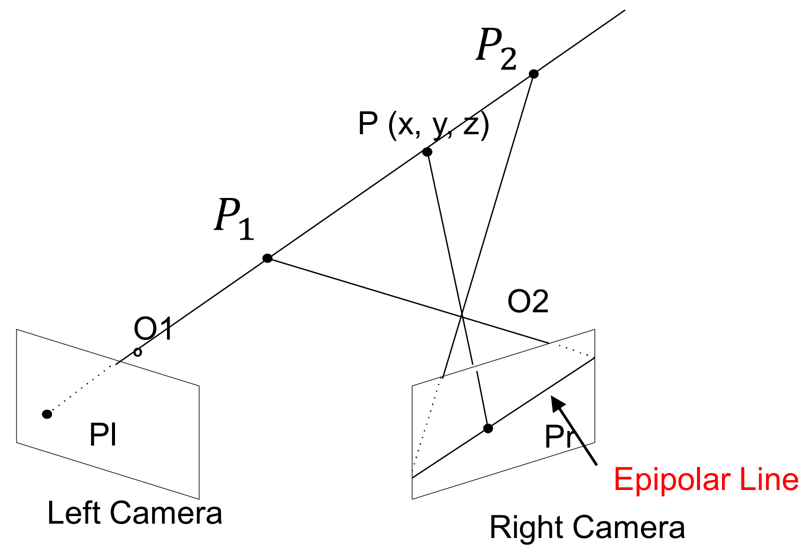


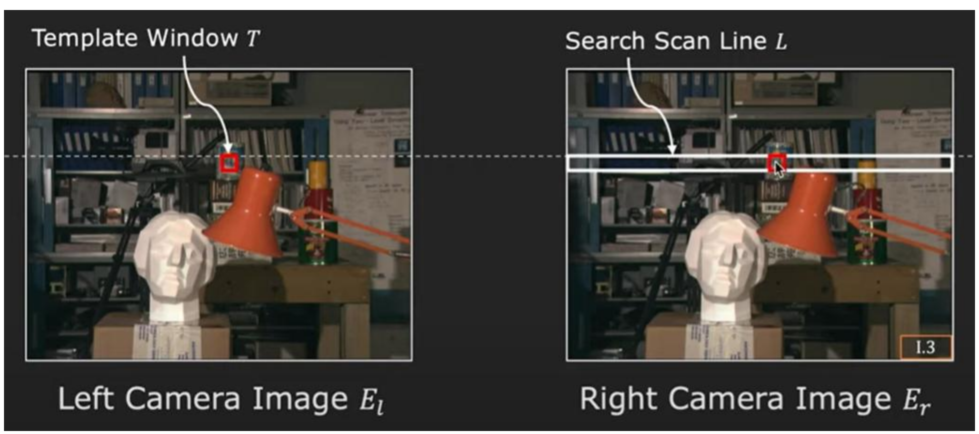
Epipolar Plane of Scene Point P: The plane formed by camera origins (O_l and O_r), epipoles ($\mathbf{e}_l$ and $\mathbf{e}_r$) and scene point P.

More About Equation of Epipolar Line:

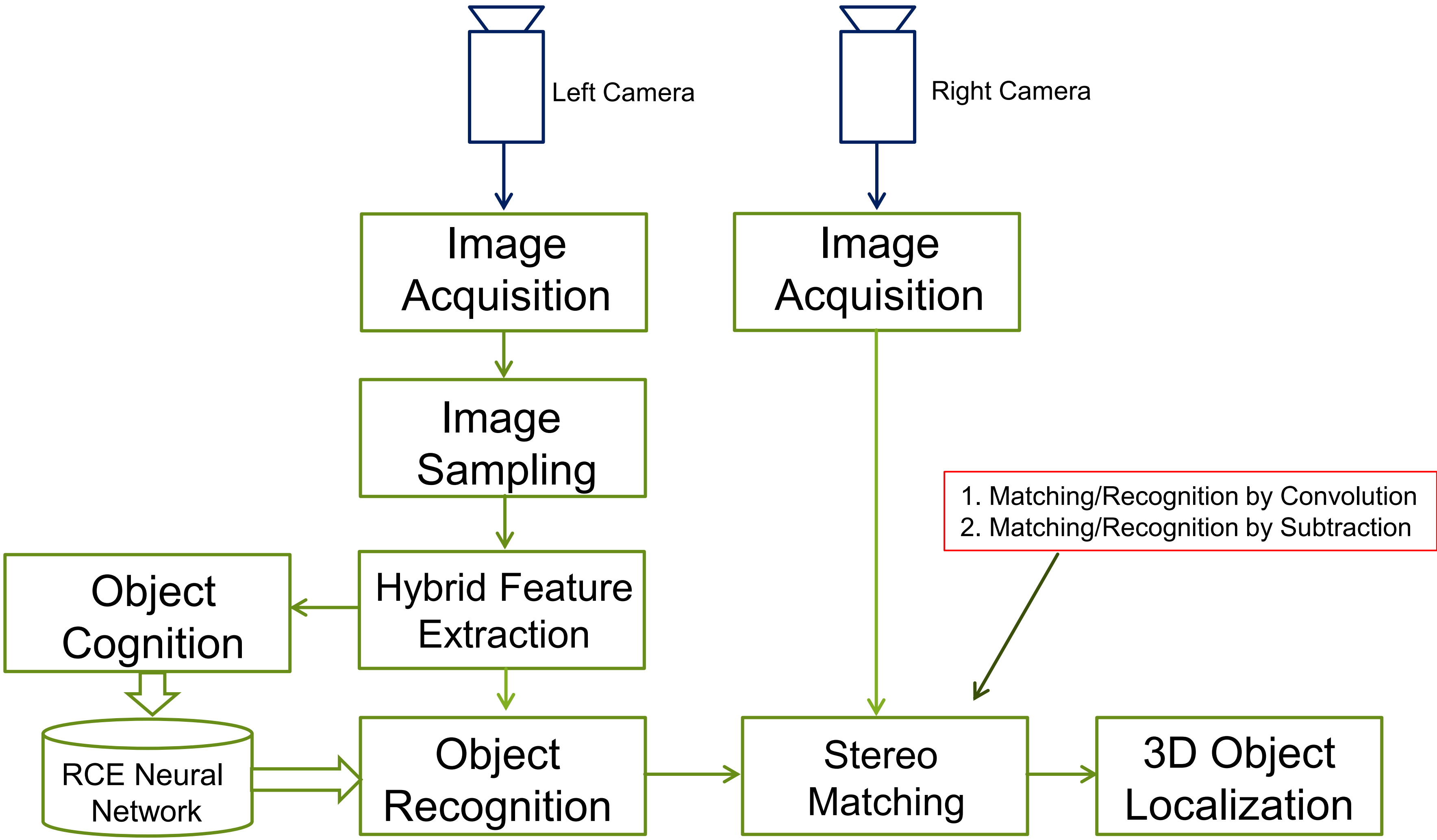
- Choose any pair of two possible value of Z coordinate: (Z1, Z2), which correspond to two 3D points (P1, P2).
- Compute the 3D coordinates of (P1, P2) by using the knowledge about the image coordinates in left camera.
- Project these two locations (P1, P2) onto the image plane of right camera.
- Compute the equation of epipolar line on image plane of right camera.

$$[u_l \quad v_l \quad 1]\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0$$





Future Research:
Solution Toward Human-like Binocular Vision

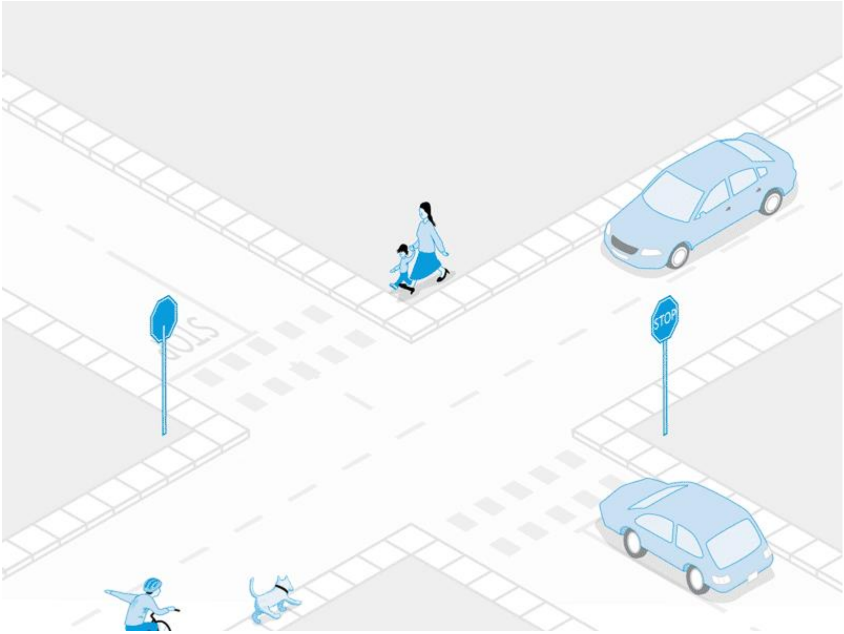


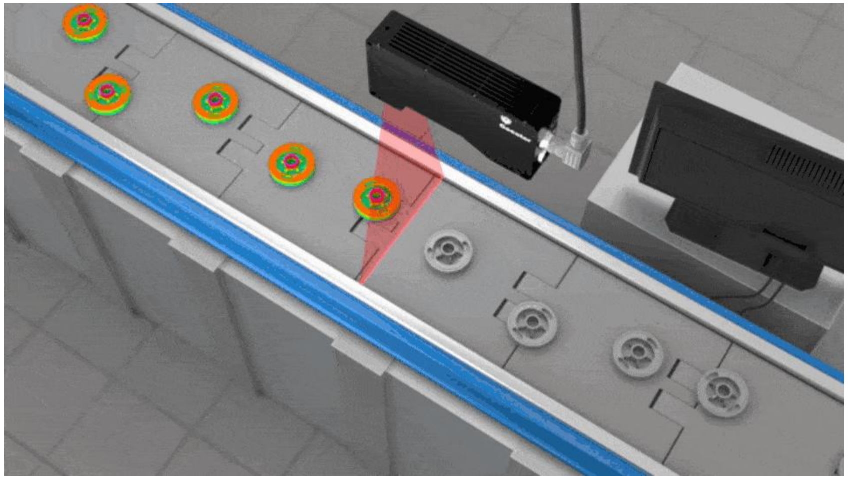
Summary of Lecture 3

- Basics of 3D Geometry
- Parameters of 3D Geometry
- Measurement of 3D Geometry

Summary of Module 2

- Perception of Photometry
- Perception of 2D Geometry
- Perception of 3D Geometry





"Ask not what your country can do for you – ask what you can do for your country," - John F. Kennedy

"Do not think that you are needy – think that you are needed in the world", - Manis Friedman

"Study will make you knowledgeable, resourceful, and hence more needed", - Xie Ming

Thank You for Listening!